PRAYAS

JEE 2025

Lecture- 01 | Mathematics

Limit, Continuity and Differentiability



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Topics *to be covered*

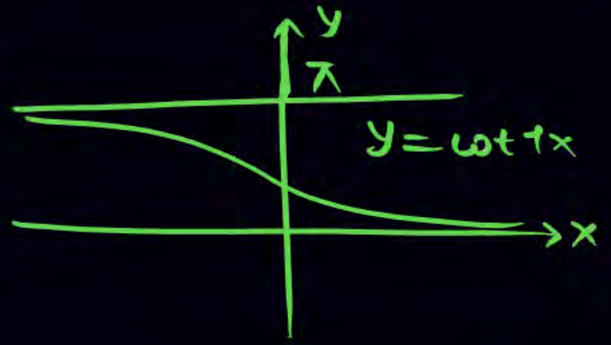
1. Two More Properties of ITF
2. Introduction to Limits

Homework Discussion

Home Challenge - 04

(KCLS) (ASRQ)

Let $f : R \to \left(0, \frac{3\pi}{4}\right]$ be defined as $f(x) = \cot^{-1}(x^2 + x + a)$. If $f(x)$ is surjective, then find the range of a.



f is onto $\Rightarrow$ Range of f = codomain of $f = (0, \frac{3\pi}{4}]$

$f(x) = \cot^{-1}\left((x + 1/2)^2 + a - 1/4\right)$

$(-\infty, \infty) = (-\infty, 0] \cup [0, \infty)$

$[0, \infty)$

$[a - 1/4, \infty)$

$(\cot^{-1}\infty, \cot^{-1}(a - 1/4)]$

$=$

Range $(0, \cot^{-1}(a - 1/4)] = (0, 3\pi/4]$

$$\cot^{-1}\left(a-\frac{1}{4}\right)=\frac{3\pi}{4}$$

$$a-\frac{1}{4}=\cot 3\pi/4=-1$$

$$a=-1+1/4$$

$$a=-3/4$$

QUESTION [JEE Advance 2022]

$\frac{\pi}{1.414} = \frac{3.14}{1.414} > 1$

Considering only the principal values of the inverse trigonometric functions, the value of $\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}} + \frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2} + \tan^{-1}\frac{\sqrt{2}}{\pi}$ **is :**

$\frac{\pi}{\sqrt{2}} > 1$

$\pi/\sqrt{2} > \tan\pi/4$

$\tan\theta > \tan\frac{\pi}{4}$

$\theta \in (\pi/4, \pi/2)$

$$E = \frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}} + \frac{1}{4}\sin^{-1}\frac{2\cdot\pi/\sqrt{2}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2} + \tan^{-1}\frac{1}{\frac{\pi}{\sqrt{2}}}$$

let $\frac{\pi}{\sqrt{2}} = \tan\theta \quad \theta \in (\pi/4, \pi/2)$

$2\theta \in (\pi/2, \pi)$

$$E = \frac{3}{2}\cos^{-1}(\cos\theta) + \frac{1}{4}\sin^{-1}(\sin 2\theta) + \tan^{-1}(\cot\theta)$$

$$= \frac{3}{2}\theta + \frac{1}{4}(\pi - 2\theta) + \pi/2 - \cot^{-1}(\cot\theta)$$

$$= 3\pi/4 - \theta/2 + \pi/2 - \theta = 3\pi/4$$



QUESTION

TAH 4a LEC 06 ITF

Maximum & Minimum value of

(i) $\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3$

(ii) $\left(\sec^{-1}x\right)^2+\left(\operatorname{cosec}^{-1}x\right)^2$

Tah-4a

$y = (\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2$

$y = (\sec^{-1}x + \operatorname{cosec}^{-1}x)^2 - 2\sec^{-1}x \operatorname{cosec}^{-1}x$

$y = \frac{\pi^2}{4} - 2\sec^{-1}x\left(\frac{\pi}{2} - \sec^{-1}x\right)$

$y = \frac{\pi^2}{4} + 2\left[(\sec^{-1}x)^2 - \frac{\pi}{2}\sec^{-1}x\right]$

$y = \frac{\pi^2}{4} + 2\left[(\sec^{-1}x)^2 - \frac{\pi}{2}\sec^{-1}x + \left(\frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right]$

$y = \frac{\pi^2}{4} + 2\left[\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right]$

$y = \frac{\pi^2}{4} - \frac{\pi^2}{8} + 2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2$

$y = \frac{\pi^2}{8} + 2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2$

$\sec^{-1}x \in [0,\pi] - \{\frac{\pi}{2}\}$

$\sec^{-1}x - \frac{\pi}{4} \in [-\frac{\pi}{4}, \frac{3\pi}{4}] - \{\frac{\pi}{4}\}$ ← $-\pi/4$

$\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 \in [0, \frac{9\pi^2}{16}] - \{\frac{\pi^2}{16}\}$ ← $(\ )^2$

$2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 \in [0, \frac{9\pi^2}{8}] - \{\frac{\pi^2}{8}\}$ ← $2(\ )$

$\frac{\pi^2}{8} + 2\left(\sec^{-1}x - \frac{\pi}{4}\right)^2 \in [\frac{\pi^2}{8}, \frac{10\pi^2}{8}] - \{\frac{2\pi^2}{8}\}$ ← $\frac{\pi^2}{8} + (\ )$

$y \in [\frac{\pi^2}{8}, \frac{5\pi^2}{4}] - \{\frac{\pi^2}{4}\}$

$[-\frac{\pi}{4}, 0] \cup [0, \frac{\pi}{4}) \cup (\pi/4, \frac{3\pi}{4}]$

Bhavesh Upadhyay
Delhi

TAH-4a

$(\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2$

$\Rightarrow y = (\sec^{-1}x + \operatorname{cosec}^{-1}x)^2 - 2\sec^{-1}x \operatorname{cosec}^{-1}x$

$= \frac{\pi^2}{4} + 2\left[(\sec^{-1}x)^2 - \frac{\pi}{2}\sec^{-1}x\right]$

$= \frac{\pi^2}{4} + 2\left[(\sec^{-1}x - \frac{\pi}{4})^2\right] - \frac{\pi^2}{8}$

$= \frac{\pi^2}{8} + 2\left[\sec^{-1}x - \frac{\pi}{4}\right]^2$

$\sec^{-1}x \in [0,\pi] - \{\frac{\pi}{2}\}$

$\sec^{-1}x - \frac{\pi}{4} \in [-\frac{\pi}{4}, \frac{3\pi}{4}] - \{\frac{\pi}{4}\} = [-\frac{\pi}{4}, 0) \cup (0, \frac{3\pi}{4}] - \{\frac{\pi}{4}\}$

$\left[\sec^{-1}x - \frac{\pi}{4}\right]^2 \in [0, \frac{9\pi^2}{16}] - \{\frac{\pi^2}{16}\}$

$2\left[\sec^{-1}x - \frac{\pi}{4}\right]^2 \in [0, \frac{9\pi^2}{8}] - \{\frac{\pi^2}{8}\}$

$\frac{\pi^2}{8} + 2\left[\sec^{-1}x - \frac{\pi}{4}\right]^2 \in [\frac{\pi^2}{8}, \frac{10\pi^2}{8}] - \{\frac{\pi^2}{4}\}$

$\therefore y_{max} = \frac{10\pi^2}{8} = \frac{5\pi^2}{4}$

$y_{min} = \frac{\pi^2}{8}$

Paritosh Ruidas
Asansol,
West Bengal

QUESTION

TAH 4 LEC 4 ITF

Find range of

(a) $f(x) = \cot^{-1} \log_{\frac{4}{5}}(5x^2 - 8x + 4)$

(b) $f(x) = \cot^{-1}\left(\log_{5/4}(5x^2 - 8x + 4)\right)$

TAM-4

(9) $f(x) = \cot^{-1} \log_{\frac{4}{5}} (5x^2 - 8x + 4)$

$f(x) = \cot^{-1} \log_{\frac{4}{5}} (5x^2 - 8x + 4)$

$\left[\frac{4}{5}, \infty\right)$

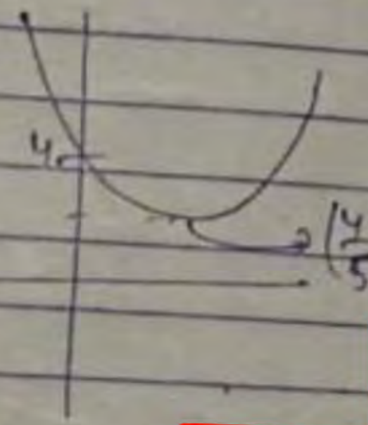
$(-\infty, 1]$

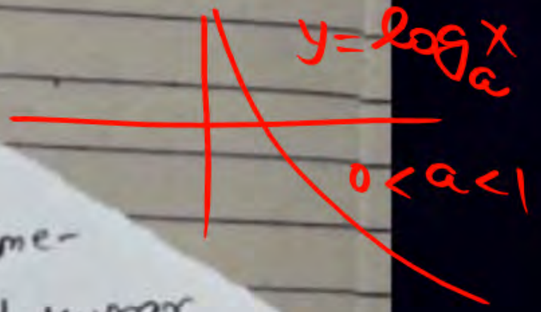
$y \in (0, \cot(-1)]$ Ans:

$y \in [\cot 1, \pi)$

$y \in [\pi/4, \pi)$

$D < 0$





Name-
Ankit Kumar
from - U.P
V.HS

(b) $f(x) = \cot^{-1}\left(\log_{5/4}(5x^2 - 8x + 4)\right)$

$D < 0$

$\left[\frac{4}{5}, \infty\right)$

$[-1, \infty)$

$y \in [0, \pi - \cot^{-1}1]$

$y \in [0, \frac{3\pi}{4}]$ Ans.

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Aao Machay Dhmaal Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2024 (31 Jan)]

For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta$, then γ equals

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$, $\sin^{-1}\gamma = C$

Clearly $A, B, C \in (0, \pi/2]$

$A+B+C = \pi$ they are angles of a triangle.

$\sin A = \alpha$, $\sin B = \beta$, $\sin C = \gamma$

By Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$\frac{a}{\alpha} = \frac{b}{\beta} = \frac{c}{\gamma} = k$

$a = \alpha k$, $b = \beta k$, $c = \gamma k$

$\sin C = \gamma$

$\sqrt{3}/2 = \gamma$

$(\alpha+\beta)^2 - \gamma^2 = 3\alpha\beta$

$\alpha^2+\beta^2+2\alpha\beta-\gamma^2 = 3\alpha\beta$

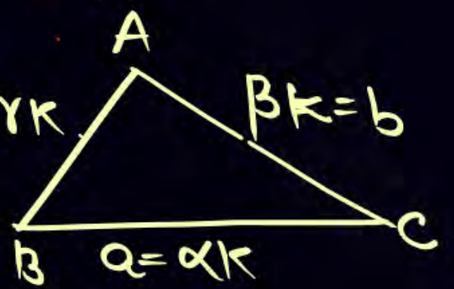
$\alpha^2+\beta^2-\gamma^2 = \alpha\beta$

$\frac{\alpha^2+\beta^2-\gamma^2}{\alpha\beta} = 1$

$\frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta} = \frac{1}{2}$

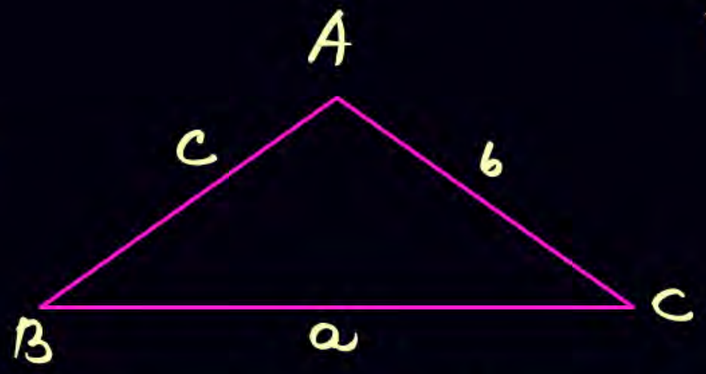
Num & Den multiply by k^2

$\frac{(k\alpha)^2+(k\beta)^2-(k\gamma)^2}{2k\alpha \cdot k\beta} = \frac{1}{2}$



$\frac{a^2+b^2-c^2}{2ab} = 1/2 \Rightarrow \cos C = 1/2 \Rightarrow C = 60^\circ$

Ans. B



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$

Cosine Rule: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$.

$\cos C = \frac{b^2 + a^2 - c^2}{2ab}$.

QUESTION

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) \text{ is equal to}$$

Tan⁻¹

$$T_r = \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1+2^r \cdot 2^{r-1}}\right)$$

observe!!

$$2^r - 2^{r-1} = 2^{r-1}(2-1) = 2^{r-1}$$

$$= \tan^{-1} 2^r - \tan^{-1} 2^{r-1}$$

A $\tan^{-1}(2^n)$

B $\tan^{-1}(2^n) - \frac{\pi}{4}$

C $\tan^{-1}(2^{n+1})$

D $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$

QUESTION [JEE Mains 2019]

Tan2

If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots$ upto 100 terms, then α is

(A) 1.01

(B) 1.00

(C) 1.02

(D) 1.03

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \tan^{-1}\frac{1}{32} + \ldots$ — up 100 terms

2, 8, 18, 32 - - - - -

6 10 14

4 4

$T_r' = ar^2 + br + c$ $\quad T_1 = 2$

$T_2 = 8$

$T_3 = 18$

$T_r = \tan^{-1}\left(\frac{1}{T_r'}\right)$

QUESTION [JEE Mains 2020]

Tan3

If S is the sum of the first 10 terms of the series

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \_\_\_\_\_$

then tan(S) is equal to:

A 10/11

B -6/5

C 5/6

D 5/11

QUESTION [JEE Mains 2022 (June)]

Tan4

The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

Tan5

4. $S = \sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$$T_n = \tan^{-1} \frac{4n}{n^4-2n^2+2}$$

$$= \tan^{-1}\left(\frac{4n}{1 + n^4-2n^2+1}\right)$$

$$= \tan^{-1}\left(\frac{4n}{1+(n^2-1)^2}\right)$$

$$= \tan^{-1}\left(\frac{(n+1)^2-(n-1)^2}{1+(n-1)^2(n+1)^2}\right)$$

$$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2 .$$

QUESTION [JEE Mains 2022 (June)]

Tah 6

Let $a_1 = 1, a_2, a_3, a_4, \ldots\ldots$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to :

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$

(B) $\frac{\pi}{4} - \tan^{-1}(2022)$

(C) $\cot^{-1}(2022) - \frac{\pi}{4}$

(D) $\tan^{-1}(2022) - \frac{\pi}{4}$

3. $\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{325} + \cdots \infty$

H
P
B

$S = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} + \cdots \infty$

2. $\tan^{-1}\frac{x}{1+(1\times 2)x^2}+\tan^{-1}\frac{x}{1+(2\times 3)x^2}+\cdots+\tan^{-1}\frac{x}{1+n(n+1)x^2}$, $x>0$

Tan 7

Two more Properties

P-6

Tan θ

$$\text{(i) } \sin^{-1}x+\sin^{-1}y=\begin{cases}\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } x\geq 0;y\geq 0 \text{ and } x^2+y^2\leq 1\\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } x\geq 0;y\geq 0 \text{ and } x^2+y^2>1\end{cases}$$

$$\text{(ii) } \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right), x>0;y>0$$

(Usable)

Proof: $\sin^{-1}x=\alpha$, $\sin^{-1}y=\beta$ $\quad \alpha,\beta\in(0,\pi/2]$

$x=\sin\alpha$, $y=\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

$\sin(\alpha-\beta)=x\sqrt{1-y^2}-y\sqrt{1-x^2}$

Taking $\sin^{-1}$ on both sides.

$$\sin^{-1}(\sin(\alpha-\beta)) = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$$

$0 < \alpha \leq \frac{\pi}{2} \Rightarrow 0 < \alpha \leq \pi/2$

$0 < \beta \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \leq -\beta < 0$

$$-\frac{\pi}{2} < \alpha-\beta < \frac{\pi}{2}$$

$$\alpha-\beta = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$$

$$\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$$

P-7

(useable)

(i) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$ where $x > 0, y > 0$

(ii) $\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right); & x < y, x, y > 0 \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right); & x > y, x, y > 0 \end{cases}$

QUESTION [JEE Mains 2019]

Tan9

If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ then x is equal to:

A $\frac{\sqrt{145}}{12}$

B $\frac{\sqrt{145}}{10}$

C $\frac{\sqrt{146}}{12}$

D $\frac{\sqrt{145}}{11}$

QUESTION [JEE Mains 2019]

Tan10

The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is

(A) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

(B) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(C) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

(D) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

QUESTION [JEE Mains 2024 (4 April)]

Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that

$\cos^{-1} x - \sin^{-1} y = \alpha, \frac{-\pi}{2} \leq \alpha \leq \pi.$

Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

(A) 0

(B) -1

(C) 1/2

(D) -1/2

$\cos^{-1}x - \sin^{-1}y = \alpha \qquad \alpha \in [-\frac{\pi}{2}, \pi] \quad , x, y \in [-1, 1]$

$\cos^{-1}x - (\pi/2 - \cos^{-1}y) = \alpha$

$\cos^{-1}x + \cos^{-1}y = \pi/2 + \alpha$.

$A \qquad B \in [0, \pi]$

$A + B = \pi/2 + \alpha$

$\cos(A+B) = \cos(\pi/2 + \alpha)$

$\cos A \cos B - \sin A \sin B = -\sin\alpha$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$.

$xy + \sin\alpha = \sqrt{1-x^2}\cdot\sqrt{1-y^2}$) S.B.S

$x^2y^2 + 2xy\sin\alpha + \sin^2\alpha = 1 - y^2 - x^2 + x^2y^2$

$$2xy\sin\alpha + \sin^2\alpha = 1 - x^2 - y^2$$

$$x^2 + 2xy\sin\alpha + y^2 = 1 - \sin^2\alpha = \cos^2\alpha$$

$$x^2 + 2xy\sin\alpha + y^2 \Big|_{MIN} = \cos^2\alpha \Big|_{MIN} = 0$$

QUESTION [JEE Mains 2019]

Tan II

If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(A) $2\sin^2\alpha$

(B) $4\cos^2\alpha + 2x^2y^2$

(C) $4\sin^2\alpha$

(D) $4\sin^2\alpha - 2x^2y^2$

LCD

आरंभ

LCD

Three Important Questions

- **Limit of What?** ⟶ function
- **Where & Why?** ⟶ At a point ie at a value of x generally where f is takes indertiminate form.
- **How?**

To know about behaviour of fn in viccinity of the point



LIMIT IS An Investigative tool which helps us to make Inferences for point where fn takes indeterminate form

Seven Indeterminate Forms

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 1^{\infty}, 0^{0}$ and ∞^{0} ← Read all as tending values

★ Indeterminate
★ not defined $\frac{1}{0}, \frac{0}{0}, 0^{0}$ - -
★ $\infty, -\infty$.

Murkho ki Indeterminate form

$\infty \times \infty$, ∞^{∞} , $\underline{\underline{0^{\infty}}}$, 1^{0}

(Exact 0) $\times \infty = 0$

(Exact 1)$^{\infty} = 1$

$1^{\infty - \infty}$

1^{∞} ← Ans $\to \infty$

How to find lim at $x=a$

Ex: $f(x)=\frac{x^2-4}{x-2}$

as x approached 2

$f(x)=\frac{\text{tending to } 0}{\text{tending to } 0}$

⇓

Indeterminate form $\left(\frac{0}{0}\right)$

$$f(x)=\frac{x^2-4}{x-2}=\begin{cases} x+2 & x\neq 2 \\ \text{N.D} & x=2 \end{cases}$$

x	$f(x)$
1.9	3.9
1.99	3.99
1.999	3.999
1.9999	3.9999
2 from L.H.S	4

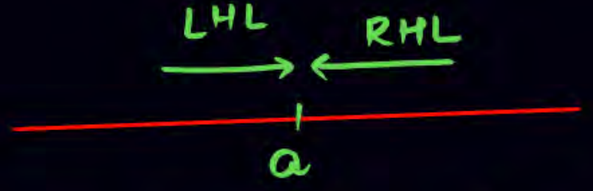
LHL

$\lim_{x\to 2^-} f(x)=4$

x	$f(x)$
2.1	4.1
2.01	4.01
2.001	4.001
2.0001	4.0001
2 from RHS	4

RHL

$\lim_{x\to 2^+} f(x)=4$



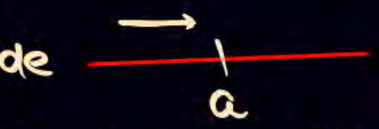
Since $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = 4$

$\Downarrow$

$\lim_{x \to a} f(x) = 4$

Left hand limit at $x = a$

As x approaches 'a' from left side



then the value towhich function approaches is called LHL.

$\lim_{x \to a^-} f(x) = \ell_1 = \text{LHL}$ — LHL

$\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) \quad (h > 0)$

Right hand limit RHL

As x approaches 'a' from right hand side



the value to which f_n approaches is called RHL

RHL $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) \quad (h>0)$

$\lim_{x \to a} f(x)$ is said to Exist if $\underset{(\text{at } x=a)}{\text{LHL}} = \underset{(\text{at } x=a)}{\text{RHL}} = \text{a.f.q}\left(\begin{matrix}\text{a finite}\\ \text{quantity}\end{matrix}\right)$

i.e if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \text{a f q}$

i.e if $\lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(a+h) = \text{a f q}$.

Left hand Limit (L. H. L.)

A real number l_1 is said to be the left hand limit of a function f(x) if, as x approaches 'a' from left hand side the value of function f approaches the value l_1 i.e. as x moves nearer and nearer to 'a' from left the value of f(x) move nearer and nearer to l_1.

Right hand Limit (R. H. L.)

A real number l_2 is said to be the right-hand limit of a function f(x) if, as x approaches 'a' from right hand-side the value of function f approaches the value l_2 i.e as x moves nearer and nearer to 'a' from right the value of f(x) move nearer and nearer to l_1.

Procedure to find L.H.L and R.H.L

To evaluate left-hand limit of function, we substitute $x = a - h$ and take the limit $h \to 0$

i.e., $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(h \to 0)$ (Where $h > 0$)

To evaluate right-hand limit of a function, we substitute $x = a + h$ and take the limit $h \to 0$

i.e., $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a + h)$

Existence of Limit of a Function

The limit of function at x = a, is said to exist if

(i) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = l$

(ii) l is a finite real number

One Sided Limit

★ If a fn is not defined in left nbd (neighbourhood / vicinity) of $x=a$ &

if $\lim_{x \to a^+} f(x) = a$ fq then we say $\lim_{x \to a} f(x) = \lim_{x \to a^+} f(x) = a$ fq

★ If a fn is not defined in right nbd (neighbourhood / vicinity) of $x=a$ &

if $\lim_{x \to a^-} f(x) = a$ fq then we say $\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = a$ fq

$\lim_{x \to a} f(x) \Rightarrow x \neq a$

$\lim_{x \to a} f(x)$ may Exists even if the f_n is not defined at $x=a$

i.e we can talk of limit at $x=a$ even if $a \notin D_f$ but has a graph in the atleast one viccinity of $x=a$

★ $\frac{1}{\text{a very small +ve quantity tending to } 0} \longrightarrow \infty$ i.e $\frac{1}{0^+} \to \infty$

★ $\frac{1}{\text{very small -ve quantity tending to } 0} \longrightarrow -\infty$ i.e $\frac{1}{0^-} \to -\infty$

★ $\frac{1}{\text{tending to } \infty \text{ or } -\infty} \longrightarrow 0$

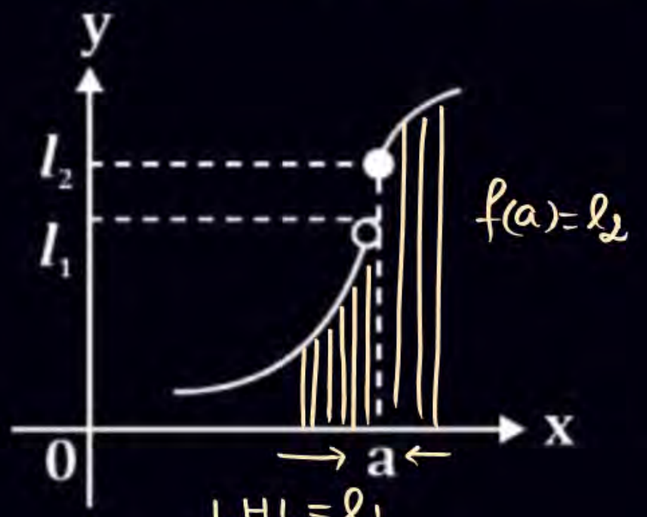
★ (Ano: b/w 0 & 1) $^{\infty} \to 0$

$0 < a < 1$

$y = a^x$

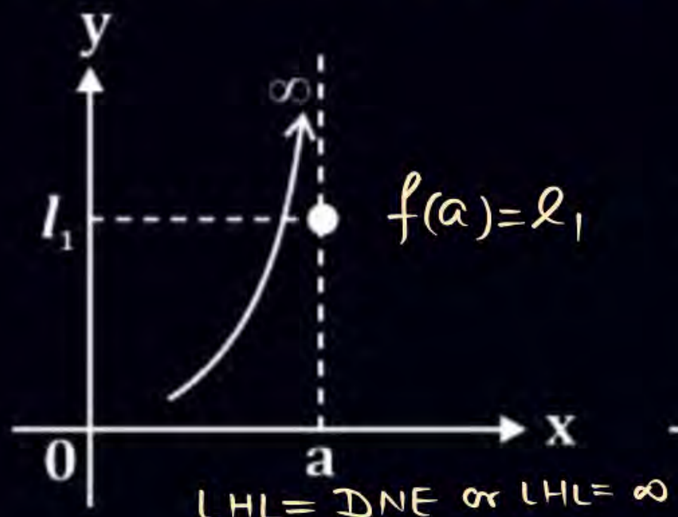
as $x \to \infty$ $a^x \to 0$

In which of the following limit exists at x = a



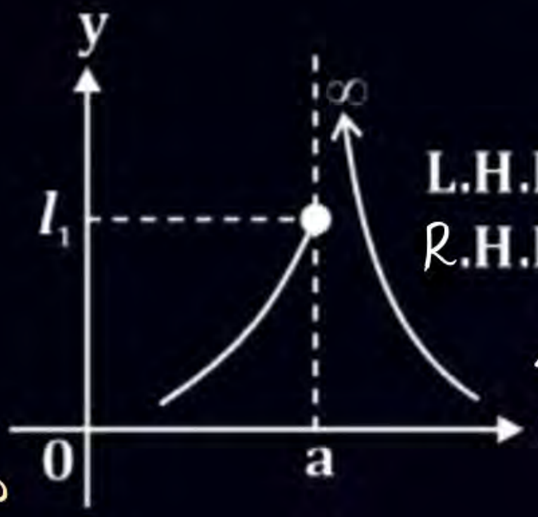
$f(a) = l_2$

LHL $= l_1$
RHL $= l_2$
$\lim_{x \to a} f(x) =$ D.N.E



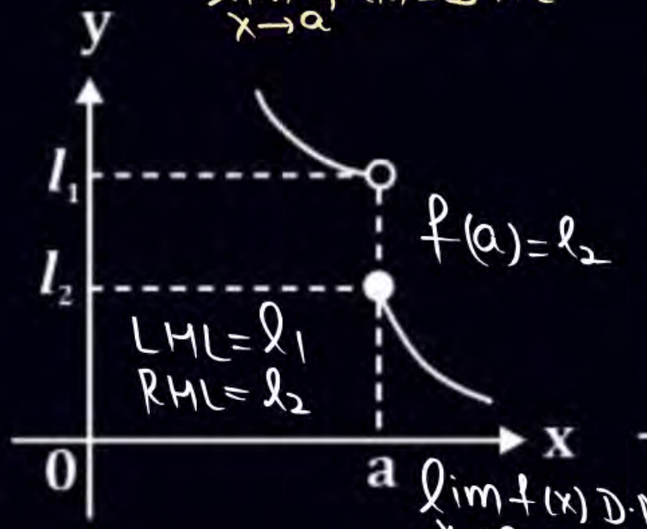
$f(a) = l_1$

LHL = DNE or LHL = ∞
RHL = N.A
$\lim_{x \to a} f(x) =$ LHL = DNE/∞



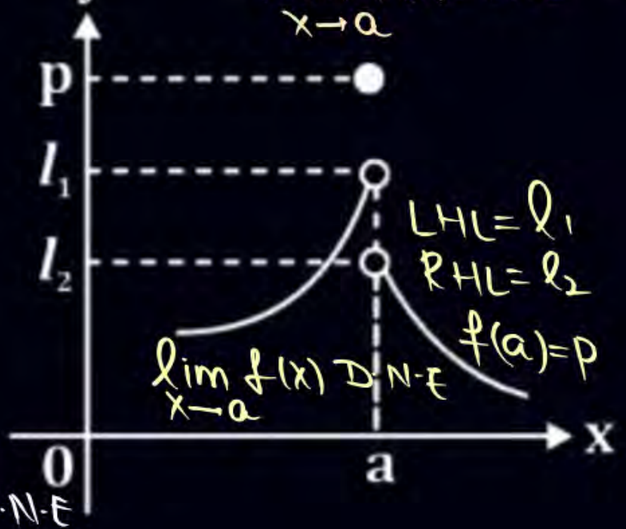
$f(a) = l_1$

L.H.L. $= l_1$
R.H.L. $= \infty$ / DNE
$\lim_{x \to a} f(x)$ D.N.E

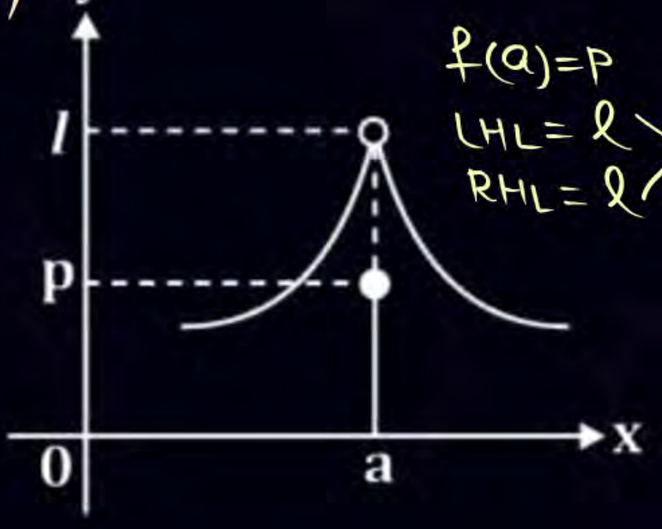


$f(a) = l_2$

LHL $= l_1$
RHL $= l_2$
$\lim_{x \to a} f(x)$ D.N.E



LHL $= l_1$
RHL $= l_2$
$f(a) = p$
$\lim_{x \to a} f(x)$ D.N.E



$f(a) = p$
LHL $= l$
RHL $= l$
$\Big\} \lim_{x \to a} f(x) = l \neq p$

If $f(a) = p \not\Rightarrow \lim_{x \to a} f(x) = p$

QUESTION

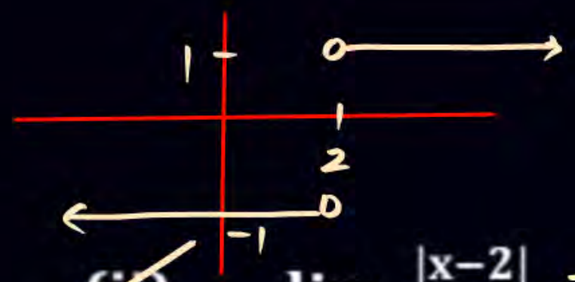
Evaluate of the following one sided limits:

(i) $\lim_{x\to 2^+} \frac{|x-2|}{(x-2)} = \lim_{h\to 0} \frac{|2+h-2|}{2+h-2}$

$= \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{K}{K} = 1.$

(ii) $\lim_{x\to 2^-} \frac{|x-2|}{(x-2)} = \lim_{h\to 0} \frac{|2-h-2|}{2-h-2}$

$= \lim_{h\to 0} \frac{|-h|}{-h} = \lim_{h\to 0} \frac{h}{h} = -1$



$y = \frac{|x-2|}{x-2}$

(iii) $\lim_{x\to 3^+} [x]$; [.] is gint function

$\lim_{h\to 0} [3+h] = 3$

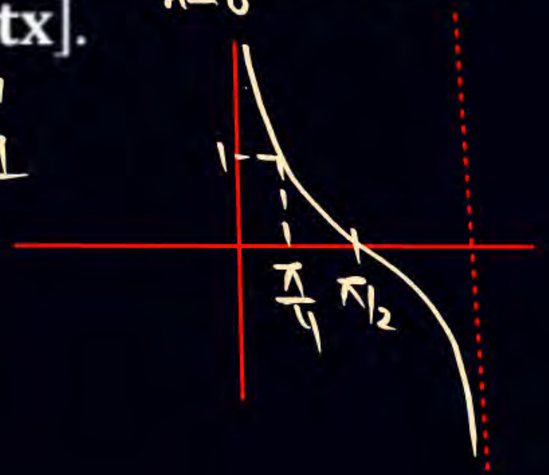
(iv) $\lim_{x\to 3^-} [x]$, [.] is gint function

$\lim_{h\to 0} [3-h] = 2$

(v) $\lim_{x\to \pi/4^-} [\cot x]$.

$\cot x \to 1^+$

$= 1$



(vi) $\lim_{x\to \pi/4^+} [\cot x] = 0$

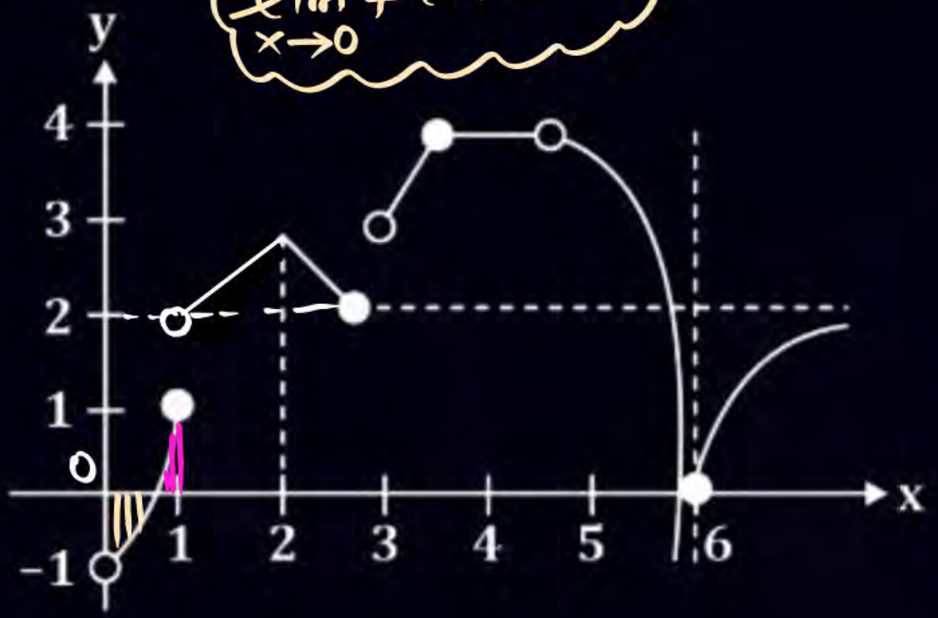
$\cot x \to 1^-$

QUESTION

Tan 12a

Consider the adjacent graph of y = f(x) Find the following:

$\lim_{x\to 0} f(x) = -1$



(a) $\lim_{x\to 0^-} f(x)$ (ND)

(b) $\lim_{x\to 0^+} f(x) = -1$

(c) $\lim_{x\to 1^-} f(x) = 1$

(d) $\lim_{x\to 1^+} f(x) = 2$

(e) $\lim_{x\to 2^-} f(x)$

(f) $\lim_{x\to 2^+} f(x)$

(g) $\lim_{x\to 3^-} f(x)$

(h) $\lim_{x\to 3^+} f(x)$

(i) $\lim_{x\to 4^-} f(x)$

(j) $\lim_{x\to 4^+} f(x)$

(k) $\lim_{x\to \infty} f(x)$

(l) $\lim_{x\to 6^-} f(x)$

QUESTION

Tan 12(b)

Determine the following limits.

(a) $\lim_{x\to -1^+} f(x)$ (b) $\lim_{x\to -1^-} f(x)$

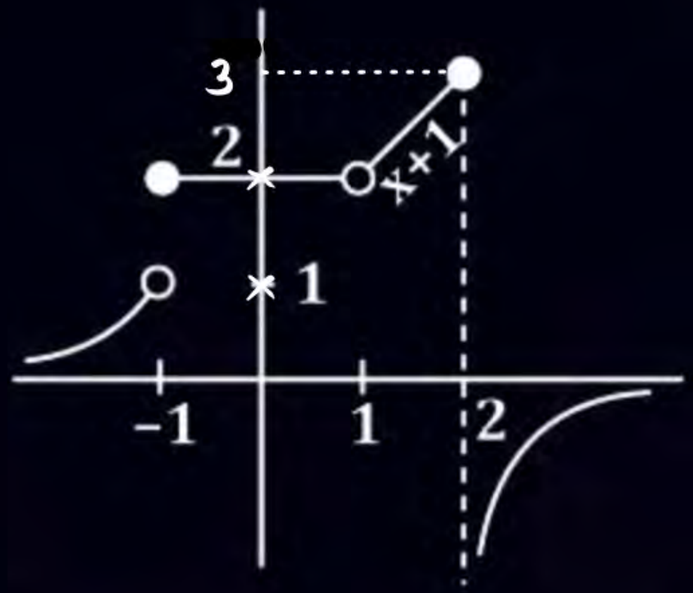
(c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 1^+} f(x)$

(e) $\lim_{x\to 1^-} f(x)$ (f) $\lim_{x\to 1} f(x)$

(g) $\lim_{x\to 2^+} f(x)$ (h) $\lim_{x\to 2^-} f(x)$

(i) $\lim_{x\to 2} f(x)$ (j) $\lim_{x\to -3} f(x)$

(k) $\lim_{x\to 5} f(x)$ (l) $\lim_{x\to 1.5} f(x)$



QUESTION

Which of the following statements about the function y = f(x) graphed here are true, and which are false?

(a) $\lim_{x \to -1^+} f(x) = 1$ (T)

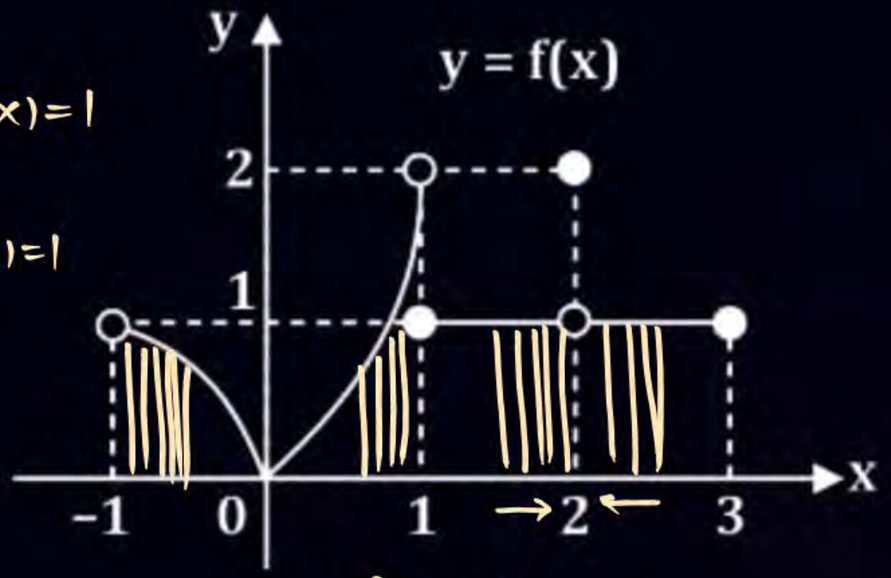
(b) $\lim_{x \to 2} f(x)$ does not exist (F)

(c) $\lim_{x \to 2} f(x) = 2$ (F)

$\lim_{x \to 2^+} f(x) = 1$

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2} f(x) = 1$



(d) $\lim_{x \to 1^-} f(x) = 2$ (T)

(e) $\lim_{x \to 1} f(x)$ does not exist (T)

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1} f(x)$ D.N.E

(f) $f(2) = 2$ (T)

QUESTION

Tan 12©

Which of the following statements about the function y = f(x) graphed here are true, and which are false?

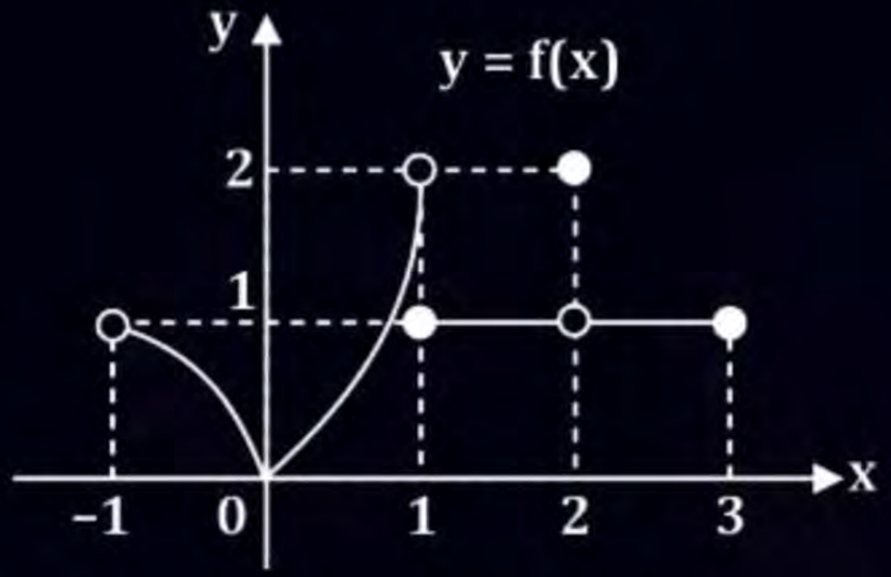
(f) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

(g) $\lim_{x\to c} f(x)$ exists at every $c \in (-1, 1)$

(h) $\lim_{x\to c} f(x)$ does not exist.

(i) $\lim_{x\to 1^-} f(x) = 0$

(j) $\lim_{x\to 3^+} f(x)$ does not exist.



QUESTION

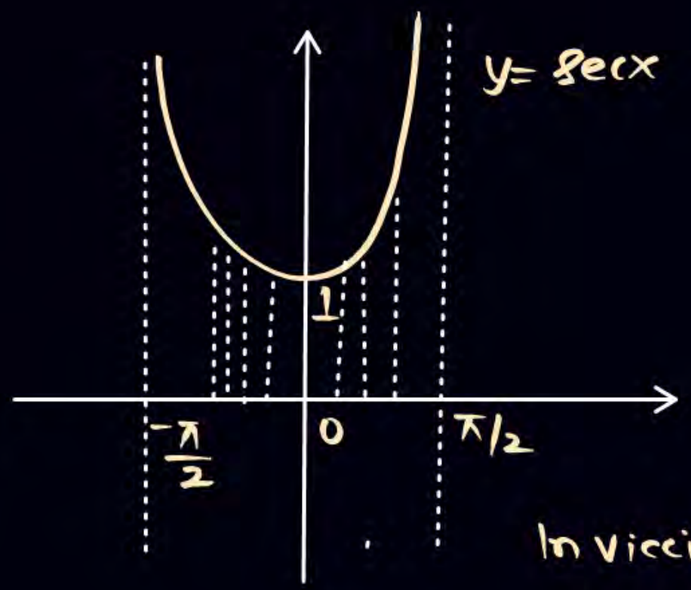
$$\lim_{x\to 0} \sin^{-1}(\sec x)$$

A $\pi/2$

B Does not exist

C $-\pi/2$

D 1



$\sin^{-1}x$ is defined only if $x \in [-1,1]$

In viccinity of 0, $\sec x > 1$

$\sin^{-1}(\sec x)$ is not defined

In both left & right nbd

of $x=0 \Rightarrow \lim_{x\to 0} \sin^{-1}(\sec x)$ is not defined.

1 se baada

QUESTION

$$\lim_{x\to 0} \tan^{-1}\left(\frac{1}{[x]}\right)$$

clearly in right nbd of $x=0$
$[x]=0 \Rightarrow f(x)=\tan^{-1}\left(\frac{1}{[x]}\right)$ is not defined in right nbd of $x=0$

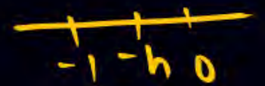
LHL

$$\lim_{x\to 0^-} \tan^{-1}\left(\frac{1}{[x]}\right)$$

$$= \lim_{h\to 0} \tan^{-1}\frac{1}{[0-h]}$$

$$= \lim_{h\to 0} \tan^{-1}\left(\frac{1}{[-h]}\right)$$

$$= \lim_{h\to 0} \tan^{-1}(-1) = -\pi/4.$$



RHL

$$\lim_{x\to 0^+} \tan^{-1}\left(\frac{1}{[x]}\right)$$

$$\lim_{h\to 0} \tan^{-1}\left(\frac{1}{[h]}\right)$$

$$\lim_{h\to 0} \tan^{-1}\left(\frac{1}{0}\right) = \pi/2$$

(Gadho/Gadhiyoo aisaa naa karo)

$$\lim_{x\to 0} \tan^{-1}\left(\frac{1}{[x]}\right) = \lim_{x\to 0^-} \tan^{-1}\left(\frac{1}{[x]}\right) = -\pi/4$$

QUESTION

Show that $\lim_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist.



$e^{\infty} \to \infty$

$e^{-\infty} \to 0$

AT $x=0$ RHL

$$\lim_{h\to 0}\left(\frac{e^{\frac{1}{0+h}}-1}{e^{\frac{1}{0+h}}+1}\right) = \lim_{h\to 0}\frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1} = \lim_{h\to 0}\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}} = 1$$

($e^{1/h} \to 0$)

LHL

$$\lim_{h\to 0}\left(\frac{e^{\frac{1}{0-h}}-1}{e^{\frac{1}{0-h}}+1}\right) = \lim_{h\to 0}\frac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1} = -1$$

($e^{-1/h} \to 0$)

LHL ≠ RHL

limit DNE.

QUESTION

$\lim_{x\to\infty} \dfrac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\dfrac{2x+1}{x-1}\right)^{x}\right\}}$ is equal to

(A) 1

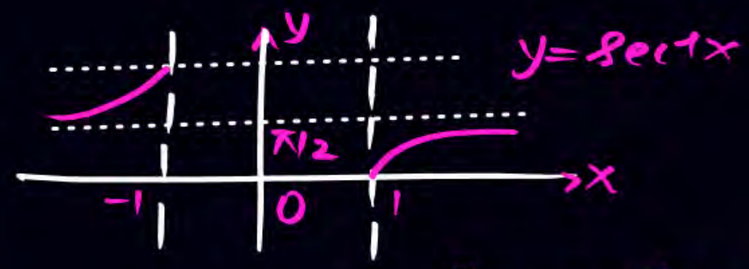
(B) 0

(C) $\pi/2$

(D) non existent

$\lim_{x\to a} f(x) \Rightarrow x \neq a$

$[x], \lceil x \rceil, |x|$



$\lim_{x\to\infty} \sec^{-1}x = \pi/2$

$\lim_{x\to-\infty} \sec^{-1}x = \pi/2$

$$\lim_{x\to-\infty} \frac{\cot^{-1}\left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)}{\sec^{-1}\left\{\left(\frac{2+1/x}{1-1/x}\right)^{x}\right\}} = \frac{\pi/2}{\pi/2} = 1$$

($\frac{1}{\sqrt{x+1}+\sqrt{x}} \to 0$, $x \to -\infty$; $\left(\frac{2+1/x}{1-1/x}\right)^{x} \to 2^{\infty}$)

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge -05

The product of all real values of x satisfying the equation $\cos^{-1}\left(\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)\right)+\cot^2\left(\cot^{-1}\left(\frac{2}{9|x|}-2\right)\right)=0$ is

A 9

B – 9

C 3

D – 3

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION [JEE Mains 2019]

KTK 1

Considering only the principal values of inverse functions, the set

$$A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$$

(A) is an empty set

(B) contains more than two elements

(C) contains two elements

(D) is a singleton

QUESTION

KTK 2

Let $f(x) = \begin{cases} x^2 - 4 & \text{if } |x| \leq 3 \\ 5 \operatorname{sgn} |x-3| & \text{if } |x| > 3 \end{cases}$ and $g(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ for all $x \in R$, then which of the following is wrong ?
(where sgn (x) denotes signum function of x)

A f(x) is an even function

B gof(x) is an even function

C g(x) is an odd function

D fof(x) is an odd function

QUESTION

KTK 3

Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is :

A $\left(-\infty, \frac{1}{4}\right]$

B $\left[-\frac{1}{4}, \infty\right)$

C $(-1/3, \infty)$

D $\left(-\infty, \frac{1}{3}\right]$

Ans : B

QUESTION

KTK 4

Determine the inverse function and its domain of definition if

$$f(x) = \begin{cases} x; & -\infty < x < 1 \\ x^2; & 1 \le x \le 4 \\ 2^x; & 4 < x < \infty \end{cases}.$$

Ans. $f^{-1}(x) = \begin{cases} x; & -\infty < x < 1 \\ \sqrt{x}; & 1 \le x \le 16 \\ \log_2 x; & 16 < x < \infty \end{cases}$

QUESTION

KTK 5

If $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is given as

A $\frac{1}{2}\left[1 - \sqrt{1 + 4\log_2 x}\right]$

B $\frac{1}{2}\left[1 + \sqrt{1 + 4\log_2 x}\right]$

C $\left[1 - \sqrt{1 + 4\log_2 x}\right]$

D $\left[1 - \sqrt{1 + 4\log_2 x}\right]$

Ans. B

QUESTION [JEE Mains 2024 (5 April)]

KTK 6

Let $f, g : R \to R$ be defined as: $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x + 1, & x \leq 0. \end{cases}$

Then the function $f(g(x))$ is

(A) neither one-one nor onto.

(B) one-one but not onto.

(C) both one-one and onto.

(D) onto but not one-one.

Ans. A

QUESTION [JEE Mains 2021]

KTK 7

If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \pi/4$, then the value of $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^8+b^8}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots\ldots$ is:

A $\log_e 2$

B e^{2-1}

C e

D $\log_e(e/2)$

QUESTION

KTK 8

Find the simplest value of

(i) $f(x) = \arccos x + \arccos\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right), x \in \left(\frac{1}{2}, 1\right)$

(ii) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \in R - \{0\}$

QUESTION

KTK 9

(i) If $\tan^{-1} 4 + \tan^{-1} 5 = \cot^{-1}(\lambda)$, then find λ.

(ii) Which is greater $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$ or $\cot^{-1}(-1)$

Homework From Module

INVERSE TRIGONOMETRIC FUNCTIONS

Prarambh : Q1 to Q23

Prabal : Q1 to Q17

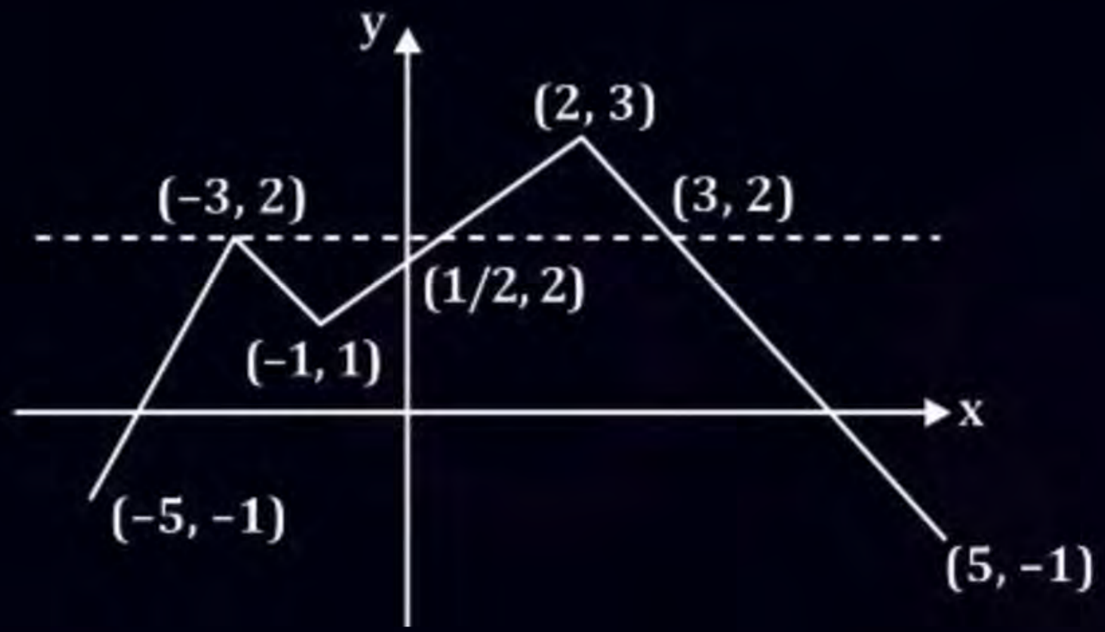
Parikshit : Q1 to Q6

(Revision Practice Problems)

QUESTION

(RPP 1)

The graph of $y = f(x)$ is shown, then number of solutions of the equation $f(f(x)) = 2$ is equal to



Ans. 3

QUESTION

(RPP 2)

Let f be a real valued function such that for any real x,
$f(15 + x) = f(15 - x)$ and $f(30 + x) = -f(30 - x)$.
Then which of the following statement is true?

A f is odd

B f is even

C f is periodic

D f is not periodic

Ans. A, C

QUESTION

(RPP 3)

In a kite ABCD, $AB = AD$ and $CB = CD$. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$ can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

A (1, 2, 1)

B (2, 1, 1)

C (4, 3, 2)

D (1, 1, 2)

Ans. D

(Solution to TAH)

QUESTION [JEE Advance 2022]

Considering only the principal values of the inverse trigonometric functions, the value of $\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$ is :

26 July Hcp

Adv: 22.

Tah-1

$$X = \frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}} + \frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2} + \tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$X = \frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}} + \frac{1}{4}\sin^{-1}\left(\frac{2\pi/\sqrt{2}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}\right) + \tan^{-1}\frac{\sqrt{2}}{\pi}$$

let $\tan\theta = \frac{\pi}{\sqrt{2}}$

$$X = \frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\tan^2\theta}} + \frac{1}{4}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \tan^{-1}(\cot\theta)$$

$$X = \frac{3}{2}\cos^{-1}(\cos\theta) + \frac{1}{4}\sin^{-1}(\sin 2\theta) + \frac{\pi}{2} - \cot^{-1}(\cot\theta)$$

$$X = \frac{3}{2}\times\theta + \frac{\theta}{2} + \frac{\pi}{2} - \theta$$

$$X = \theta + \frac{\pi}{2}$$

$$X = \frac{\pi}{2} + \tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right) \quad \text{Ans}$$

tah-1
kshitij from up

Page No.:

Adv. TAH (1)

Considering only the principal values of the inverse trignometric function the value of

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$x=\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$x=\frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\frac{\pi^2}{2}}}+\frac{1}{4}\sin^{-1}\left(\frac{2\pi/\sqrt{2}}{1+\frac{\pi^2}{2}}\right)+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

(b) $\frac{\pi}{\sqrt{2}}=\tan\theta \Rightarrow \pi=\sqrt{2}\tan\theta$

$$=\frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\tan^2\theta}}+\frac{1}{4}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\tan^{-1}(\cot\theta)$$

$$=\frac{3}{2}\cos^{-1}(\cos\theta)+\frac{1}{4}\sin^{-1}(\sin 2\theta)+\tan^{-1}(\cot\theta)$$

$$=\frac{3}{2}\theta+\frac{1}{4}(\pi-2\theta)+\frac{\pi}{2}-\cot^{-1}(\cot\theta)\ \tan^{-1}\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)$$

$$=\frac{3}{2}\theta+\frac{\pi}{4}-\frac{\theta}{2}+\frac{\pi}{2}-\theta$$

$$=\frac{3\pi}{4}\quad \text{Ans}$$

Here, $2\theta>\frac{\pi}{2}$

So, $\pi-2\theta$ (taken)

Sundaram Tubey
From - U P

Date
Page

26 July

Tah-① consider only Principle values –

Que

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}}+\frac{1}{4}\sin^{-1}\left(\frac{2\times\frac{\pi}{\sqrt{2}}}{1+\left(\frac{\pi}{\sqrt{2}}\right)^2}\right)+\tan^{-1}\left(\frac{1}{\pi/\sqrt{2}}\right)$$

Let $\frac{\pi}{\sqrt{2}}=\tan\theta$

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{1}{1+\tan^2\theta}}+\frac{1}{4}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)+\tan^{-1}\left(\frac{1}{\tan\theta}\right)$$

$$\frac{3}{2}\cos^{-1}(\cos\theta)+\frac{1}{4}\sin^{-1}(\sin 2\theta)+\tan^{-1}(\cot\theta)$$

$$\frac{3}{2}\theta+\frac{1}{4}(\pi-2\theta)+\frac{\pi}{2}-\theta$$

Shivanshi Mishra
Mr. anoop Mishra
Mrs smita Mishra

$$=\frac{3\pi}{4}\quad \text{Ans}$$

QUESTION

The number of solution of the equations $2\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \pi x^3 = 0$ is equal to

A 0

B 1

C 2

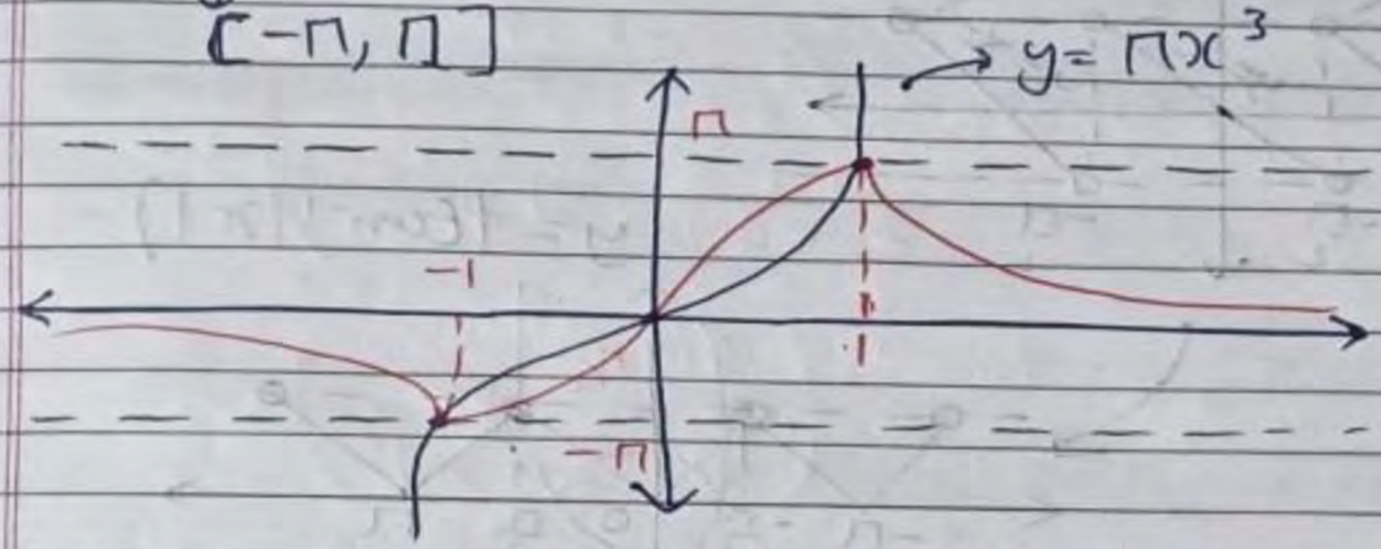
D 3

Tah-2

Himanshu Swami

$$2\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi x^3$$

$[-\pi, \pi]$



$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -(\pi + 2\tan^{-1}x) & x < -1 \\ 2\tan^{-1}x & x \in (-1, 1) \\ \pi - 2\tan^{-1}x & x \in (1, \infty) \end{cases}$$

$\frac{\pi}{2}$

So, no. of solutions = 3.

OPPO A79 5G

26/7/24

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Tah-2

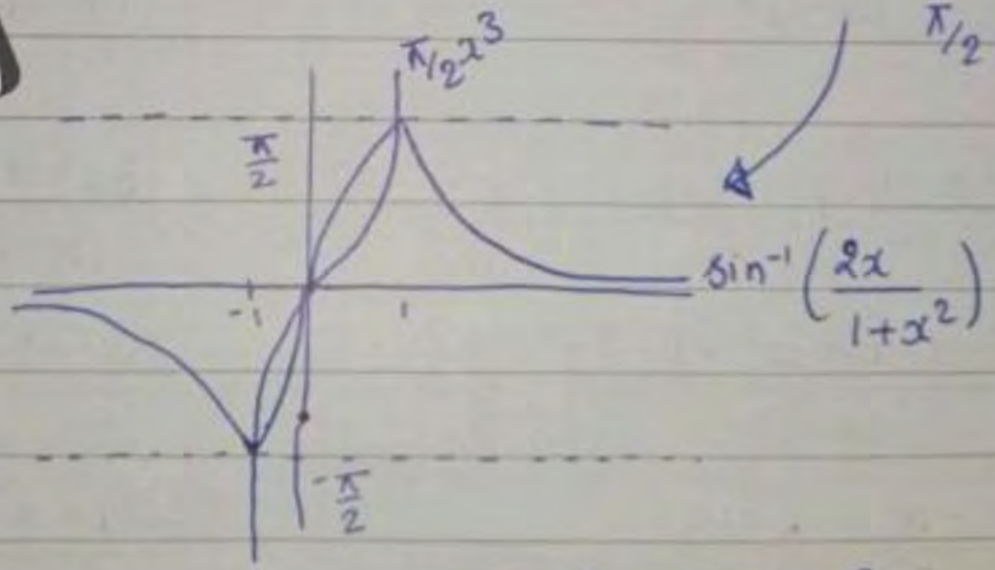
$$2\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \pi x^3 = 0$$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \frac{\pi}{2}x^3 -$$

let $x=1$

$\sin^{-1}(1) = \frac{\pi}{2}$

$\pi/2 = \pi/2$



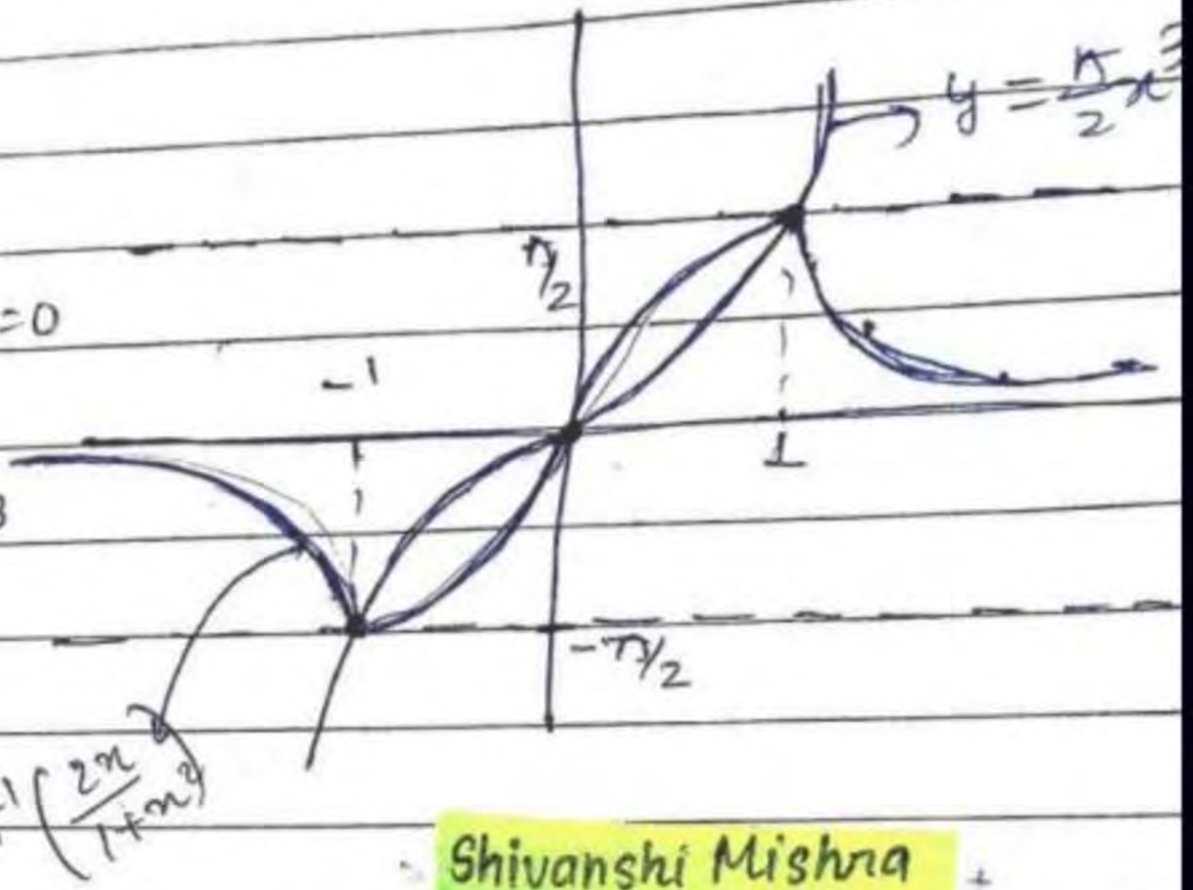
no of soln = 3

Bhavesh Upadhyay
Delhi

Tah-8

Que $2\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \pi x^3 = 0$

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \frac{\pi}{2}x^3$



no. of sol^n $= 3$ $\{-1, 0, 1\}$

Shivanshi Mishra
Mr.anoop Mishra
Mrs smita Mishra

Tah-9

QUESTION

The number of solutions of the equation $\left|\tan^{-1}|x|\right| = \sqrt{(x^2+1)^2 - 4x^2}$ is

Tah 3

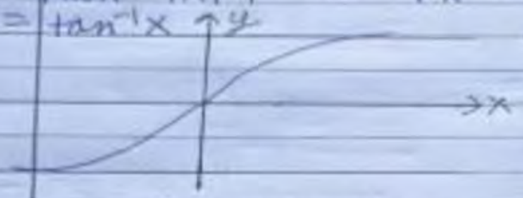
Tah-3.

$|\tan^{-1}|x||| = \sqrt{(x^2+1)^2 - 4x^2}$

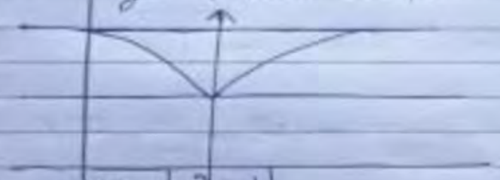
$|\tan^{-1}|x|| = \sqrt{(x^2-1)^2}$

$|\tan^{-1}|x|| = |x^2-1|$

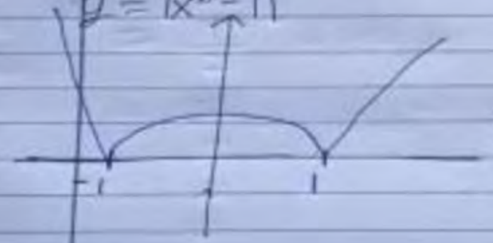
$y = \tan^{-1}x$



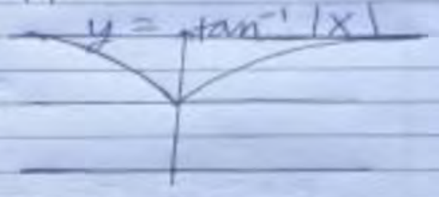
$y = |\tan^{-1}|x||$



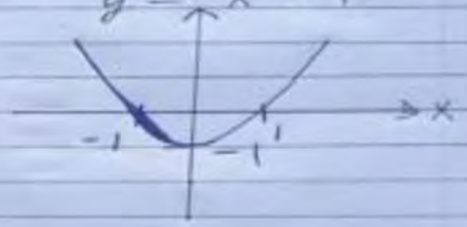
$y = |x^2-1|$



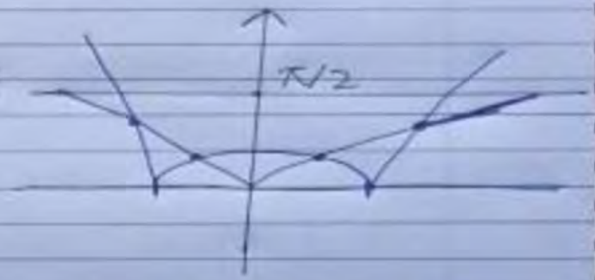
$y = \tan^{-1}|x|$



$y = x^2-1$



≡



= 4 Solution Ans.

Hemant Mewada
Madhya pradesh

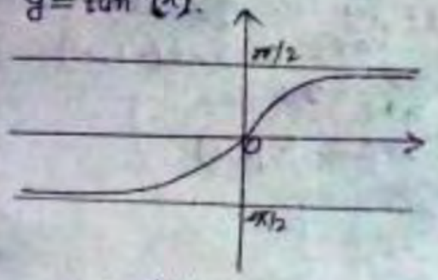
Tah-3

$|\tan^{-1}|x|| = \sqrt{(x^2+1)^2 - 4x^2}$

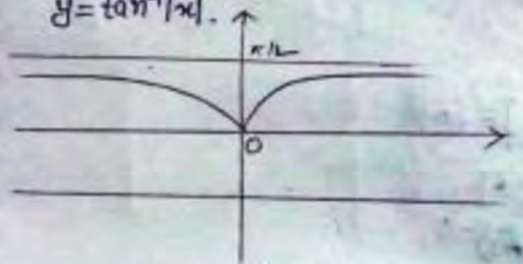
$|\tan^{-1}|x|| = \sqrt{(x^2-1)^2}$

$|\tan^{-1}|x|| = |x^2-1|$

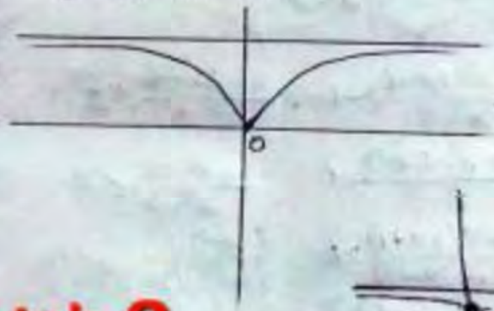
$y = \tan^{-1}(x)$.



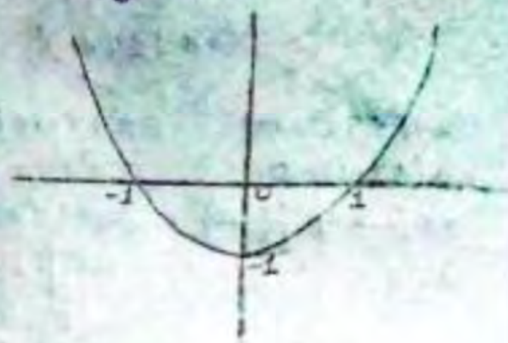
$y = \tan^{-1}|x|$.



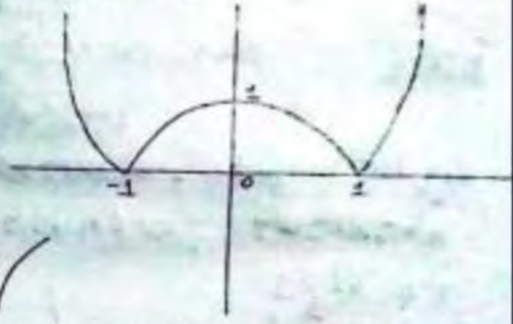
$y = |\tan^{-1}|x||$.



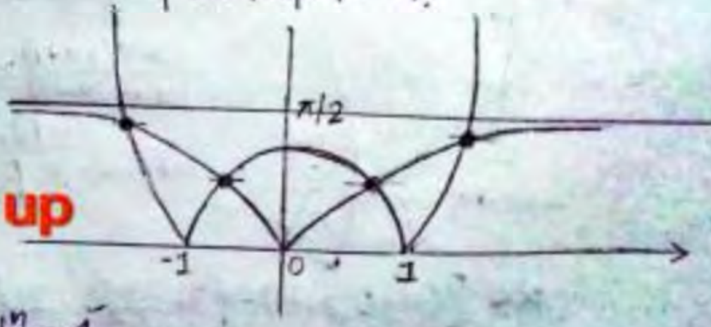
$y = x^2-1$



$y = |x^2-1|$.



$|\tan^{-1}|x|| = |x^2-1|$.



tah-3
kshitij from up

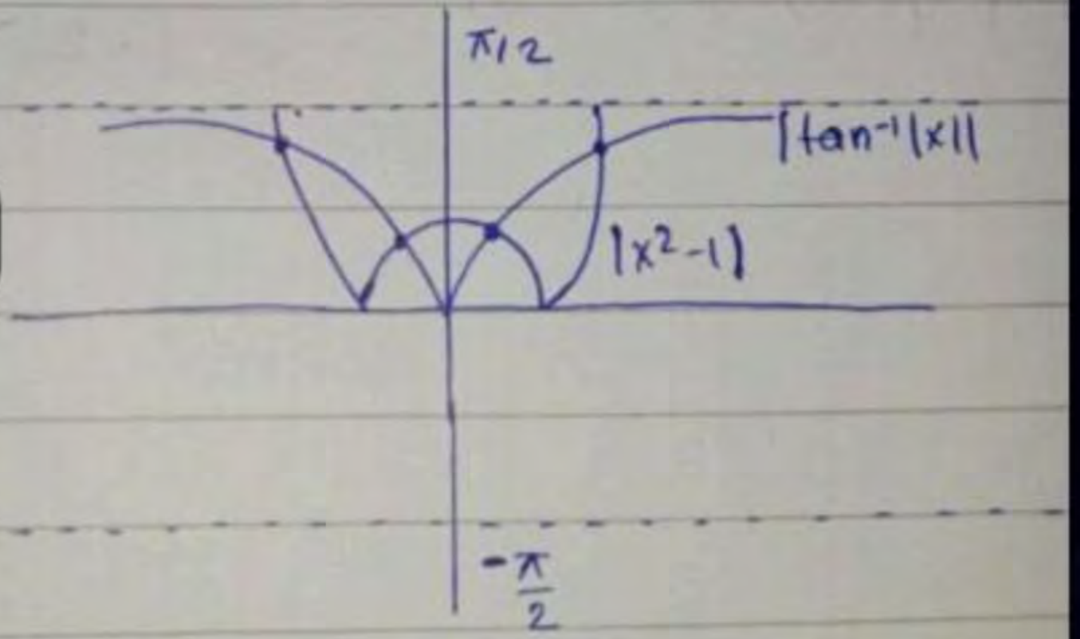
No of Soln = 4

Tah-3

$$|\tan^{-1}|x|| = \sqrt{(x^2+1)^2 - 4x^2}$$

$$|\tan^{-1}|x|| = \sqrt{(x^2-1)^2}$$

$$|\tan^{-1}|x|| = |x^2-1|$$



"no. of solutions = 4"

Bhavesh Upadhyay
Delhi

Home challenge :- (4)

$f: R \to (0, \frac{3\pi}{4}]$. $f(x) = \cot^{-1}(x^2+x+a) \cdot 2f - f(x)$ is surjective then find the range of a.

codomain = Range
b'coz f is onto.

$\cot^{-1}(x^2+x+a)$ has Range $(0, \frac{3\pi}{4}]$

(x^2+x+a) has Range $[\cot\frac{3\pi}{4}, \infty)$

$$\left[\frac{-D}{4a}, \infty\right) = \left[\cot\frac{3\pi}{4}, \infty\right)$$

$$\frac{-D}{4a} = \cot\frac{3\pi}{4}$$

$$-\frac{(1-4a)}{4} = \cot\left(\pi - \frac{\pi}{4}\right)$$

$$\frac{1-4a}{4} = 1$$

Home challenge
kshitij from up

$$1-4a = 4.$$

$$-4a = 3$$

$$a = \frac{-3}{4} \quad \text{Ans}$$

(Solution to KTK)

QUESTION

KTK 1

The domain of the function $f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ is :

A $\left[0, \frac{1}{4}\right]$

B $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

C $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

D $\left[0, \frac{1}{2}\right]$

Ans:C

Date / / KTK(01) Page No.:

The domain of the function

$f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ is

$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1$ × & $-1 \le \frac{x-1}{x+1} \le 1$

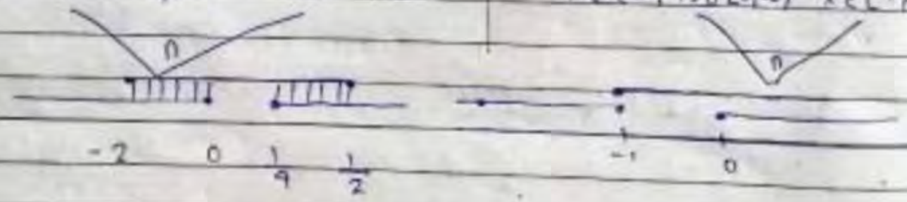
$-(x-1)^2 \le 3x^2+x-1 \le (x-1)^2$ | $\frac{x-1}{x+1}+1 \ge 0$ $\frac{x-1}{x+1}-1 \le 0$

$-x^2-1+2x \le 3x^2+x-1 \le x^2+1-2x$ | $\frac{2x}{x+1} \ge 0$ $\frac{-2}{x+1} \le 0$

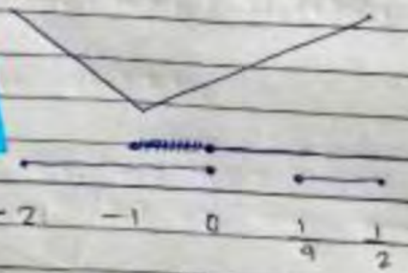
$4x^2-x \ge 0$ $2x^2+3x-2 \le 0$ | $\frac{2x}{x+1} \ge 0$ $\frac{2}{x+1} > 0$

$x(4x-1) \ge 0$ $(2x-1)(x+2) \le 0$ | $x \in (-\infty,-1] \cup [0,\infty)$ $x \in [-1,\infty)$

$x \in (-\infty,0] \cup [\frac{1}{4},\infty)$ $x \in [-2,\frac{1}{2}]$



$x \in [-2,0] \cup [\frac{1}{4},\frac{1}{2}]$ $x \in [0,\infty)$



$x \in [\frac{1}{4},\frac{1}{2}] \cup \{0\}$ Answer

Sundaram Dubey
From: UP

Esha
From: Karnataka

PAGE NO.:
DATE: / /

KTK-1

Domain of $f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$

Soln.-

$-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1$ and $-1 \le \frac{x-1}{x+1} \le 1$

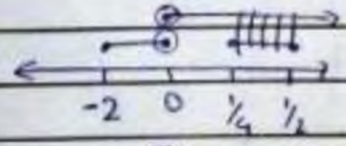
$-x^2+2x-1 \le 3x^2+x-1 \le x^2-2x+1$ | $\frac{x-1+x+1}{x+1} \ge 0$ $\frac{x-1-x-1}{x+1} \le 0$

$4x^2-x \ge 0$ $2x^2+3x-2 \le 0$ | $\frac{x}{x+1} \ge 0$ $\frac{1}{x+1} > 0$

$x(4x-1) \ge 0$ $(2x-1)(x+2) \le 0$ | $x \in (-\infty,0] \cup (0,\infty)$ $x \in (-1,\infty)$

$x \in (-\infty,0] \cup [\frac{1}{4},\infty)$ $x \in [-2,\frac{1}{2}]$ $x \in [0,\infty)$

$x \in [-2,0] \cup [\frac{1}{4},\frac{1}{2}]$



Domain:- $x \in [\frac{1}{4},\frac{1}{2}] \cup \{0\}$

$[\frac{1}{4},\frac{1}{2}] \cup \{0\}$

QUESTION [JEE Mains 2022 (June)]

KTK 2

The domain of the function $f(x)=\frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e\left(x^2-3x+2\right)}$ is:

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Ans : D

M:-22

KTK-2

$$f(x)=\frac{\cot^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}.$$

$\log_e(x^2-3x+2)\neq 0$, $x^2-3x+2>0$. , $-1\le\frac{x^2-5x+6}{x^2-9}\le 1$

$x^2-3x+2\neq 1$ $\quad x\in(-\infty,1)\cup(2,\infty)$

$x\neq\frac{3\pm\sqrt{5}}{2}$

$\frac{x^2-5x+6}{x^2-9}\ge -1$, $\frac{x^2-5x+6}{x^2-9}\le 1$

$\frac{2x^2-5x-3}{x^2-9}\ge 0$ $\quad \frac{5x-15}{x^2-9}\ge 0$

$\frac{(x-3)(2x+1)}{(x-3)(x+3)}\ge 0$ $\quad \frac{(x-3)}{(x+3)(x-3)}\ge 0$

$(x\neq 3)$ $\quad x\in(-3,\infty)\ (x\neq 3)$

$x\in(-\infty,-3)\cup\left[-\frac{1}{2},\infty\right)-\{3\}$

$x\in\left[-\frac{1}{2},1\right)\cup(2,\infty)-\left\{3,\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2}\right\}$

ktk2

kshitij from up

(Solution to RPP)

QUESTION

(RPP 1)

Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$

for the matrix $S = ABCD$, consider the two statements.

I. $S^3 = S$

II. $S^2 = S^4$

A. II is true but not I

B. I is true but not II

C. both I and II are true

D. both I and II are false

Ans. C

RPP-1 $A^T = BCD$, $B^T = CDA$, $C^T = DAB$, $D^T = ABC$.

$S = ABCD$.

$A^T = BCD$

$A = D^T C^T B^T$.

$A = ABC \cdot DAB \cdot CDA$

$A = S^2 A$.

$A(BCD) = S^2 \cdot A(BCD)$

$\boxed{S = S^3}$ → I st true.

$S^2 = ABCD \cdot ABCD$

$S^4 = S^3 \cdot S$

↓

$S^4 = S \cdot S$.

$\boxed{S^4 = S^2}$ — II nd true.

rpp1
kshitij from up

both statement are true

Boby. Hr

Rpp 1

RPP1 $A^T = ABC$

RPP1 $A^T = BCD$ $B^T = CDA$ $C^T = DAB$ $D^T = AB$

$S = ABCD$ St ① $S^3 = S$

St ② $S^4 = S^2$ (A, B, C, D not ness. sq. M).

Ans $S = ABCD$

Taking Transpose both side

$A = D^T C^T B^T$

$B = A^T D^T C^T$

$C = B^T A^T D^T$

$D = C^T B^T A^T$

$S^T = D^T C^T B^T A^T$

$S^T = (ABC)(DAB)(CDA)(BCD)$

$S^T = A(BCD)(ABC)(DAB)CD$

$S^T = A A^T D^T C^T CD$

$S^T = A B C D$

$S^T = ABCD$

$\therefore (S^T = S)$

$S^3 = (ABCD)(ABCD)(ABCD)$

$S^3 = D^T C^T B^T A^T$

$S^3 = S^T = S$ (True)

Both side (Pre) Multiple by S

$S^4 = S^2$ —— (True).

QUESTION

(RPP 2)

$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then let us define a function $f(x) = \det.(A^T A^{-1})$ then which of the following can not be the value of $\underbrace{f(f(f(f \ldots \ldots . . f(x))))}_{n \text{ times}}$ is $(n \geq 2)$

A $f^n(x)$

B 1

C $f^{n-1}(x)$

D $nf(x)$

Ans. D

RPP2 $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ Then let us define a fn $f(x) = |A^T . A^{-1}|$ Then which of the following can not be the values of $f(f(f\cdots f(x)))$ is ($n \geq 2$) (f n times)

a) $f^n(x) = (f(x))^n = 1^n = 1$

b) 1

c) $f^{n-1}(x) = (f(x))^{n-1} = 1$

d) $n f(x) = n \cdot 1 = n$

$f(x) = |A^T| . |A^{-1}|$

$f(x) = \frac{|A^T|}{|A|} = 1$ $(\because |A^T| = |A|$

$f(x) = 1$

$f(f(f\cdots f(x))) = 1$ (f n times)

Priyanka Keshri W.B

RPP3 $\begin{bmatrix} & & \end{bmatrix}$ & $10B = \begin{bmatrix} 4 & 2 & 2 \end{bmatrix}$

RPP-2 $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, $f(x)=\det(A^TA^{-1})$.

$\underbrace{f(f(f(f\cdots f(x))))}_{n \text{ times}}=?$ $(n\geq 2)$.

$f(x)=\det(A^TA^{-1})$

we know $\det(A^T)=\det A$

$f(x)=|A^TA^{-1}|$

$=|A^T||A^{-1}|$

$f(x)=|A|\cdot\frac{1}{|A|}=1$, $\underbrace{f(f(f(f\cdots f(x))))}_{n \text{ times}}=1$

(A) $f^n(x)$

(B) 1

(C) $f^{n-1}(x)$

(D) $nf(x)$

rpp2
kshitij from up

QUESTION

(RPP 3)

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10\,B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$.

If B is the inverse of matrix A, then α is

A -2

B -1

C 2

D 5

Ans. D

KPP3

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix} \quad \& \quad 10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$$

$|A| = 1(1+3)+1(2+3)+1(2-1)$

$|A| = 10$

$|10B| = +5(6+4)-\alpha(-8-2)$

$10^{3\cdot 2}\cdot|B| = 50+10\alpha$

$\frac{100}{|A|} = 5+\alpha \Rightarrow \frac{100}{10} = 5+\alpha \Rightarrow \alpha = 10-5 = 5$ (D)

Priyanka Keshri W.B

RPP-3 $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ & $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$

If B is the inverse of matrix A, then α is.

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix},\quad B=\begin{bmatrix}2/5 & 1/5 & 1/5\\ -1/2 & 0 & \alpha/10\\ 1/10 & -1/5 & 3/10\end{bmatrix}$$

B is the inverse of matrix A

then $AB=I$

$$\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}2/5 & 1/5 & 1/5\\ -1/2 & 0 & \alpha/10\\ 1/10 & -1/5 & 3/10\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & \frac{1}{5}-\frac{\alpha}{10}+\frac{3}{10}\\ 0 & 1 & \frac{2}{5}+\frac{\alpha}{10}-\frac{9}{10}\\ 0 & 0 & \frac{1}{5}+\frac{\alpha}{10}+\frac{3}{10}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\frac{1}{5}-\frac{\alpha}{10}+\frac{3}{10}=0 \Rightarrow \alpha=5$

$\frac{2}{5}+\frac{\alpha}{10}-\frac{9}{10}=0 \Rightarrow \alpha=5$

$\frac{1}{5}+\frac{\alpha}{10}+\frac{3}{10}=1 \Rightarrow \alpha=5$

rpp3
kshitij from up

Ans $\alpha=5$.

R.P.P-3

Let $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ and $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & a\\ 1 & -2 & 3\end{bmatrix}$ If B is the inverse of matrix A, then a is.

(A) −2
(B) −1
(C) 2
(D) 5

$B=A^{-1}$

$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ $B=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & a\\ 1 & -2 & 3\end{bmatrix}$

$AB=I$

$$\frac{1}{10}\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & a\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\frac{1}{10}[3+a+3]=0$

$a=5$

Hence, a will be 5.

ARYAN FROM GHAZIPUR (UP)


THANK
YOU

PRAYAS

JEE 2025

Lecture-02 | Mathematics

Limit, Continuity and Differentiability

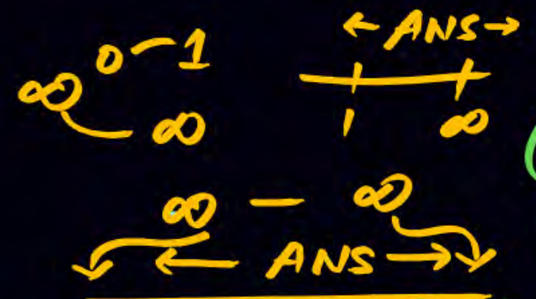


By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Method to Evaluate limits
2. Practice Problems.

Recap *of previous lecture*



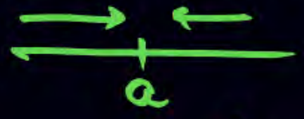
$a^0 = 1$, $a \neq 0$

1. Which of the following is not an indeterminate form

$\frac{\infty}{\infty}, \infty - \infty, 0^0, 1^\infty, \infty + \infty, \infty \times \infty, \infty^\infty, \infty^0, 0^\infty, \frac{0}{0}$



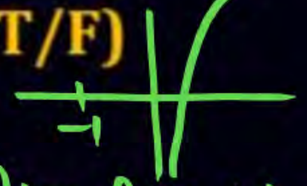
2. In order to talk about limit of a function f at x = a it should have a graph in atleast one nbd of x = a. (T/F)



$0^0 = (\text{tending to } 0)^{\text{tending to } 0}$

(Exact 0)$^{\text{Exact 0}}$ is n.d

3. $\lim_{x \to -1} \ln x$ is meaningless to talk about. (T/F)



4. LHL (at x = a) is defined as $\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h), \; h > 0$

5. RHL (at x = a) is defined as $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h), \; h > 0$

Recap *of previous lecture*

6. A function $y = f(x)$ has a limit at $x = a$ if LHL = RHL = afq i.e. if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
i.e. if $\lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(a+h)$

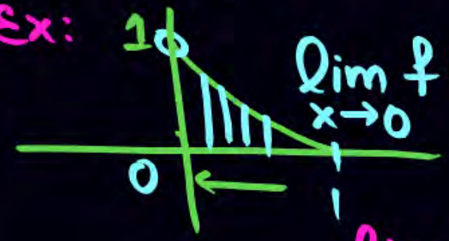
7. If function is defined in only right nbd of $x = a$ then $\lim_{x \to a} f(x) = \lim_{x \to a^+} f(x)$

8. If function is defined in only left nbd of $x = a$ then $\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x)$

9. If $f(a) = l$ then $\lim_{x \to a} f(x) = l$. (T/F)

10. If $\lim_{x \to a} f(x) = l$ then $f(a) = l$. (T/F)

Ex:



$\lim_{x \to 0} f(x) = \lim_{x \to 0^+} f(x) = 1$

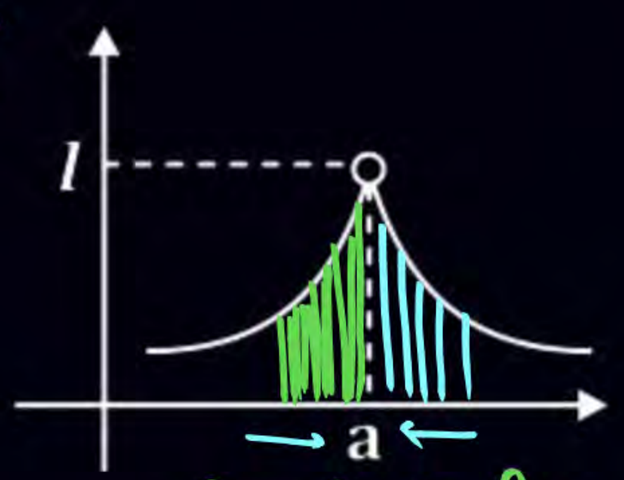
$\lim_{x \to 1} f(x) = \lim_{x \to 1^-} f(x) = 0$.

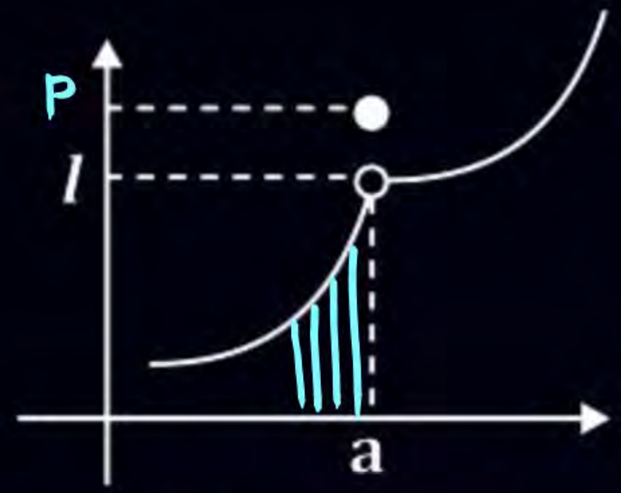
Recap *of previous lecture*

$f(x) = \frac{1}{[x]}$; $R - [0,1)$

11. Limit of a function 'f' can not exist at x = a if 'f' is not defined at x = a. (T/F)

12.





$\lim_{x \to a^-} f(x) = \ell$

$\lim_{x \to a^+} f(x) = \ell$

$\Rightarrow \lim_{x \to a} f(x) = \ell.$

Then find

(a) $f(a) = P$

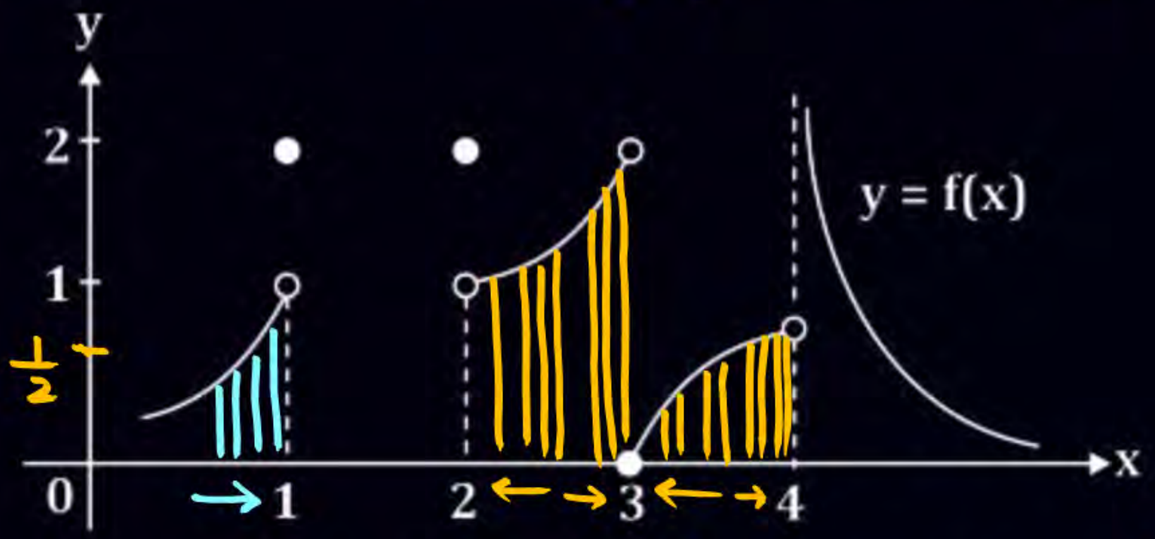
(b) $\lim_{x \to a} f(x) = \ell$

i.e. $\lim_{x \to a^-} f(x) = \ell$

$\lim_{x \to a^+} f(x) = \ell.$

Recap *of previous lecture*

13.



(iii) Smj

(i) $\lim_{x\to 1^+} f(x) =$ Not defined D.N.E

(ii) $\lim_{x\to 1^-} f(x) = 1$

(iii) $\lim_{x\to 1} f(x) = \lim_{x\to 1^-} f(x) = 1$

(iv) $\lim_{x\to 2^-} f(x) =$ Not def

(v) $\lim_{x\to 2^+} f(x) = 1$

(vi) $\lim_{x\to 2} f(x) = \lim_{x\to 2^+} f(x) = 1$

(vii) $\lim_{x\to 3^-} f(x) = 2$

(viii) $\lim_{x\to 3^+} f(x) = 0$

(ix) $\lim_{x\to 3} f(x) =$ DNE

(x) $\lim_{x\to 4^-} f(x) = 1/2$

(xi) $\lim_{x\to 4^+} f(x) = \infty$/DNE

Recap of previous lecture

$\cos^{-1}(\square)$ is def $\Rightarrow -1 \le \square \le 1$

([.] denote G.I.F.)

14. $\lim_{x\to\pi/2} \cos^{-1}(\text{cosec } x) =$ Not defined

15. $\lim_{x\to 0} \cot^{-1}\left(\frac{1}{[x]}\right) = \frac{3\pi}{4}$

16. $\lim_{x\to 0} \tan^{-1}\left(\frac{1}{x}\right) =$ DNE

RHL $= \lim_{h\to 0} \tan^{-1}\left(\frac{1}{0+h}\right) \to \infty = \pi/2$

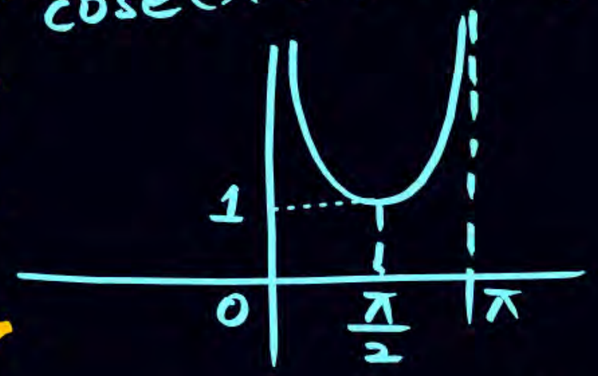
LHL $\lim_{h\to 0} \tan^{-1}\left(\frac{1}{0-h}\right) \to -\infty = -\pi/2$

17. $\lim_{x\to 1^-} \sin^{-1}([x]) = \lim_{h\to 0} \sin^{-1}[1-h] = \lim_{h\to 0} \sin^{-1} 0 = 0$ & $\lim_{x\to 1^+} \sin^{-1}([x]) = \lim_{h\to 0} \sin^{-1}[1+h] = \lim_{h\to 0} \sin^{-1} 1 = \pi/2$

18. State sine rule & cosine rule in a triangle.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\text{cosec}\, x \in (-\infty, -1] \cup [1, \infty)$



$\pi/2$ se aagay $\text{cosec} x > 1$

$\pi/2$ se peechay $\text{cosec} x > 1$

$\Rightarrow \cos^{-1}(\text{cosec} x)$ is not def in right as well as left of $\pi/2$

$f(x)=\cot^{-1}\left(\frac{1}{[x]}\right)$ is not defined $x\in[0,1)$

$x\in[0,1)\Rightarrow[x]=0$

$f(x)$ is not defined in right nbd of $x=0$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\cot^{-1}\left(\frac{1}{[x]}\right)$$

$$=\lim_{h\to 0}\cot^{-1}\left(\frac{1}{[0-h]}\right)$$

$$=\lim_{h\to 0}\cot^{-1}\left(\frac{1}{-1}\right)$$

$$=\cot^{-1}(-1)=3\pi/4$$

LHL RHL kab check karnaa chahiyay

$\lim_{x \to a} f(x)$ if value of fn changes abruptly in nbd of $x=a$

we should check for LHL & RHL

Ex: $[x]$ at $x \in I$

Ex: $\{x\}$ at $x \in I$

Ex: $\frac{1}{x}$ at $x=0$

Ex: $\operatorname{sgn} x$ at $x=0$

Ex: $\tan^{-1}\tan x$ at $(2n+1)\frac{\pi}{2}$

Panch Parmeshwar of Limits

If $\lim_{x \to c} f(x) = L$; $\lim_{x \to c} g(x) = M$ where L and M are finite quantities then

(a) **Sum rule** : $\lim_{x \to c} (f(x) + g(x)) = L + M$

(b) **Difference rule** : $\lim_{x \to c} (f(x) - g(x)) = L - M$

(c) **Product rule** : $\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$

(d) **Quotient rule** : $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, M \neq 0$

(e) **Constant Multiple rule** : $\lim_{x \to c} K.\, f(x) = KL$

If $\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M$, L, M are finite then reals.

① $\lim_{x\to a}(f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M.$

② $\lim_{x\to a}(f(x)\cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = L \cdot M$

③ $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ (provided $M \neq 0$)

④ $\lim_{x\to a} k f(x) = k \lim_{x\to a} f(x) = k \cdot L.$

⑤ if $f(x) \geq g(x)$ in nbd of $x=a$ then $\lim_{x\to a} f(x) \geq \lim_{x\to a} g(x)$

$L \geq M$

QUESTION

Determine whether the statement is true of false.

1. $\lim_{x\to4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right)=\lim_{x\to4}\frac{2x}{x-4}-\lim_{x\to4}\frac{8}{x-4}$. (F)

2. $\lim_{x\to1}\frac{x^2+6x-7}{x^2+5x-6}=\frac{\lim_{x\to1}(x^2+6x-7)}{\lim_{x\to1}(x^2+5x-4)}$ (F) $\because \lim_{x\to1}(x^2+5x-6)=0$

3. $\lim_{x\to1}\frac{x-3}{x^2+2x-4}=\frac{\lim_{x\to1}(x-3)}{\lim_{x\to1}(x^2+2x-4)}$ (T)

4. If $\lim_{x\to5} f(x)=2$ and $\lim_{x\to5} g(x)=0$, then $\lim_{x\to5}\frac{f(x)}{g(x)}$ does not exist (True) $\left(\frac{2}{0}\right)$

5. If $\lim_{x\to5} f(x)=0$ and $\lim_{x\to5} g(x)=2$, then $\lim_{x\to5}\frac{f(x)}{g(x)}$ does not exist (F) $\left(\frac{0}{2}\right)$

QUESTION

If $\lim_{x\to 0} \frac{f(x)}{x^2} = 1$, find

(i) $\lim_{x\to 0} f(x)$

M① $\lim_{x\to 0} \frac{f(x)}{x^2} = 1$ (given)

$\lim_{x\to 0} x^2 = 0$ (we know)

By (P.P) $\lim_{x\to 0}\left(\frac{f(x)}{x^2}\cdot x^2\right) = \lim_{x\to 0}\frac{f(x)}{x^2}\cdot\lim_{x\to 0} x^2$

$\lim_{x\to 0} f(x) = 1\cdot 0 = 0$

(ii) $\lim_{x\to 0} \frac{f(x)}{x}$

Tan(a)

M② $\lim_{x\to 0} \frac{f(x)}{x^2} = 1$

$x^2 \to 0$ (non zero ✗)

$\Downarrow$

$\lim_{x\to 0} f(x) = 0$

QUESTION

Jan 16

If $\lim_{x \to a}[f(x) + g(x)] = 2$ and $\lim_{x \to a}[f(x) - g(x)] = 1$, then $\lim_{x \to a} f(x)\, g(x)$

A need not exist

B exist and is $\frac{3}{4}$

C exists and is $\frac{-3}{4}$

D exists and is $\frac{4}{3}$

Ans: B

QUESTION

If $\lim_{x\to -2} \frac{f(x)}{x^2} = 1$, find

(i) $\lim_{x\to -2} f(x)$

(ii) $\lim_{x\to -2} \frac{f(x)}{x}$

M②

$\lim_{x\to -2} \frac{f(x)}{x^2} = 1$ (4 over $f(x)$, 4 over x^2)

$\Downarrow$

$\lim_{x\to -2} f(x) = 4$

Now $\lim_{x\to -2} \frac{f(x)}{x}$ (4 over $f(x)$, -2 over x) $= -2$

M①

$\lim_{x\to -2} x \cdot \frac{f(x)}{x^2}$

$= \lim_{x\to -2} x \cdot \lim_{x\to -2} \frac{f(x)}{x^2}$

$= -2 \cdot 1 = -2$

$\Rightarrow \lim_{x\to -2} \frac{f(x)}{x} = -2$

$\lim_{x\to -2} x = -2$ we know

$\lim_{x\to -2} \frac{f(x)}{x^2} = 1$ (given)

Ans: 4,-2

QUESTION

The value of a for which $\lim_{x\to -2} \frac{3x^2+ax+a+3}{x^2+x-2}$ exists. Also find the value of the limit.

non zero ✗

0

0

$$\lim_{x\to -2}(3x^2+ax+a+3)=0$$

$$3(4)-2a+a+3=0$$

$$12-a+3=0$$

$$a=15$$

$$\lim_{x\to -2}\frac{3x^2+15x+18}{(x+2)(x-1)}$$

$$\lim_{x\to -2}\frac{3x^2+6x+9x+18}{(x+2)(x-1)}\left(\frac{0}{0}\right)$$

$$\lim_{x\to -2}\frac{(3x+9)(x+2)}{(x+2)(x-1)}=\frac{-6+9}{-3}=-1$$

Various Strategies to Evaluate Limits

1. Factorization/Algebraic Identities
2. Rationalization/Double Rationalization
3. Using substitution
4. Using Law of LOVE
5. Binomial Theorem
6. Expansions

Using Factorization

- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^2 - b^2 = (a + b)(a - b)$

QUESTION

$$\lim_{x\to -3} \frac{x^4 - 6x^2 - 27}{x^3 + 3x^2 + x + 3}$$

$\left(\frac{0}{0}\right)$

If not indeterminate form Evaluate directly

Ex: $\lim_{x\to 0} \frac{x+3}{x-2}$ — Not indt form

$= \frac{3}{-2}$

$$\lim_{x\to -3} \frac{x^4 - 9x^2 + 3x^2 - 27}{x^2(x+3) + 1(x+3)}$$

$$\lim_{x\to -3} \frac{(x^2-9)(x^2+3)}{(x+3)(x^2+1)}$$

$$\lim_{x\to -3} \frac{(x-3)(x+3)(x^2+3)}{(x+3)(x^2+1)} \left(\frac{0}{0}\right)$$

$$= \frac{-6 \cdot 12}{10} = -36/5 \text{ Ans}$$

QUESTION

Tah 2

Evaluate: $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-5x+4}\right)$ Ans. 1/3

Evaluate: $\lim_{x\to 1}\left(\frac{1-x^{-\frac{1}{3}}}{1-x^{-\frac{2}{3}}}\right)$ Ans. 1/2

$\lim_{x\to 1}\frac{1-x^{-1/3}}{1-(x^{-1/3})^2}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 1}\frac{1-x^{-1/3}}{(1-x^{-1/3})(1+x^{-1/3})}$

$= \frac{1}{2}$.

Evaluate: $\lim_{x\to 2}\left(\frac{x^3+7x^2-36}{x^2+2x-8}\right)$ Ans. 28/5

QUESTION

Tan 3

For $a > 0$, $\lim_{x \to a^2} \frac{\sqrt{x} + x - a - a^2}{x\sqrt{x} + x\sqrt{a} - a^2\sqrt{x} - a^2\sqrt{a}}$ equals

$\lim_{x \to a^2} \frac{\sqrt{x} + x - a - a^2}{x\sqrt{x} + x\sqrt{a} - a^2\sqrt{a} - a^2\sqrt{a}}$ $\left(\frac{0}{0}\right)$

A $\frac{2a+1}{2a(a+\sqrt{a})}$

B $\frac{2a+1}{4a^{3/2}}$

C $\frac{a+2}{a^{3/2}}$

D None

Using Substitution

Agar koi quantity Question mai sabhi jagah dekhay toh usko t put karle

QUESTION

$$\lim_{x\to\frac{\pi}{4}} \frac{1-\cot^3 x}{2-\cot x-\cot^3 x}$$

put $\cot x = t$

as $x \to \pi/4$, $t \to 1$

$$= \lim_{t\to 1} \frac{1-t^3}{2-t-t^3} = \lim_{t\to 1} \frac{(1-t)(1+t^2+t)}{-(t^3+t-2)}$$

$\left(\frac{0}{0}\right)$

$$= \lim_{t\to 1} \frac{(1-t)(1+t^2+t)}{-[t^2(t-1)+t(t-1)+2(t-1)]}$$

$$= \lim_{t\to 1} \frac{(1-t)(t^2+t+1)}{-(t^2+t+2)(t-1)} = 3/4 \text{ Ans}$$

QUESTION

Tan 4

$$\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$$

$2^x = t$

as $x \to 2$, $t \to 4$

Using Rationalization

QUESTION

(i) $\lim_{x\to 9} \frac{3-\sqrt{x}}{4-\sqrt{2x-2}}$ $\left(\frac{0}{0}\right)$

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{4-\sqrt{2x-2}} \cdot \frac{4+\sqrt{2x-2}}{4+\sqrt{2x-2}} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$\lim_{x\to 9} \frac{(9-x)}{16-(2x-2)} \cdot \frac{4+\sqrt{2x-2}}{3+\sqrt{x}}$$

$$\lim_{x\to 9} \frac{9-x}{18-2x} \cdot \frac{4+\sqrt{2x-2}}{3+\sqrt{x}}$$

$$\lim_{x\to 9} \frac{9-x}{2(9-x)} \cdot \frac{4+\sqrt{2x-2}}{3+\sqrt{x}}$$

$$= \frac{1}{2} \cdot \frac{8}{6} = \frac{2}{3}$$

(ii) $\lim_{x\to 1} \frac{\sqrt{5-x}-2}{\sqrt{2-x}-1}$ $\left(\frac{0}{0}\right)$

$$\lim_{x\to 1} \frac{5-x-4}{2-x-1} \cdot \frac{\sqrt{2-x}+1}{\sqrt{5-x}+2}$$

$$\lim_{x\to 1} \frac{1-x}{1-x} \cdot \frac{\sqrt{2-x}+1}{\sqrt{5-x}+2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

QUESTION

(i) $\lim_{x\to 0} \frac{x}{\sqrt{5-x}-\sqrt{5+x}}$

Tan 4θ

ASRQ

(ii) $\lim_{x\to \frac{\pi}{2}} \tan^2 x\left\{\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right\}$ $0\times\infty$.

$$\lim_{x\to \pi/2} \frac{\tan^2 x\left(2\sin^2 x+3\sin x+4-\sin^2 x-6\sin x-2\right)}{\left(\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}\right)\to 6}$$

$$\lim_{x\to \pi/2} \frac{\sin^2 x\left(\sin^2 x-3\sin x+2\right)}{\cos^2 x\ (6)}$$

$$\lim_{x\to \pi/2} \frac{\sin^2 x\,(\sin x-1)(\sin x-2)}{1-\sin^2 x\ (\cdot)} = \lim_{x\to \pi/2} \frac{\sin^2 x\,(\sin x-1)(\sin x-2)}{(1-\sin x)(1+\sin x)\ (6)} = \frac{1}{12}$$

QUESTION

ASRQ

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$$ $\left(\frac{0}{0}\right)$

$(a-b)(a^2+b^2+ab)=a^3-b^3$

$$=\lim_{x\to 0}\frac{\left(\sqrt[3]{1+x}\right)^3-\left(\sqrt[3]{1-x}\right)^3}{\left(\left(\sqrt[3]{1+x}\right)^2+\left(\sqrt[3]{1-x}\right)^2+\sqrt[3]{1+x}\cdot\sqrt[3]{1-x}\right)\cdot x}$$

$$=\lim_{x\to 0}\frac{1+x-(1-x)=2x}{x\left((1+x)^{2/3}+(1-x)^{2/3}+(1-x^2)^{\frac{1}{3}}\right)}$$

$$=\frac{2}{3}$$ Ans

QUESTION

Tan 5

Evaluate: $\lim_{x\to 0} \frac{x}{\sqrt{a+x}-\sqrt{a-x}}$ Ans $\sqrt{a}$

Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ Ans. 1/2

Evaluate: $\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ Ans. $2/3\sqrt{3}$

Evaluate: $\lim_{x\to -1} \frac{\sqrt[3]{7-x}-2}{(x+1)}$ Ans. $-\frac{1}{12}$

Evaluate: $\lim_{x\to 1} \left(\frac{\sqrt{x^2+8}-\sqrt{10-x^2}}{\sqrt{x^2+3}-\sqrt{5-x^2}}\right)$ Ans. $\frac{2}{3\sqrt{3}}$

Evaluate: $\lim_{x\to 0} \left(\frac{\sqrt{1+x}-1}{x}\right)$ Ans. 1/2

Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$ Ans. 1/3

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Homework From Module

INVERSE TRIGONOMETRIC FUNCTIONS: Complete

LCD

(Solution to TAH)

QUESTION

TAH 1

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) \text{ is equal to}$$

A $\tan^{-1}(2^n)$

B $\tan^{-1}(2^n) - \frac{\pi}{4}$

C $\tan^{-1}(2^{n+1})$

D $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$

(Tah-1)

Q $\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$ is equal to

A) $\tan^{-1}(2^n)$

B) $\tan^{-1}(2^n) - \frac{\pi}{4}$

(C) $\tan^{-1}(2^{n+1})$

D) $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$

$\Rightarrow T_r = \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1+2^r \cdot 2^{r-1}}\right)$

$T_r = \tan^{-1}(2^r) - \tan^{-1}(2^{r-1})$

Observe!!

$2^r - 2^{r-1}$

$\Rightarrow 2^{r-1}(2-1)$

$\Rightarrow 2^{r-1}$

$T_1 = \tan^{-1}(2) - \tan^{-1}(2^0)$

$T_2 = \tan^{-1}(2^2) - \tan^{-1}(2^1)$

$T_3 = \tan^{-1}(2^3) - \tan^{-1}(2^2)$

$T_4 = \tan^{-1}(2^4) - \tan^{-1}(2^3)$

⋮

$T_n = \tan^{-1}(2^n) - \tan^{-1}(2^{n-1})$

$S_n = \tan^{-1}(2^n) - \tan^{-1}(1)$

$S_n = \tan^{-1}(2^n) - \frac{\pi}{4}$

Manish Kumar
बिहार

HW (1)

TAH-1

10

$\sum_{r=1}^{} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$

→ observe $2^r - 2^{r-1} = 2^{r-1}$

↳ OPStep

$\Sigma \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1 + 2^r \cdot 2^{r-1}}\right)$ → let $\tan x = 2^r$ & $\tan y = 2^{r-1}$

~~$\Sigma \tan^{-1}(\tan(2^r - 2^{r-1})$~~ → $\Sigma \tan^{-1} \tan(x-y)$

$\Rightarrow \sum_{r=1}^{n} \tan^{-1} 2^r - \tan^{-1} 2^{r-1}$

now, $T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 2^2 - \tan^{-1} 2$

$T_3 = \tan^{-1} 2^3 - \tan^{-1} 2^2$

⋮ ⋮ ⋮

$T_n = \tan^{-1} 2^n - \tan^{-1} 2^{n-1}$

$S_n = \tan^{-1} 2^n - \tan^{-1} 1$

$= \tan^{-1} 2^n - \frac{\pi}{4}$

Akshit

QUESTION [JEE Mains 2019]

TAH 2

If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots$ upto 100 terms, then α is

A 1.01

B 1.00

C 1.02

D 1.03

Q. If $\cot^{-1}(a) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots$ upto 100 terms,

then a is -

(A) 1.01 (B) 1.00 (C) 1.02 (D) 1.03

$\Rightarrow \tan^{-1}\frac{1}{1} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots$ upto 100 terms

2, 8, 18, 32, ...

6 10 14

4 4

TAN-2

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 2$

$T_2 = 4a + 2b + c = 8$ $\Rightarrow 3a + b = 6$

$T_3 = 9a + 3b + c = 18$ $\Rightarrow 5a + b = 10$

$\Rightarrow 2a = 4$

$a = 2$

$b = 0$ and $c = 0$

$T_n' = 2n^2$

$T_n = \tan^{-1}\left(\frac{1}{2n^2}\right)$

$T_n = \tan^{-1}\left(\frac{2}{4n^2}\right)$

$T_n = \tan^{-1}\left(\frac{2}{1 + 4n^2 - 1}\right)$

$T_n = \tan^{-1}\left(\frac{2}{1 + (2n-1)(2n+1)}\right)$

$T_n = \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n-1)(2n+1)}\right)$

$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$T_1 = \tan^{-1}(3) - \tan^{-1}(1)$

$T_2 = \tan^{-1}(5) - \tan^{-1}(3)$

$T_3 = \tan^{-1}(7) - \tan^{-1}(5)$

$\vdots$

$T_{100} = \tan^{-1}(201) - \tan^{-1}(199)$

$S_{100} = \tan^{-1}(201) - \tan^{-1}(1)$

$S_{100} = \tan^{-1}\left(\frac{201 - 1}{1 + 201}\right)$

$S_{100} = \tan^{-1}\left(\frac{200}{202}\right)$

$S_{100} = \tan^{-1}\left(\frac{200}{202}\right)$

$S_{100} = \cot^{-1}\left(\frac{202}{200}\right)$

Now,

$\cot^{-1}a = \cot^{-1}\left(\frac{202}{200}\right)$

$a = \frac{202}{200}$

$a = \frac{101}{100}$

$a = 1.01$ [A] [Ans.]

TAN-2

Shubhajit Roy
From WB.

QUESTION [JEE Mains 2020]

TAH 3

If S is the sum of the first 10 terms of the series

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \_\_\_\_\_$

then tan(S) is equal to:

A 10/11

B -6/5

C 5/6

D 5/11

Q If Λ is the sum of the first 10 terms of the series -

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \cdots\cdots$

then $\tan(\Lambda)$ is equal to :

(A) $11/11$ (B) $-6/5$ (C) $5/6$ (D) $5/11$

$\Rightarrow \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \cdots\cdots$

3, 7, 13, 21, ...

4 6 8

2 2

TAN-3

$T_n = an^2 + bn + c$

$T_1 = a + b + c = 3$

$T_2 = 4a + 2b + c = 7$ $\Big\rangle$ $3a + b = 4$

$T_3 = 9a + 3b + c = 13$ $\Big\rangle$ $5a + b = 6$ $\Big\rangle$ $2a = 2$, $a = 1$, $b = 1$ and $c = 1$

$T_n' = n^2 + n + 1$

$T_n = \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$

$T_n = \tan^{-1}\left(\frac{1}{1 + n(n+1)}\right)$

$T_n = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$T_n = \tan^{-1}(n+1) - \tan^{-1}(n)$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

$\vdots$

$T_{10} = \tan^{-1}11 - \tan^{-1}10$

$\Lambda_{10} = \tan^{-1}11 - \tan^{-1}1$

$\Lambda_{10} = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\Lambda_{10} = \tan^{-1}\left(\frac{10}{12}\right)$

$\Lambda_{10} = \tan^{-1}\left(\frac{5}{6}\right)$

Now, $\tan(\Lambda) = \tan\left(\tan^{-1}\frac{5}{6}\right) = \frac{5}{6}$ [C] [Ans.]

TAN-3

Shubhajit Roy

From W.B.

TAN 3° ~~tan(S)~~

$\rightarrow S = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + \cdots$

upto 10 terms.

3, 7, 13, 21, ---

4, 6, 8

2, 2 → 2nd order diff constant

$T_n = an^2 + bn + c$

Nitu Singh
Haryana

$T_1 = a+b+c = 3$

$T_2 = 4a+2b+c = 7$ → $3a+b = 4$

$T_3 = 9a+3b+c = 13$ → $5a+b = 6$ → $2a = 2$

$\boxed{a = 1}$

$3a+b = 4 \Rightarrow \boxed{b = 1}$

$a+b+c = 3 \rightarrow \boxed{c = 1}$

$T_n = n^2+n+1.$

$T_r = \tan^{-1}\left(\frac{1}{r^2+r+1}\right) = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$

$= \tan^{-1}\left(\frac{(r+1)-r}{1+r(r+1)}\right) = \tan^{-1}(r+1) - \tan^{-1}(r)$

$T_1 = \tan^{-1}(2) - \tan^{-1}(1)$

$T_2 = \tan^{-1}(3) - \tan^{-1}(2)$

$T_3 = \tan^{-1}(4) - \tan^{-1}(3)$

$\vdots$

$T_{10} = \tan^{-1}(11) - \tan^{-1}(10)$

$\tan(S) = \tan^{-1}(11) - \tan^{-1}(1)$

$\tan(S) = \tan^{-1}\left(\frac{11-1}{1+11}\right)$

$\tan(S) = \tan^{-1}\left(\frac{10}{12}\right)$

$S = \tan^{-1}\left(\frac{5}{6}\right)$

$\tan S = \tan\left(\tan^{-1}\left(\frac{5}{6}\right)\right)$

$\boxed{\tan S = \frac{5}{6}}$

QUESTION [JEE Mains 2022 (June)]

TAH 4

The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

Q. The value of $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is -

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$ (C) $\frac{51}{52}$ (D) $\frac{52}{51}$

$\Rightarrow$ $T_n = \tan^{-1}\left(\frac{1}{1+n+n^2}\right)$

$T_n = \tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$

$T_n = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$T_n = \tan^{-1}(n+1) - \tan^{-1}(n)$

$T_1 = \tan^{-1}(2) - \tan^{-1}(1)$

$T_2 = \tan^{-1}(3) - \tan^{-1}(2)$

$T_3 = \tan^{-1}(4) - \tan^{-1}(3)$

TAH-4
Shatbhaj it Roy
From W.B.

$T_{50} = \tan^{-1}(51) - \tan^{-1}(50)$

$S_{50} = \tan^{-1}(51) - \tan^{-1}(1)$

$S_{50} = \tan^{-1}\left(\frac{50}{52}\right)$

$S_{50} = \cot^{-1}\left(\frac{52}{50}\right)$

Now, $\cot\left(\cot^{-1}\left(\frac{52}{50}\right)\right) = \frac{52}{50} = \frac{26}{25}$ [A] [Ans.]

Ques The value of $\cot \sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)$ is

$\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)\right)$

$\tan^{-1}(n+1) - \tan^{-1}n$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_{50} = \tan^{-1}51 - \tan^{-1}50$

$S_{50} = \tan^{-1}51 - \tan^{-1}1$

$= \tan^{-1}\left(\frac{50}{52}\right)$

TAH-4
Aastha Gupta
Aligarh Up

$= \cot\left(\tan^{-1}\left(\frac{25}{26}\right)\right)$

concept

We know $\tan^{-1}\theta = \cot^{-1}\left(\frac{1}{\theta}\right)$

if $\theta > 0$

$= \cot\left(\cot^{-1}\left(\frac{26}{25}\right)\right)$

Ans = $\frac{26}{25}$

TAH 5

4. $S = \sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

Q. $A = \sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$\Rightarrow T_n = \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$T_n = \tan^{-1} \frac{4n}{1 + n^4 - 2n^2 + 1}$

$T_n = \tan^{-1} \frac{4n}{1 + (n^2 - 1)^2}$

$T_n = \tan^{-1} \frac{(n^2+1)^2 - (n-1)^2}{1 + (n^2+1)^2 (n^2-1)^2}$

$T_n = \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$

$T_1 = \tan^{-1}(2^2) - \tan^{-1} 0$

$T_2 = \tan^{-1}(3^2) - \tan^{-1}(1)^2$

$T_3 = \tan^{-1}(4^2) - \tan^{-1}(2)^2$

$T_4 = \tan^{-1}(5^2) - \tan^{-1}(3^2)$

$\vdots \quad \vdots$

$T_n = \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$

$S_n = \tan^{-1}(n+1) - \tan^{-1}(0) - \tan^{-1}(1)^2$

Now,

$A = \sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1}(0) - \tan^{-1} 1$

$A = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ [Ans.]

TAH-5

Shubhojit Roy
From W.B

$S = \sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$T_n = \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2} = \tan^{-1} \frac{4n}{1 + (n^4 - 2n^2 + 1)}$

$= \tan^{-1} \frac{4n}{1 + (n^2-1)^2} = \tan^{-1} \frac{4n}{1 + (n-1)^2 (n+1)^2}$

$= \tan^{-1} \left(\frac{(n+1)^2 - (n-1)^2}{1 + (n+1)^2 (n-1)^2} \right)$

$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2.$

TAH-5
Nitu Singh
from haryana

$T_1 = \tan^{-1}(2)^2 - \tan^{-1}(0)^2$

$T_2 = \tan^{-1}(3)^2 - \tan^{-1}(1)^2$

$T_3 = \tan^{-1}(4)^2 - \tan^{-1}(2)^2$

$\vdots$

$T_n = \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$

$T_1 = \tan^{-1}(2)^2 - \tan^{-1}(0)^2$

$T_2 = \tan^{-1}(3)^2 - \tan^{-1}(1)^2$

$T_3 = \tan^{-1}(4)^2 - \tan^{-1}(2)^2$

$\vdots \quad \vdots \quad \vdots$

$T_{n-1} = \tan^{-1}(n)^2 - \tan^{-1}(n-2)^2$

$T_n = \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$

$S_n = \tan^{-1}(n+1)^2 + \tan^{-1}(n)^2 - \tan^{-1}(0)^2 - \tan^{-1}(1)$

$S_n = \tan^{-1}(n^2+1+2n) + \tan^{-1}(n)^2 - \tan^{-1}(0) - \tan^{-1}(1)$

$S_\infty = \lim_{n \to \infty} S_n = \lim_{n \to \infty} [\tan^{-1}(n^2+1+2n) + \tan^{-1}(n)^2 - \tan^{-1}(0) - \tan^{-1}(1)]$

$S_\infty = \frac{\pi}{2} + \frac{\pi}{2} - 0 - \frac{\pi}{4}$

$= \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

Ques $S = \sum_{n=1}^{\infty} \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$T_n = \tan^{-1} \frac{4n}{n^4 - 2n^2 + 2}$

$= \tan^{-1} \frac{4n}{1 + (n^4 - 2n^2 + 1)}$

$= \tan^{-1} \frac{4n}{1 + (n^2 - 1)^2}$

$= \tan^{-1} \frac{4n}{1 + (n^2 + 1 - 2n)(n^2 + 1 - 2n)}$

$= \tan^{-1} \left[\frac{(n+1)^2 - (n-1)^2}{1 + (n+1)^2 (n-1)^2} \right]$

$= \tan^{-1} (n+1)^2 - \tan^{-1} (n-1)^2$

$T_1 = \tan^{-1} 2^2 - \tan 0$

$T_2 = \tan^{-1} 3^2 - \tan^{-1} 1^2$

$T_3 = \tan^{-1} 4^2 - \tan^{-1} 2^2$

$T_4 = \tan^{-1} 5^2 - \tan^{-1} 3^2$

$T_{(n-4)} = \tan^{-1} (n-3)^2 + \tan^{-1} (n-5)^2$

$T_{(n-3)} = \tan^{-1} (n-2)^2 + \tan^{-1} (n-4)^2$

$T_{(n-2)} = \tan^{-1} (n-1)^2 + \tan^{-1} (n-3)^2$

$T_{(n-1)} = \tan^{-1} (n)^2 + \tan^{-1} (n-2)^2$

$T_n = \tan^{-1} (n+1)^2 + \tan^{-1} (n-1)^2$

$S_n = \tan^{-1} n^2 + \tan^{-1} (n+1)^2 - \tan^{-1} 0 - \tan^{-1} 1$

$S_n = \tan^{-1} n^2 + \tan^{-1} (n+1)^2 - 0 - \frac{\pi}{4}$

$S_\infty = \tan^{-1} (\infty) + \tan^{-1} (\infty) - \frac{\pi}{4}$

$= \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$ Ans

TAH-5

Aastha Gupta

Aligarh Up

QUESTION [JEE Mains 2022 (June)]

TAH 6

Let $a_1 = 1, a_2, a_3, a_4, \ldots\ldots$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to :

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$

(B) $\frac{\pi}{4} - \tan^{-1}(2022)$

(C) $\cot^{-1}(2022) - \frac{\pi}{4}$

(D) $\tan^{-1}(2022) - \frac{\pi}{4}$

Q. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to -

(A) $\frac{\pi}{4} - \cot^{-1}(2022)$

(B) $\frac{\pi}{4} - \tan^{-1}(2022)$

(C) $\cot^{-1}(2022) - \frac{\pi}{4}$

(D) $\tan^{-1}(2022) - \frac{\pi}{4}$

=> Let,

$$S = \tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$

$$S = \tan^{-1}\left(\frac{1}{1+1\cdot2}\right) + \tan^{-1}\left(\frac{1}{1+2\cdot3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+2021\cdot2022}\right)$$

$$S = \tan^{-1}\left(\frac{2-1}{1+1\cdot2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\cdot3}\right) + \ldots + \tan^{-1}\left(\frac{2022-2021}{1+2021\times2022}\right)$$

$$S = \tan^{-1}2 - \tan^{-1}1 + \tan^{-1}3 - \tan^{-1}2 + \ldots + \tan^{-1}2022 - \tan^{-1}(2021)$$

$$S = \tan^{-1}(2022) - \tan^{-1}1$$

$$S = \tan^{-1}(2022) - \frac{\pi}{4} \quad [\text{Ans.}]$$

TAN-6

Shubhajit Roy

TAH-6 $\tan^{-1}\frac{1}{1+a_1a_2} + \tan^{-1}\frac{1}{1+a_2a_3} + \ldots\ldots + \tan^{-1}\frac{1}{1+a_{2021}a_{2022}}$

Since $a_1 = 1$ & $a_2, a_3 \ldots$ are consecutive natural no.

then,

$\Rightarrow \tan^{-1}\frac{a_2 - a_1}{1+a_1a_2} + \tan^{-1}\frac{a_3 - a_2}{1+a_2a_3} + \ldots\ldots + \tan^{-1}\frac{a_{2022} - a_{2021}}{1+a_{2022}a_{2021}}$

$\Rightarrow \cancel{\tan^{-1}a_2} - \tan^{-1}a_1 + \cancel{\tan^{-1}a_3} - \cancel{\tan^{-1}a_2} + \ldots\ldots + \tan^{-1}a_{2022} - \cancel{\tan^{-1}a_{2021}}$

$\Rightarrow \tan^{-1}a_{2022} - \tan^{-1}a_1$

$\Rightarrow \tan^{-1}2022 - \frac{\pi}{4}$

Akshit

3. $\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{325} + \cdots \infty$

Date : / /
Page No. :
MUNIM JI

Ans(3) $\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{325} \cdots \infty$

$S = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} \cdots \infty$

$= \tan^{-1}\left(\frac{1\times 2}{2n^2+2}\right) \Rightarrow \tan^{-1}\left(\frac{2}{1+4n^2-1}\right) \Rightarrow \tan^{-1}\left[\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right]$

$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$T_1 = \tan^{-1}3 - \tan^{-1}1$

$T_2 = \tan^{-1}5 - \tan^{-1}3$

$\vdots$

$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$S_n = \tan^{-1}(2n+1) - \tan^{-1}1$

$S_n = \tan^{-1}(2n+1) - \frac{\pi}{4}$

$S_\infty = \lim_{n\to\infty} \tan^{-1}(2n+1) - \frac{\pi}{4}$

$= \frac{\pi}{2} - \frac{\pi}{4} \Rightarrow \boxed{\frac{\pi}{4} = Ans}$

TAH -6 b
Aastha Gupta
Aligarh Up

TAH 7

2. $\tan^{-1}\frac{x}{1+(1\times 2)x^2}+\tan^{-1}\frac{x}{1+(2\times 3)x^2}+\cdots+\tan^{-1}\frac{x}{1+n(n+1)x^2}$

$$y = \tan^{-1}\left(\frac{x}{1+(1\times2)x^2}\right) + \tan^{-1}\left(\frac{x}{1+(2\cdot3)x^2}\right) - \tan^{-1}\left(\frac{x}{1+n(n+1)x^2}\right)$$

, $x>0$

$$T_n = \tan^{-1}\left(\frac{(n+1)x - nx}{1+(n+1)nx^2}\right)$$

$$y = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}\left(\tan^{-1}(n+1)x - \tan^{-1}(nx)\right)$$

$$y = \tan^{-1}2x - \tan^{-1}x$$
$$\tan^{-1}3x - \tan^{-1}2x$$
$$\tan^{-1}4x - \tan^{-1}3x$$
$$\vdots$$
$$\tan^{-1}(n+1)x - \tan^{-1}(nx)$$

$$\boxed{y = \tan^{-1}(n+1)x - \tan^{-1}x}$$

TAH 7
__hashtag_sonu__
from bihar siwan

(Tah-7)

Q. $\tan^{-1}\frac{x}{1+(1\times2)x^2} + \tan^{-1}\frac{x}{1+(2\times3)x^2} + \cdots + \tan^{-1}\frac{x}{1+n(n+1)x^2}$

$$T_r = \tan^{-1}\frac{x}{1+r(r+1)x^2}$$

observe!!
$x(r+1) - rx = x$

$$T_r = \tan^{-1}\left(\frac{(r+1)x - rx}{1+rx\cdot(r+1)x}\right)$$

$$T_r = \tan^{-1}(r+1)x - \tan^{-1}(rx)$$

Manish Kumar
बिहार

$$T_1 = \tan^{-1}(2x) - \tan^{-1}(x)$$
$$T_2 = \tan^{-1}(3x) - \tan^{-1}(2x)$$
$$T_3 = \tan^{-1}(4x) - \tan^{-1}(3x)$$
$$T_4 = \tan^{-1}(5x) - \tan^{-1}(4x)$$
$$\vdots$$
$$T_n = \tan^{-1}(n+1)x - \tan^{-1}(nx)$$

$$S_n = \tan^{-1}(n+1)x - \tan^{-1}(x)$$

$$S_n = \tan^{-1}\left(\frac{(n+1)x - x}{1+(n+1)x\cdot x}\right)$$

$$\boxed{S_n = \tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)}$$ Ans

Q. $\tan^{-1}\frac{x}{1+(1\times2)x^2} + \tan^{-1}\frac{x}{1+(2\times3)x^2} + \cdots + \tan^{-1}\frac{x}{1+n(n+1)x^2}$, $x>0$.

=> Let,

$$S = \tan^{-1}\frac{x}{1+(1\times2)x^2} + \tan^{-1}\frac{x}{1+(2\times3)x^2} + \cdots + \tan^{-1}\frac{x}{1+n(n+1)x^2}$$

$$S = \tan^{-1}\left(\frac{2x-1x}{1+(1\times2)x^2}\right) + \tan^{-1}\left(\frac{3x-2x}{1+(2\times3)x^2}\right) + \cdots + \tan^{-1}\left(\frac{(n+1)x-nx}{1+n(n+1)x^2}\right)$$

$$S = \tan^{-1}(2x) - \tan^{-1}(1x) + \tan^{-1}(3x) - \tan^{-1}(2x) + \cdots + \tan^{-1}(n+1)x - \tan^{-1}(nx)$$

$$S = \tan^{-1}(n+1)x - \tan^{-1}(x)$$

$$S = \tan^{-1}\left(\frac{nx+x-x}{1+(nx+x)x}\right)$$

$$S = \tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)$$ [Ans.]

TAN-7

Shubhajit Roy

TAH 8

P-6

(i) $\sin^{-1}x+\sin^{-1}y=\begin{cases}\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } x\geq 0; y\geq 0 \text{ and } x^2+y^2\leq 1\\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } x\geq 0; y\geq 0 \text{ and } x^2+y^2>1\end{cases}$

(ii) $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right), x>0; y>0$

QUESTION [JEE Mains 2019]

TAH 9

If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ then x is equal to:

A $\frac{\sqrt{145}}{12}$

B $\frac{\sqrt{145}}{10}$

C $\frac{\sqrt{146}}{12}$

D $\frac{\sqrt{145}}{11}$

21. $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}$ $\left(x>\frac{3}{4}\right)$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right)=\frac{\pi}{2}-\cos^{-1}\left(\frac{3}{4x}\right)$

$\cos^{-1}\left(\frac{2}{3x}\right)=\sin^{-1}\left(\frac{3}{4x}\right)$

$\cos^{-1}\left(\frac{2}{3x}\right)=\cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$

$\frac{2}{3x}=\frac{\sqrt{16x^2-9}}{4x}$ $(x\neq 0)$

TAH 9
__hashtag_sonu__
from bihar siwan

$\frac{8}{3}=\sqrt{16x^2-9}$

$\frac{64}{9}=16x^2-9$

$64+81=16\cdot 9x^2$

$x^2=\frac{145}{16\cdot 9}$, $x=\pm\sqrt{\frac{145}{16\cdot 9}}$ $\left(x>\frac{3}{4}\right)$

$\boxed{x=\frac{\sqrt{145}}{12}.}$

QUESTION [JEE Mains 2019]

TAH 10

The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is

(A) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

(B) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(C) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

(D) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

TAH 10
__hashtag_sonu__
from bihar siwan

(10). $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right) = y$

$y = \sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$

using. $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

$y = \sin^{-1}\left(\frac{12}{13}\sqrt{1-\left(\frac{3}{5}\right)^2} - \frac{3}{5}\sqrt{1-\left(\frac{12}{13}\right)^2}\right)$

$y = \sin^{-1}\left(\frac{12}{13}\times\frac{4}{5} - \frac{3}{5}\times\frac{5}{13}\right)$

$y = \sin^{-1}\left(\frac{33}{13\times 5}\right) = \sin^{-1}\left(\frac{33}{65}\right)$ $\left(\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}\right)$

$y = \frac{\pi}{2} - \cos^{-1}\left(\frac{33}{65}\right)$ $\left(\sqrt{(65)^2-(33)^2} = 56\right)$

$y = \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$

Page No.
Date: / /

Ques The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is :

Soln $\sin^{-1}\left(xy - \sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ $\quad (\because x, y > 0)$

$\sin^{-1}\left(\frac{12}{13} \times \frac{4}{5} - \frac{3}{5} \cdot \frac{5}{13}\right) = \sin^{-1}\left(\frac{33}{65}\right)$

$$\boxed{\text{Ans} = \frac{\pi}{2} - \cos^{-1}\left(\frac{33}{65}\right)}$$

TAH-10
Aastha Gupta
Aligarh Up

QUESTION [JEE Mains 2024 (4 April)]

TAH 11

Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that

$\cos^{-1} x - \sin^{-1} y = \alpha, \frac{-\pi}{2} \leq \alpha \leq \pi.$

Then, the minimum value of $x^2 + y^2 + 2xy \sin \alpha$ is

(A) 0

(B) -1

(C) 1/2

(D) -1/2

Ans. A

TAH - 10 (JEE)

If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$ where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$ for all x, y, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to —

Let, $A = \cos^{-1}x$, $B = \cos^{-1}\frac{y}{2}$, $A + B = \alpha$

$$\cos(A-B) = \cos\alpha$$

$$\Rightarrow \cos A\cos B + \sin A \sin B = \cos\alpha$$

$$\Rightarrow x \cdot \frac{y}{2} + \sqrt{1-x^2}\cdot\sqrt{1-\left(\frac{y}{2}\right)^2} = \cos\alpha \quad \text{S.B.S}$$

$$\Rightarrow \frac{x^2y^2}{4} + \cos^2\alpha - xy\cos\alpha = 1 - \frac{y^2}{4} - x^2 + \frac{x^2y^2}{4}$$

$$\Rightarrow 4\cos^2\alpha - 4xy\cos\alpha = 4 - y^2 - 4x^2$$

$$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4(1-\cos^2\alpha)$$

$$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha \rightsquigarrow \text{Ans}$$

Risha From WB

QUESTION

TAH 12a

Consider the adjacent graph of $y = f(x)$ Find the following:

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$

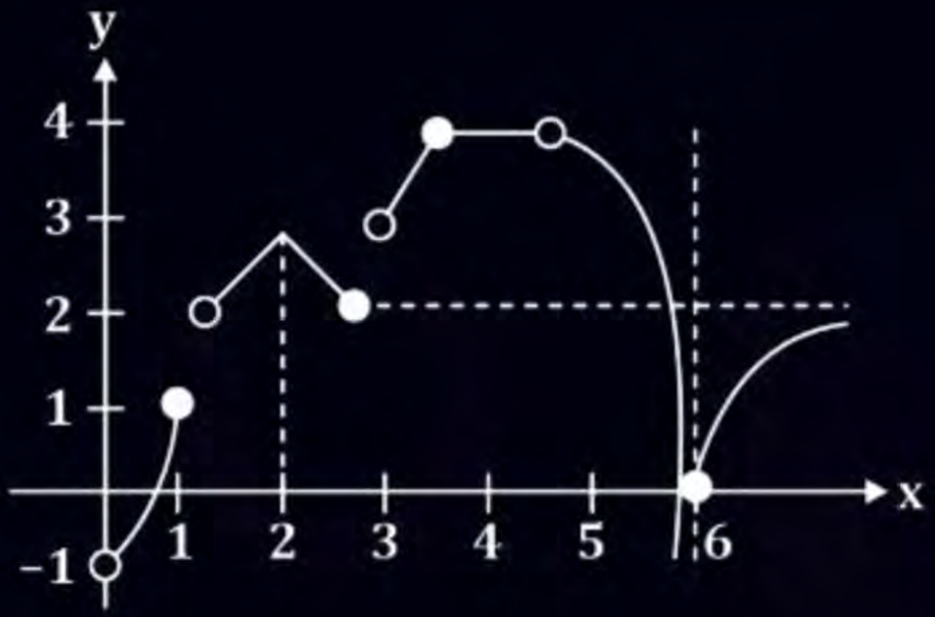
(c) $\lim_{x\to 1^-} f(x)$ (d) $\lim_{x\to 1^+} f(x)$

(e) $\lim_{x\to 2^-} f(x)$ (f) $\lim_{x\to 2^+} f(x)$

(g) $\lim_{x\to 3^-} f(x)$ (h) $\lim_{x\to 3^+} f(x)$

(i) $\lim_{x\to 4^-} f(x)$ (j) $\lim_{x\to 4^+} f(x)$

(k) $\lim_{x\to \infty} f(x)$ (l) $\lim_{x\to 6^-} f(x)$



Ques (a) $\lim_{x \to 0^-} f(x) = \text{NO}$

(b) $\lim_{x \to 0^+} f(x) = -1$

(c) $\lim_{x \to 0} f(x) = -1$

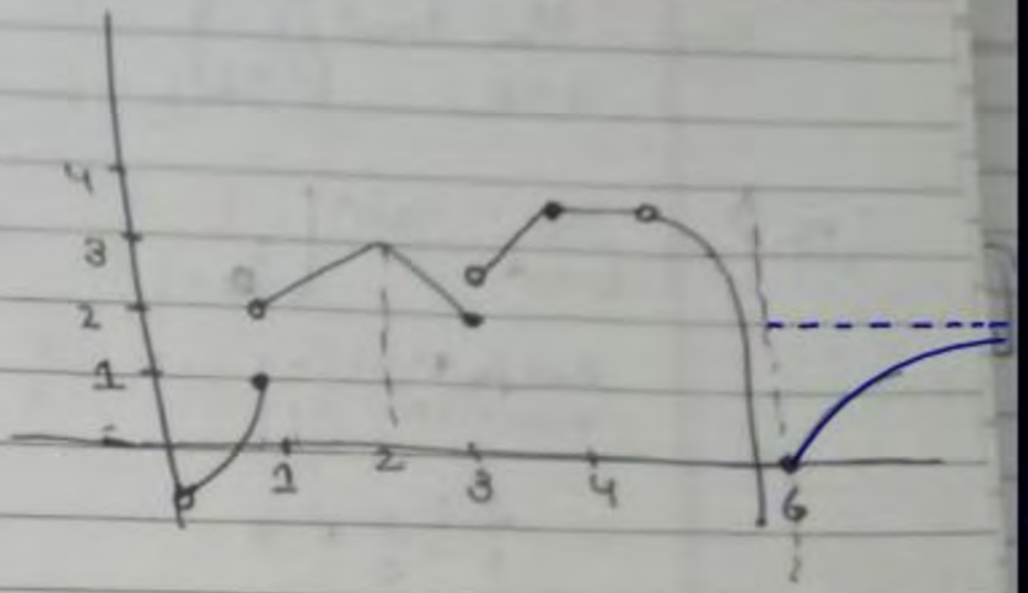
(d) $\lim_{x \to 1^-} f(x) = 1$

(e) $\lim_{x \to 1^+} f(x) = 2$

(f) $\lim_{x \to 2^-} = 3$

(g) $\lim_{x \to 2^+} = 3$

(h) $\lim_{x \to 3^-} f(x) = 2$



(i) $\lim_{x \to 4^-} f(x) = 4$

(j) $\lim_{x \to 4^+} f(x) = 4$

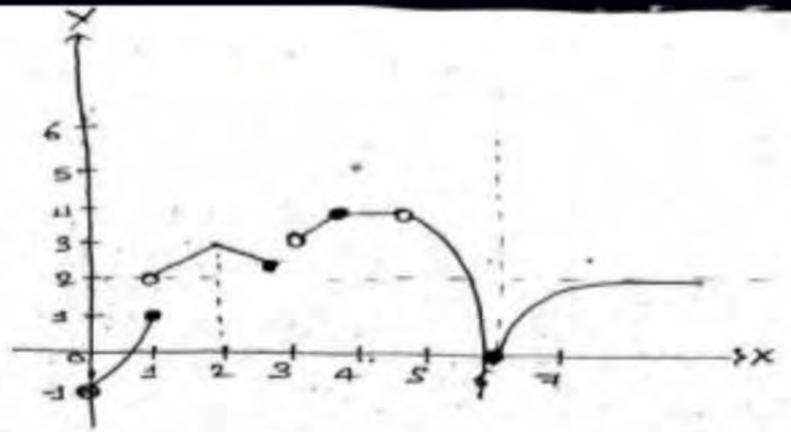
(k) $\lim_{x \to \infty} f(x) = 2$

(l) $\lim_{x \to 6^-} f(x) = \text{DNE}$

TAH-12a
Aastha Gupta
Aligarh Up

Ques

MATH 18(B)



(a) $\lim_{x \to 0} f(x) = \lim_{h \to 0} f(0-h)$ = NOT DEFINED

(b) $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} (0+h) = -1$

(c) $\lim_{x \to 1} f(x) = \lim_{h \to 0} (1-h) = 1$

(d) $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} (1+h) = 2$

(e) $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} (2-h) = 3$

(f) $\lim_{x \to 2^+} f(x) = \lim_{h \to 0} (2+h) = 3$

(g) $\lim_{x \to 5} f(x) = \lim_{h \to 0} (5-h) = 2$

(h) $= \lim_{x \to 3^+} f(x) = \lim_{h \to 0} (3+h) = 3$

(i) $= \lim_{x \to 4^-} f(x) = \lim_{h \to 0} (4-h) = 4$

(j) $\lim_{x \to 4^+} f(x) = \lim_{h \to 0} (4+h) = 4$

(k) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f(x) = 2$

(l) $= \lim_{x \to 6} f(x) = \lim_{h \to 0} (6+h) = -\infty$

Sanjeev

QUESTION

TAH 12b

Determine the following limits.

(a) $\lim_{x\to -1^+} f(x)$ (b) $\lim_{x\to -1^-} f(x)$

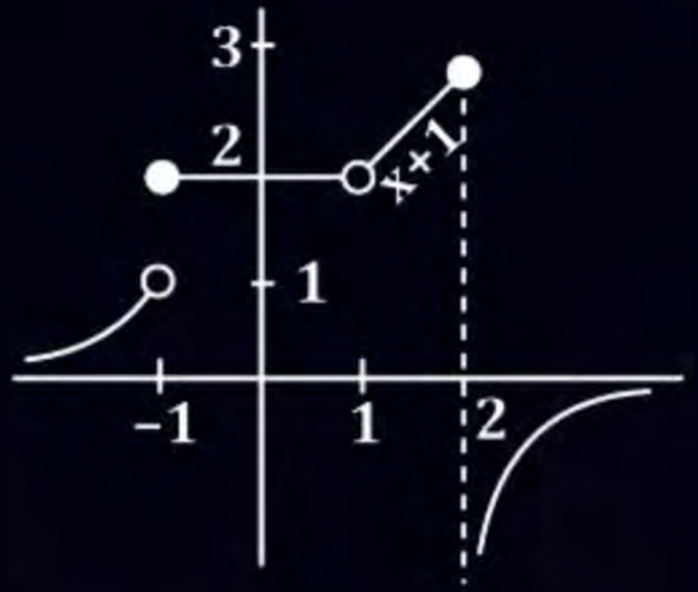
(c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 1^+} f(x)$

(e) $\lim_{x\to 1^-} f(x)$ (f) $\lim_{x\to 1} f(x)$

(g) $\lim_{x\to 2^+} f(x)$ (h) $\lim_{x\to 2^-} f(x)$

(i) $\lim_{x\to 2} f(x)$ (j) $\lim_{x\to -3} f(x)$

(k) $\lim_{x\to 5} f(x)$ (l) $\lim_{x\to 1.5} f(x)$



TAH — 12(b)

a) $\lim_{x \to -1^+} f(x) =$ 2 b) $\lim_{x \to -1^-} f(x) = 1$

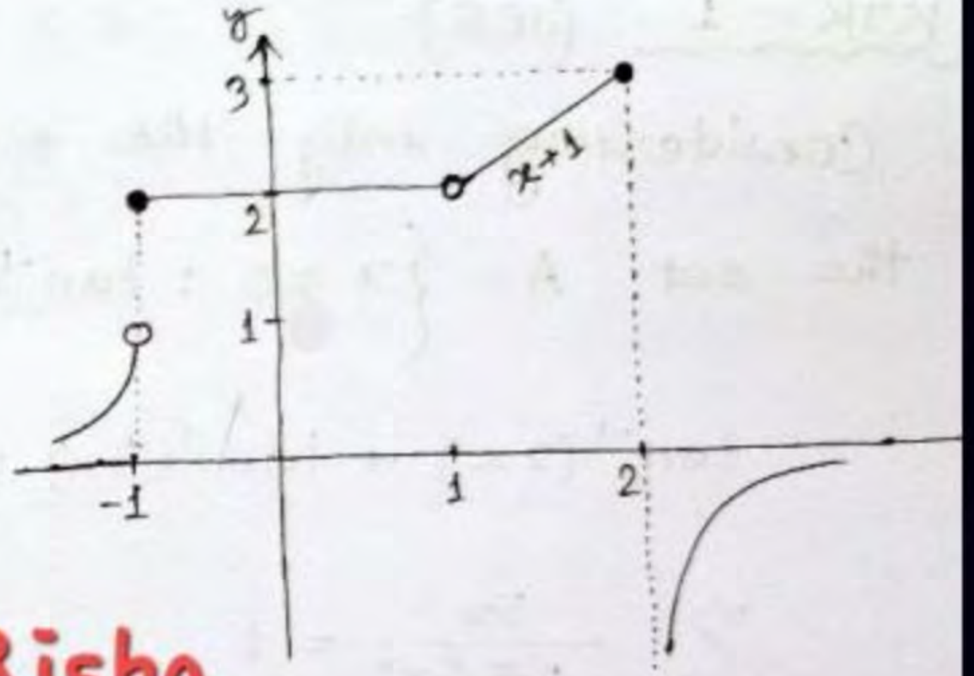
c) $\lim_{x \to -1} f(x) =$ D.N.E

d) $\lim_{x \to 1^+} f(x) = 2$ e) $\lim_{x \to 1^-} f(x) = 2$

f) $\lim_{x \to 1} f(x) = 2$

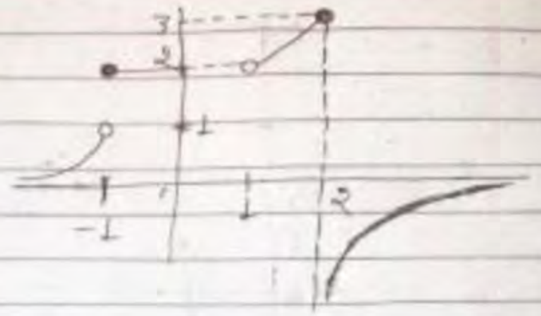
g) $\lim_{x \to 2^+} f(x) = \infty$ h) $\lim_{x \to 2^-} f(x) = 3$ i) $\lim_{x \to 2} f(x) =$ D.N.E

j) $\lim_{x \to -3} f(x) =$ N.D k) $\lim_{x \to 5} f(x) =$ N.D l) $\lim_{x \to 1.5} f(x) = 2.5$



Risha
From WB

TAH-12(a)



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to -1^-} f(x) = \infty$

(c) $\lim_{x \to -1} f(x) = \text{DNE}$

(d) $\lim_{x \to 1^+} f(x) = 2$

(e) $\lim_{x \to 1^-} f(x) = 2$

(f) $\lim_{x \to 1} f(x) = 2$

(g) $\lim_{x \to 2^+} f(x) = \infty$

(h) $\lim_{x \to 2^-} f(x) = 5$

(i) $\lim_{x \to 2} f(x) = \text{DNE}$

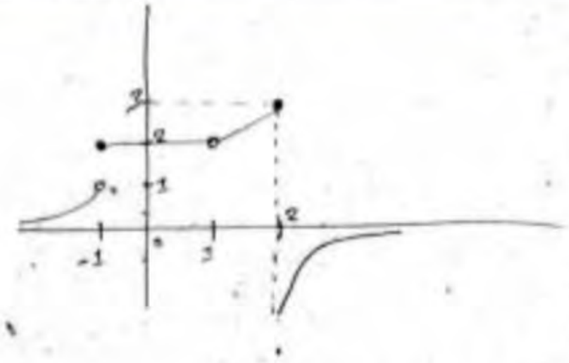
(j) $\lim_{x \to 2^-} f(x) = 0$

(k) $\lim_{x \to 5} f(x) = 0$

(l) $\lim_{x \to 2.5} f(x) = 2.5$

Name Sanjeet kumar
From - Varanasi (U.P)

TAH 12(b)



(a) $\lim_{x \to 1^+} f(x) = 2$

(b) $\lim_{x \to -1^-} f(x) = +1$

(c) $\lim_{x \to -1} f(x) = \text{DNE}$

(d) $\lim_{x \to 5^+} f(x) = 2$

(e) $\lim_{x \to 1^-} f(x) = 2$

(f) $\lim_{x \to 1} f(x) = 2$

(g) $\lim_{x \to 2^+} f(x) = -\infty$

(h) $\lim_{x \to 2^-} f(x) = 3$

(i) $\lim_{x \to 2} f(x) = \text{DNE}$

(j) $\lim_{x \to 3} f(x) = \text{N.D.}$

(k) $\lim_{x \to 5} f(x) = \text{N.D.}$

(l) $\lim_{x \to 2.5} f(x) =$

Sanjeev

QUESTION

TAH 12c

Which of the following statements about the function y = f(x) graphed here are true, and which are false?

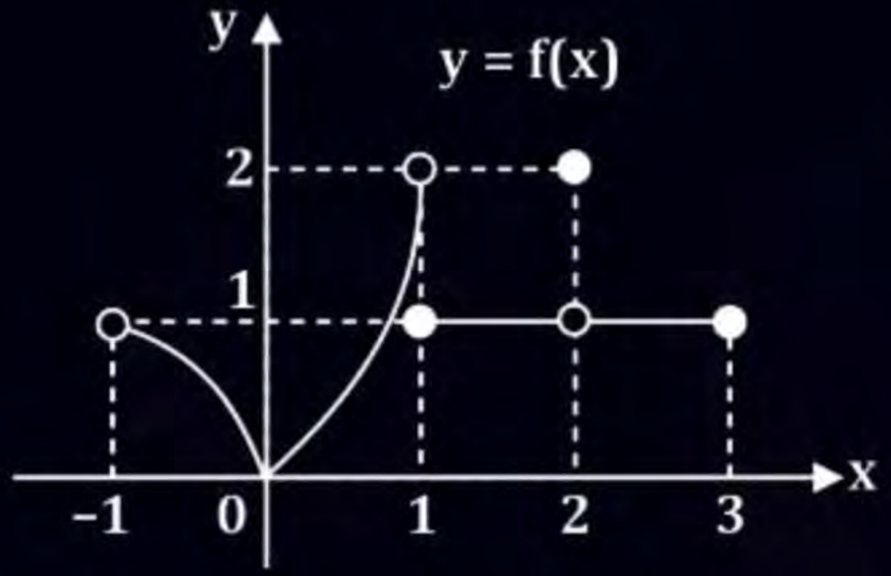
(f) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

(g) $\lim_{x\to c} f(x)$ exists at every $c \in (-1, 1)$

(h) $\lim_{x\to c} f(x)$ does not exist.for some cε(-1, 1)

(i) $\lim_{x\to 1^-} f(x) = 0$

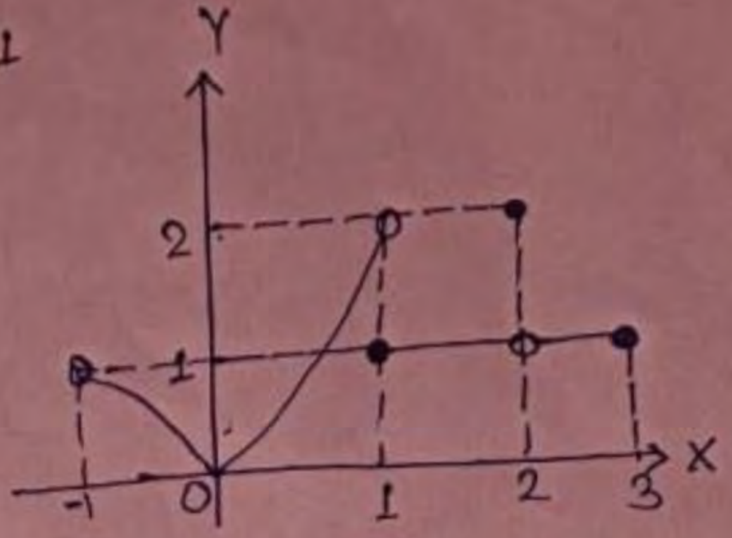
(j) $\lim_{x\to 3^+} f(x)$ does not exist.



Tab-12C

(a) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 1$

$=$ (True)



(b) $\lim_{x \to c} f(x)$ exist at every $c \in (-1, 1)$

$\Rightarrow$ (True)

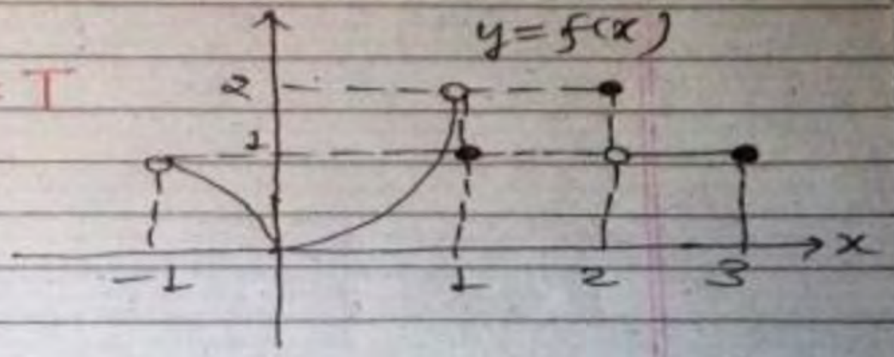
(c) $\lim_{x \to c} f(x)$ does not exist (False)

(d) $\lim_{x \to 1^-} f(x) = 0 \rightarrow$ (True)

(e) $\lim_{x \to 3^+} f(x)$ does not exist. $\rightarrow$ (True)

ROHINI SOLANKI
FROM - U.P.

TAH-12 C



(f) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ = T

(g) $\lim_{x \to c} f(x)$ exists at every $c \in (-1, 1)$ = T

(h) $\lim_{x \to c} f(x)$ does not exist = F

(i) $\lim_{x \to 1^-} f(x) = 0$ = F

(j) $\lim_{x \to 3^+} f(x)$ does not exist = T

Name Sanjeet Kumar
From – Varanasi (U.P)

(Solution to KTK)

QUESTION [JEE Mains 2019]

KTK 1

Considering only the principal values of inverse functions, the set

$$A = \left\{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\right\}$$

(A) is an empty set

(B) contains more than two elements

(C) contains two elements

(D) is a singleton

M-19 KTK-1 $A=\{x\geq 0: \tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}\}$

$\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\alpha \quad\quad \beta$

$\tan\alpha = 2x \quad\quad \tan\beta = 3x$

$\alpha+\beta=\frac{\pi}{4}$

$\tan(\alpha+\beta)=\tan\frac{\pi}{4}$

$\frac{2x+3x}{1-6x^2}=1$

$5x=1-6x^2$

$6x^2+5x-1=0$

$6x^2+6x-x-1=0$

$6x(x+1)-1(x+1)=0$

$(x+1)(6x-1)=0$

$x=\frac{1}{6}, -1$ ✗ b'coz $x\geq 0$

$x=\frac{1}{6}$ Singleton Ans

ktk1
kshitij from up

QUESTION

KTK 2

Let $f(x) = \begin{cases} x^2 - 4 & \text{if } |x| \leq 3 \\ 5 \operatorname{sgn} |x-3| & \text{if } |x| > 3 \end{cases}$ and $g(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ for all $x \in R$, then which of the following is wrong ?
(where sgn (x) denotes signum function of x)

A f(x) is an even function

B gof(x) is an even function

C g(x) is an odd function

D fof(x) is an odd function

QUESTION

KTK 3

Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is :

A $\left(-\infty, \frac{1}{4}\right]$

B $\left[-\frac{1}{4}, \infty\right)$

C $(-1/3, \infty)$

D $\left(-\infty, \frac{1}{3}\right]$

Ans : B

$f(x) = \cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$

ktk3
kshitij from up

$$-1 \le \frac{x^2-4x+2}{x^2+3} \le 1$$

$$\frac{-x^2-4x+2}{x^2+3} \ge -1$$

$$\frac{x^2-4x+2+x^2+3}{x^2+3} \ge 0$$

(always +ve)

$$2x^2-4x+5 \ge 0$$

$D < 0$

$a > 0$.

always true

$$\frac{x^2-4x+2}{x^2+3} \le 1$$

$$\frac{x^2-4x+2-x^2-3}{x^2+3} \le 0$$

(always +ve)

$$-4x-1 \le 0$$

$$x \ge \frac{-1}{4}$$

$x \in \left[\frac{-1}{4}, \infty\right)$.

$x \in \left[\frac{-1}{4}, \infty\right)$ Ans

KTK-3

Considering only the principal values of the ITF, the domain of the fn $f(x) = \cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$ is ——

$$-1 \leq \frac{x^2-4x+2}{x^2+3} \leq 1$$

$x^2-4x+2 \geq -x^2-3$

$\Downarrow$

$2x^2-4x+5 \geq 0$

$\Downarrow$

$D<0$ & $a>0$

$\Downarrow$

$x \in R$

$x^2-4x+2 \leq x^2+3$

$\Downarrow$

$4x+1 \geq 0$

$\Downarrow$

$x \geq -\frac{1}{4}$

$x \in \left[-\frac{1}{4}, \infty\right)$ ⇝ Ans

Risha From WB

Galaxy F54 5G

QUESTION

KTK 4

Determine the inverse function and its domain of definition if

$$f(x) = \begin{cases} x; & -\infty < x < 1 \\ x^2; & 1 \le x \le 4 \\ 2^x; & 4 < x < \infty \end{cases}.$$

Ans. $f^{-1}(x) = \begin{cases} x; & -\infty < x < 1 \\ \sqrt{x}; & 1 \le x \le 16 \\ \log_2 x; & 16 < x < \infty \end{cases}$

KTK-9) determine the inverse function & domain

$$f(x) = \begin{cases} x & x < 1 \\ x^2 & 1 \le x \le 4 \\ 2^x & 4 < x < \infty \end{cases}$$

$$f^{-1}(x) = \begin{cases} x = y & y \in (-\infty, 1) \\ x^2 = y & y \in [1, 16] \\ 2^x = y & y \in (16, \infty) \end{cases}$$

$$f^{-1}(x) = \begin{cases} x & x \in (-\infty, 1) \\ \sqrt{x} & x \in [1, 16] \\ \log_2 x & x \in (16, \infty) \end{cases}$$

$$f^{-1}(x) = \begin{cases} x & -\infty < x < 1 \\ \sqrt{x} & 1 \le x \le 16 \\ \log_2 x & 16 < x < \infty \end{cases}$$

KTK-4

Determine the ITF & its domain of definition if —

$$f(x) = \begin{cases} x & , -\infty < x < 1 \\ x^2 & , 1 \le x \le 4 \\ 2^x & , 4 < x < \infty \end{cases}$$

$$\therefore f^{-1}(x) = \begin{cases} x & , -\infty < x < 1 \\ \sqrt{x} & , 1 \le x \le 16 \\ \log_2 x & , 16 < x < \infty \end{cases}$$

$y = x \quad , -\infty < y < 1$

$f^{-1}(x) = x \quad , -\infty < x < 1$

$y = x^2 \quad , -1 \le x \le 4$

$\Rightarrow x = \sqrt{y} \quad , 1 \le y \le 16$

$f^{-1}(x) = \sqrt{x} \quad , 1 \le x \le 16$

$y = 2^x \quad , 4 < x < \infty$

$\Rightarrow x = \log_2 y \quad , 16 < y < \infty$

$f^{-1}(x) = \log_2 x \quad , 16 < x < \infty$

Risha
From WB

Galaxy F54 5G
28 July 2024 13:41

QUESTION

KTK 5

If $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is given as

A $\frac{1}{2}\left[1 - \sqrt{1 + 4\log_2 x}\right]$

B $\frac{1}{2}\left[1 + \sqrt{1 + 4\log_2 x}\right]$

C $\left[1 - \sqrt{1 + 4\log_2 x}\right]$

D $\left[1 - \sqrt{1 + 4\log_2 x}\right]$

Ans. B

KTK 5

Q if $f:[1,\infty) \to [1,\infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is given as.

Sol^n

$$f(x) = 2^{x(x-1)}$$

$$y = 2^{x(x-1)}$$

$$\log_2 y = x(x-1)$$

$$\left(x^2 - \frac{1}{2}\right)^2 = \frac{1}{4} + \log_2 y$$

$$x - \frac{1}{2} = \pm\frac{\sqrt{1+4\log_2 y}}{2}$$

$\because$ we take +ve only bcoz $x \in [1,\infty)$

$$x = \frac{1}{2} + \frac{1}{2}\sqrt{1+4\log_2 y}$$

$$f^{-1}(x) = \frac{1}{2}\left(1 + \sqrt{4+\log_2 x}\right)$$

Deepak from Batala punjab

RTKP) If $f:[1,\infty) \to [1,\infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ –

$y = 2^{x(x-1)} \longrightarrow$ as $x \to \infty$ $y \to \infty$

↓ Taking log base 2 both sides

$\log_2 y = x(x-1)\log_2 2 \Rightarrow x(x-1) = x^2 - x$

$= x^2 - x - \log_2 y = 0 \Rightarrow$ quad in x.

$x = \dfrac{1 \pm \sqrt{1+4\log_2 y}}{2} \rightarrow \dfrac{1+\sqrt{1+4\log_2 y}}{2} \to y \to \infty,\ x \to \infty$ ✓

$\hookrightarrow \dfrac{1-\sqrt{1+4\log_2 y}}{2} \to y \to -\infty,\ x \to -\infty$

↳ Rejected

$x = \dfrac{1+\sqrt{1+4\log_2 y}}{2}$

$y = f^{-1}(x) = \dfrac{1+\sqrt{1+4\log_2 x}}{2}$ Ans

QUESTION [JEE Mains 2024 (5 April)]

KTK 6

Let $f, g : R \to R$ be defined as: $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x + 1, & x \leq 0. \end{cases}$

Then the function $f(g(x))$ is

(A) neither one-one nor onto.

(B) one-one but not onto.

(C) both one-one and onto.

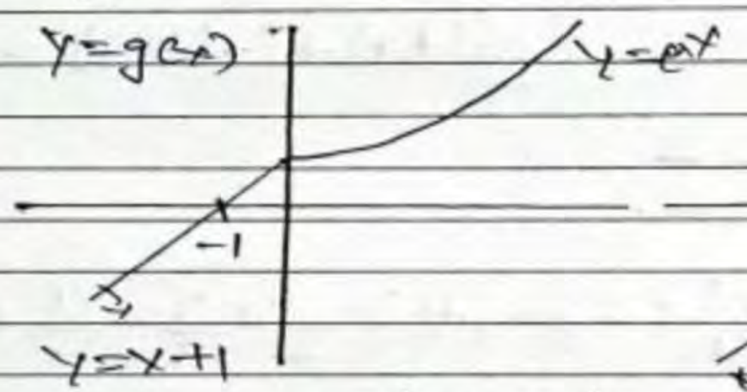
(D) onto but not one-one.

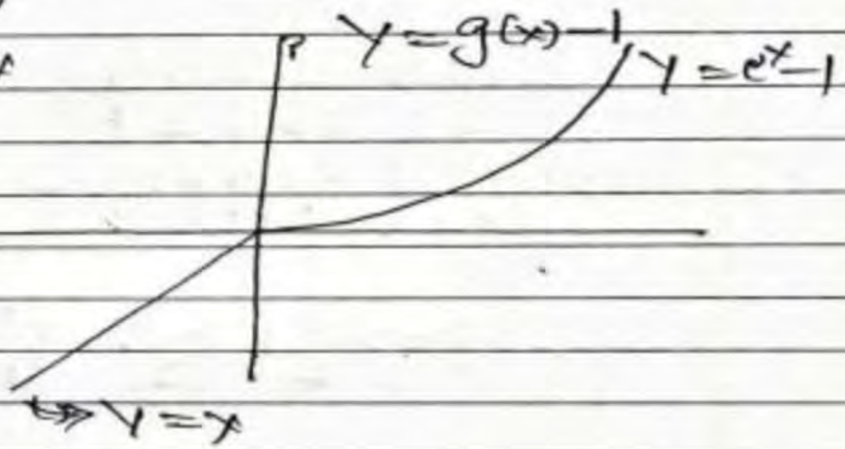
Ans. A

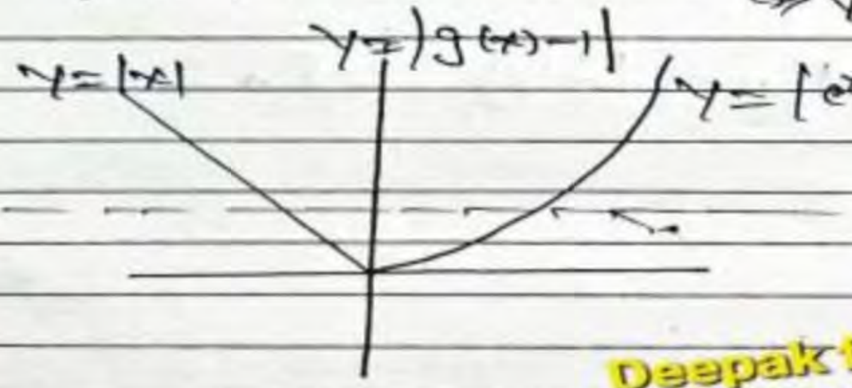
KTK 6

Q Let $f,g: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = |x-1|$ & $g(x) = \begin{cases} e^x & x \geq 0 \\ x+1 & x \leq 0 \end{cases}$ then for $f(g(x))$ is

Soln $f(g(x)) = |g(x)-1|$







f is many one &
Range $= [0, \infty) \neq \mathbb{R}$.

Deepak from Batala punjab

QUESTION [JEE Mains 2021]

KTK 7

If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \pi/4$, then the value of $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^8+b^8}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots\ldots$ is:

A $\log_e 2$

B e^{2-1}

C e

D $\log_e(e/2)$

KTK 7

____hashtag_sonu____

from Bihar siwan

KTK(9). $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{a+b}{1-ab}\right) = \frac{\pi}{4}$

$a+b = 1-ab$

$\boxed{a+b+ab=1}$

$\Rightarrow (a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) \cdots$

$\Rightarrow \left(a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} \cdots\right) + \left(b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} \cdots\right)$

$\log_e(1+a) + \log_e(1+b)$

$\log_e(1+a) + \log_e(1+b)$

$\log_e^{(1+a)(1+b)}$

$\log_e(a+ab+b+1)$

$\log_e^{(1+1)}$

$\boxed{\log_e^2}$ Ans.

QUESTION

KTK 8

Find the simplest value of

(i) $f(x) = \arccos x + \arccos\left(\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right), x \in \left(\frac{1}{2}, 1\right)$

(ii) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \in R - \{0\}$

KTK-8

find the simplest value of-

(i) $f(x) = \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right)$

Observe!! $\cos^{-1}\left[\frac{1}{2}\cdot x + \sqrt{\frac{3(1-x^2)}{4}}\right]$

$= \cos^{-1}\left[x\cdot\frac{1}{2} + \sqrt{1-\frac{1}{4}}\cdot\sqrt{1-x^2}\right]$

$\Rightarrow -\cos^{-1}x + \cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \cos^{-1}\left(\frac{1}{2}\right) - \cos^{-1}x$

$\Rightarrow \frac{\pi}{3} - \cos^{-1}(x)$

$\Rightarrow \cos^{-1}x + \frac{\pi}{3} - \cos^{-1}x \Rightarrow \frac{\pi}{3}$ Ans

(ii) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \in R - [0]$

put $x = \tan\theta$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

$\Rightarrow \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$\tan^{-1}\left[\tan\left(\frac{\theta}{2}\right)\right]$

$\Rightarrow \frac{\theta}{2} \Rightarrow \frac{1}{2}\tan^{-1}x$ Ans

ROHINI SOLANKI
FROM - U.P.

KTK8 (i) $f(x) = \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right)$, $x \in \left(\frac{1}{2}, 1\right)$

$= \cos^{-1}x + \cos^{-1}\left(x\cdot\frac{1}{2} + \sqrt{1-\frac{1}{4}}\times\sqrt{1-x^2}\right)$

(x = $\cos\theta$, $\frac{1}{2}$ = $\cos\alpha$)

$\theta = \cos^{-1}x + (\cos^{-1}\frac{1}{2} - \cos^{-1}x) = \cos^{-1}\left(\frac{1}{2}\right)$ Ans

(ii) $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \in R - \{0\}$

Put $x = \tan\theta$, & $y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\frac{1}{\cos\theta} - 1}{\sin\theta/\cos\theta}\right)$

$y \Rightarrow \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$ $\because \tan\theta \Rightarrow \frac{1-\cos 2\theta}{\sin 2\theta}$

& $y = \frac{\theta}{2}$ & $y = \frac{\tan^{-1}x}{2}$ Ans

kamran Ashraf
Muzaffarpur

QUESTION

KTK 9

(i) If $\tan^{-1} 4 + \tan^{-1} 5 = \cot^{-1}(\lambda)$, then find λ.

(ii) Which is greater $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$ or $\cot^{-1}(-1)$

KTK -9

(1) If $\tan^{-1}4 + \tan^{-1}5 = \cot^{-1}(\lambda)$ then λ will be

$\pi + \tan^{-1}\left(\frac{9}{-19}\right) = \cot^{-1}(\lambda)$

$\Rightarrow \pi + \cot^{-1}\left(\frac{19}{-9}\right) - \pi = \cot^{-1}(\lambda)$

$\Rightarrow \boxed{\lambda = \frac{19}{9}}$ Ans.

(2) Which is greater $\cos^{-1}\frac{7}{5} + \cos^{-1}\frac{3}{5}$ or $\cot^{-1}(-1)$.

$\Rightarrow \cos^{-1}\left[\frac{7}{25}\times\frac{3}{5} - \sqrt{1-\frac{49}{625}}\sqrt{1-\frac{9}{25}}\right]$

$\cos^{-1}\left[\frac{21}{125} - \frac{24}{25}\times\frac{4}{5}\right]$

$\Rightarrow \cos^{-1}\left[\frac{21-96}{125}\right]$

$\Rightarrow \cos^{-1}\left[-\frac{75}{125}\right]$

$\Rightarrow \boxed{\pi - \cos^{-1}\left(\frac{3}{5}\right)}$

ROHINI SOLANKI FROM - U.P.

Now, $\cot^{-1}(-1) = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

then, $\cot^{-1}(-1)$ is greater than $\left[\cos^{-1}\frac{7}{5} + \cos^{-1}\frac{3}{5}\right]$

$\Rightarrow \cot^{-1}(-1) > \left[\cos^{-1}\frac{7}{5} + \cos^{-1}\frac{3}{5}\right]$ Ans.

KTK-9

$\tan^{-1}(4) + \tan^{-1}(5) = \cot^{-1}(\lambda)$

$\tan^{-1}\frac{(-9)}{1-20} = \tan^{-1}\left(\frac{1}{\lambda}\right)$

So $\lambda \rightarrow \frac{-19}{9}$ Ans

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Muzaffarpur

(ii) $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$

$\tan^{-1}\frac{24}{7} + \tan^{-1}\frac{4}{3} \rightarrow \tan^{-1}\frac{28}{1-21}$

$\rightarrow \tan^{-1}\frac{28}{-20} \rightarrow \cot^{-1}\left(\frac{-20}{28}\right) \gg \cot^{-1}(-1)$ Ans

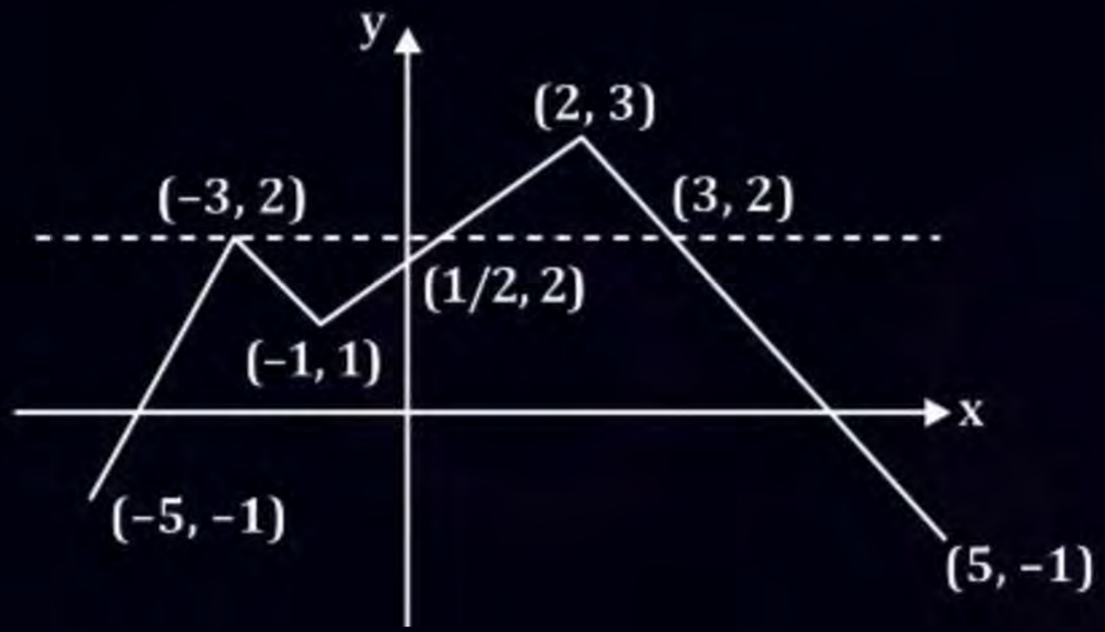
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(Solution to RPP)

QUESTION

(RPP 1)

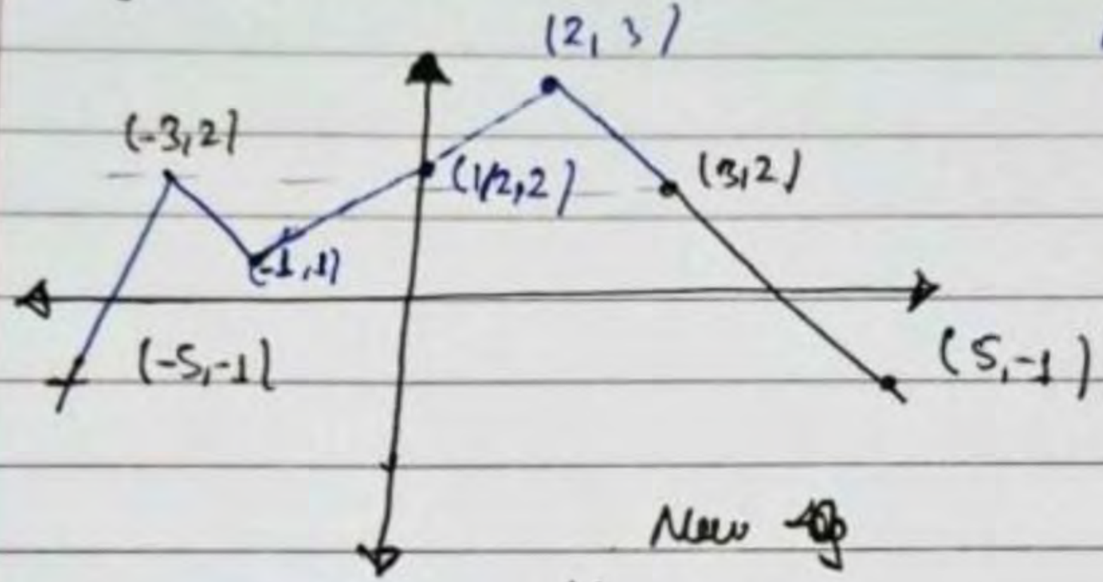
The graph of $y = f(x)$ is shown, then number of solutions of the equation $f(f(x)) = 2$ is equal to



Ans. 3

RPP1

$y = f(x)$



Now $f(f(x)) = 2$

Let $f(x) \to z$

& $f(z) = 2$

There are 3 Sol^n

$\frac{1}{2}, -3, 3$

Now $z \Rightarrow \frac{1}{2}, -3 \& 3$

If

$f(x) = \frac{1}{2}$ 2 Sol^n

$f(x) = -3$ 0 Sol^n

$f(x) = 3$ 1 Sol^n

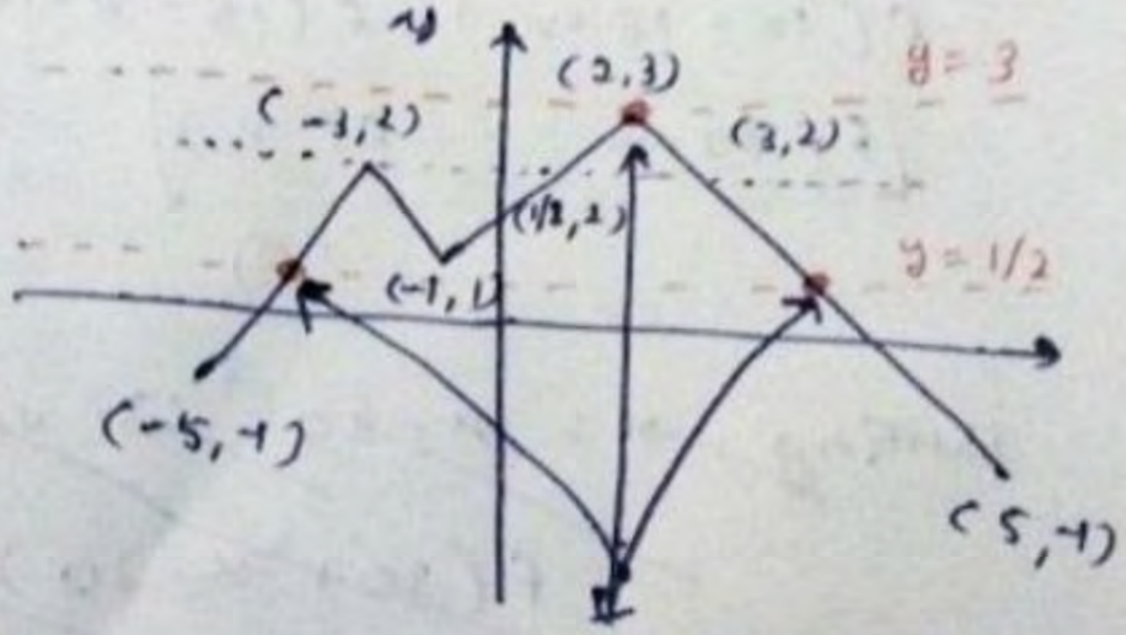
& $f(f(x))$ has 3 Sol^n Ans

kamran Ashraf
Muzaffarpur

Rpp1

Ayush
From
Jharkhand 💕

Here, $f(x) = 2$ at $x = -3, 1/2$ and 3



3 intersection points

$\therefore f(f(x)) = 2$ has 3 real soln Ans

QUESTION

(RPP 2)

Let f be a real valued function such that for any real x,
$f(15 + x) = f(15 - x)$ and $f(30 + x) = -f(30 - x)$.
Then which of the following statement is true?

A f is odd

B f is even

C f is periodic

D f is not periodic

Ans. A, C

Rpp2

$f(15+x) = f(15-x)$ and $f(30+x) = -f(30-x)$

Put $x = 15-x$

$f(15+15-x) = f(15-15+x)$

$\boxed{f(30-x) = f(x)}$ ①

Now Put $x = x+15$

$f(15+15+x) = f(15-x-15)$

$\boxed{f(30+x) = f(-x)}$ ②

$\Rightarrow \boxed{f(-x) = -f(x)}$

Proved

↳ From ① and ②

$\therefore$ f is odd (A) ✓

$\because f(x) = -f(x)$

Now, we know,

$f(30-x) = f(x)$ — from (1)

$\Rightarrow -f(30+x) = f(x)$ — From given eqⁿ

put $x \to x+30$, $-f(30+x+30) = f(x+30)$

$-f(x+60) = f(-x)$ — From (2)

$\neq f(x+60) = \neq f(x)$ (proved above)

$\Rightarrow \boxed{f(x) = f(x+60)}$

$\therefore$ f is periodic with period 60 Ans

(C)

(A) and (C) are Answers !! Ans

Ayush From Jharkhand 💕

QUESTION

(RPP 3)

In a kite ABCD, $AB = AD$ and $CB = CD$. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$ can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

A (1, 2, 1)

B (2, 1, 1)

C (4, 3, 2)

D (1, 1, 2)

Ans. D


THANK
YOU

PRAYAS

JEE 2025

Lecture-03 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Method to Evaluate Limits
2. Trigonometric Limits
3.
4.

Recap *of previous lecture*

State True or False.

1. $\lim_{x \to a} f(x) \Rightarrow x \neq a$ (T)

$$\lim_{x \to -2} \frac{(x+2)(x-1)}{(x-3)(x+2)} = \frac{-3}{-5} = \frac{3}{5}$$

(nonzero, nonzero)

$\Downarrow$

$x \neq -2$

2. If $f(2) = 3$ then $\lim_{x \to 2} f(x) = 3$ (False)

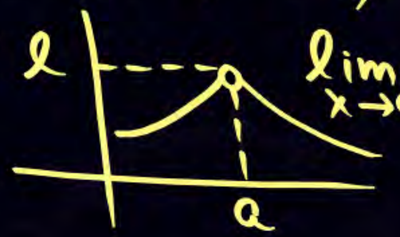
Ex:



$\lim_{x \to 2} f(x) = 2 \neq f(2)$

3. If a function f is not defined at x = a then $\lim_{x \to a} f(x)$ can not exist. (False)

Ex:

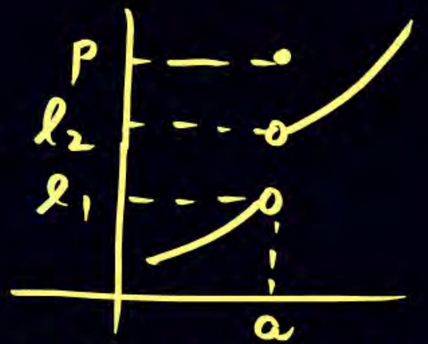


$\lim_{x \to a} f(x) = l$

4. If $\lim_{x \to a} f(x) = 3$ then $f(a) = 3$ (False)

Recap *of previous lecture*

5. If $\lim_{x \to a} f(x)$ does not exist then f(a) is not defined. (False)



$\lim_{x \to a} f(x)$ D.N.E but $f(a) = P$

6. $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$ (False)

True only if $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = M$ } finite reals

7. $\lim_{x \to 0} \sqrt{x} = \lim_{x \to 0^+} \sqrt{x}$ (True)

$f(x) = \sqrt{x}$ is not defined in left nbd of $x = 0 \Rightarrow \lim_{x \to 0} \sqrt{x} = \lim_{x \to 0^+} \sqrt{x} = 0$.

8. If a function is defined only in left NBD of x = a then $\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x)$

Recap *of previous lecture*

9. For talking about limit at x = a function should be defined in atleast one NBD of x = a. (True)

10. If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (False)

Valid only if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ → L, M are finite Reals & $M \neq 0$.

Homework Discussion

Home Challenge -05

$x^2+5|x|+3$
$\geq 0 \quad \geq 0$

The product of all real values of x satisfying the equation

$$\cos^{-1}\left(\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)\right)+\cot\left(\cot^{-1}\left(\frac{2}{9|x|}-2\right)\right)=0 \text{ is}$$

A 9

B −9

C 3

D −3

$$\cos^{-1}\left(\cos\left(\frac{2(x^2+5|x|+3)-2}{x^2+5|x|+3}\right)\right)+\frac{2}{9|x|}-2=0$$

$$\cos^{-1}\left(\cos\left(2-\frac{2}{x^2+5|x|+3}\right)\right)+\frac{2}{9|x|}-2=0$$

$\cos^{-1}(\cos x)=x$
$0\leq x\leq\pi$

$[3,\infty)$

$(0,2/3]$

$\left[\frac{4}{3},2\right)$

$$2-\frac{2}{x^2+5|x|+3}+\frac{2}{9|x|}-2=0$$

$$\frac{2}{9|x|} = \frac{2}{x^2+5|x|+3}$$

$$x^2+5|x|+3 = 9|x|$$

$$x^2-4|x|+3=0 \Rightarrow |x|^2-4|x|+3=0$$

$$(|x|-1)(|x|-3)=0$$

$$|x| = 1, 3$$

$$x = \pm 1, \pm 3.$$

QUESTION

KTK 2

Let $f(x) = \begin{cases} x^2 - 4 & \text{if } |x| \le 3 \\ 5 \operatorname{sgn} |x-3| & \text{if } |x| > 3 \end{cases}$ and $g(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ for all $x \in R$,

then which of the following is wrong ?
(where sgn (x) denotes signum function of x)

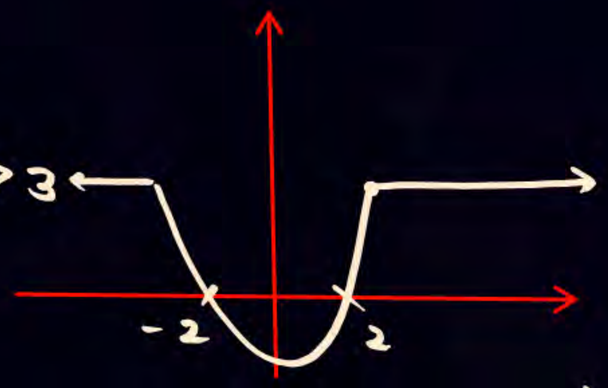
A. f(x) is an even function

B. gof(x) is an even function

C. g(x) is an odd function

D. fof(x) is an odd function

$f(x) = \begin{cases} x^2-4 & -3 \le x \le 3 \\ 5 & x<-3 \text{ or } x>3 \end{cases}$



f is even fn.

$g(x) = 2\tan^{-1}e^x - \pi/2$

$g(-x) = 2\tan^{-1}e^{-x} - \pi/2$

$= 2(\pi/2 - \cot^{-1}e^x) - \pi/2 = \pi/2 - 2\cot^{-1}\frac{1}{e^x} = \pi/2 - 2\tan^{-1}e^x$

$= -g(x)$ ⇒ odd fn

$$gof(-x) = g(f(-x)) = g(f(x)) = gof(x)$$ (Even; Even; gof is even)

$$fof(-x) = f(f(-x))$$ (Even)
$$= f(f(x))$$
$$= fof(x)$$

fof is even fn.

QUESTION

(RPP 1)

The graph of $y = f(x)$ is shown, then number of solutions of the equation $f(f(x)) = 2$ is equal to

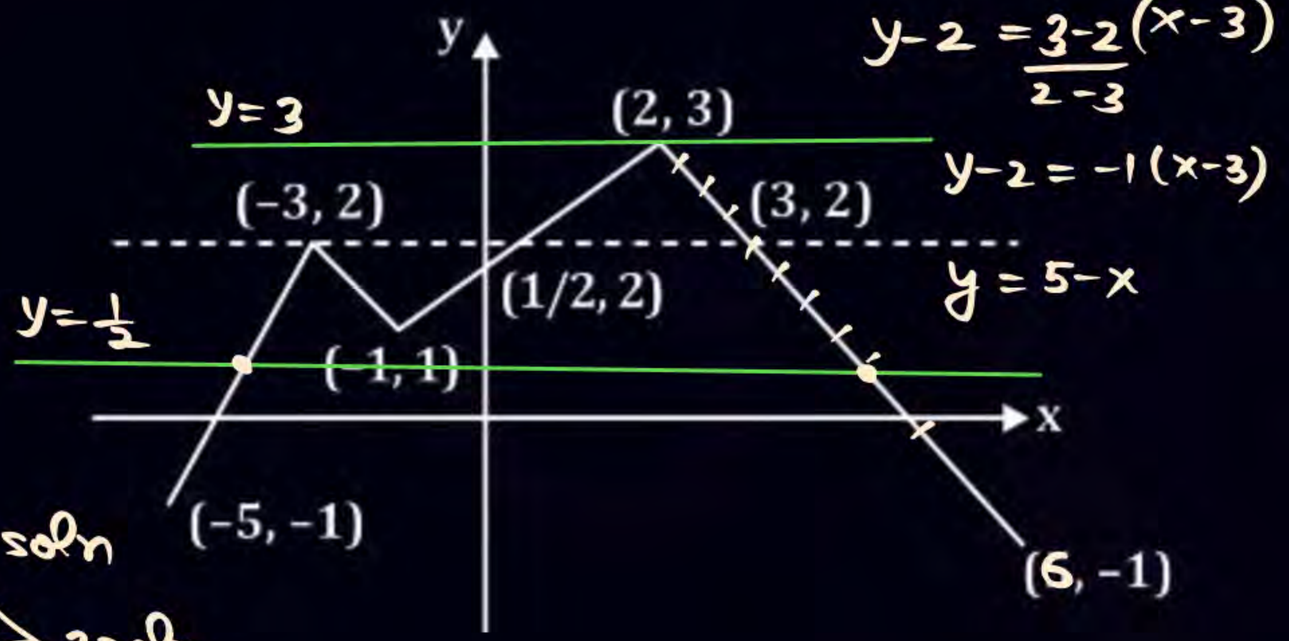
$f(f(x)) = 2$

Let $t = f(x)$ $\quad f(t) = 2$

$t = -3, \frac{1}{2}, 3$

$f(x) = -3, \frac{1}{2}, 3$ — only 1 soln

2 soln. → 3 soln



Ans. 3

QUESTION

(RPP 2)

Let f be a real valued function such that for any real x,
$f(15 + x) = f(15 - x)$ and $f(30 + x) = -f(30 - x)$.
Then which of the following statement is true?

A f is odd

B f is even

C f is periodic

D f is not periodic

$f(15+x) = f(15-x)$

$x \to 15+x$

$f(30+x) = f(-x)$

$x \to 15-x$

$f(30-x) = f(x)$

given

$f(30+x) = -f(30-x)$

$f(-x) = -f(x)$

$\Downarrow$

odd fn.

$f(30+x) = f(-x) = -f(x)$

$x \to 30+x$

$f(60+x) = -f(30+x) = -(-f(x)) = f(x)$

Ans. A, C

QUESTION

(RPP 3)

In a kite ABCD, AB = AD and CB = CD. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle$ ABD to the area of $\triangle$ CBD can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

Re-do



AC divides kite in to two congruent triangles

(A) (1, 2, 1)

(B) (2, 1, 1)

(C) (4, 3, 2)

(D) (1, 1, 2)

Ans. D

Galtiyaa Batao

Tan 1 (b) If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, then $\lim_{x \to a} f(x)\, g(x)$

splitting not allowed.

Galti Batao

$\Rightarrow \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = 2$ — (i)

$\Rightarrow \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = 1$ — (ii)

(i) + (ii)

$2 \lim_{x \to a} f(x) = 3 \Rightarrow \lim_{x \to a} f(x) = \frac{3}{2}$

(i) − (ii)

$2 \lim_{x \to a} g(x) = 1 \Rightarrow \lim_{x \to a} g(x) = \frac{1}{2}$

ATQ,

$$\lim_{x \to a} f(x)\, g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

QUESTION

TAH 12b

Determine the following limits.

(a) $\lim_{x\to -1^+} f(x)$ (b) $\lim_{x\to -1^-} f(x)$

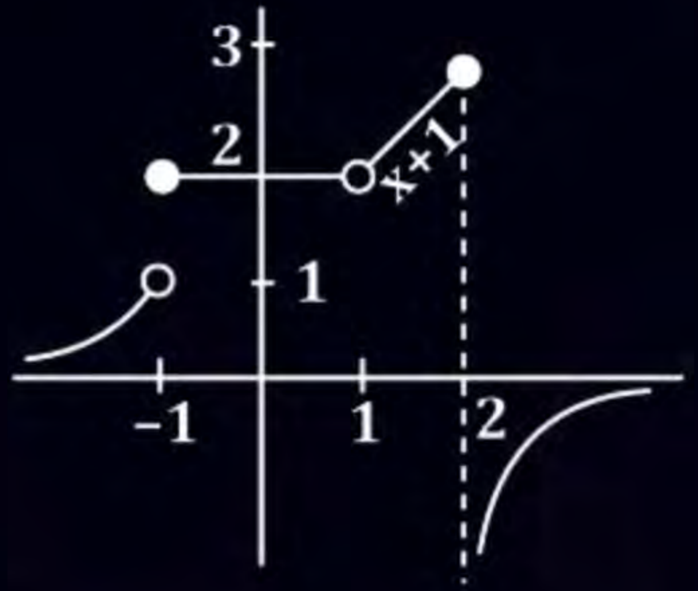
(c) $\lim_{x\to -1} f(x)$ (d) $\lim_{x\to 1^+} f(x)$

(e) $\lim_{x\to 1^-} f(x)$ (f) $\lim_{x\to 1} f(x)$

(g) $\lim_{x\to 2^+} f(x)$ (h) $\lim_{x\to 2^-} f(x)$

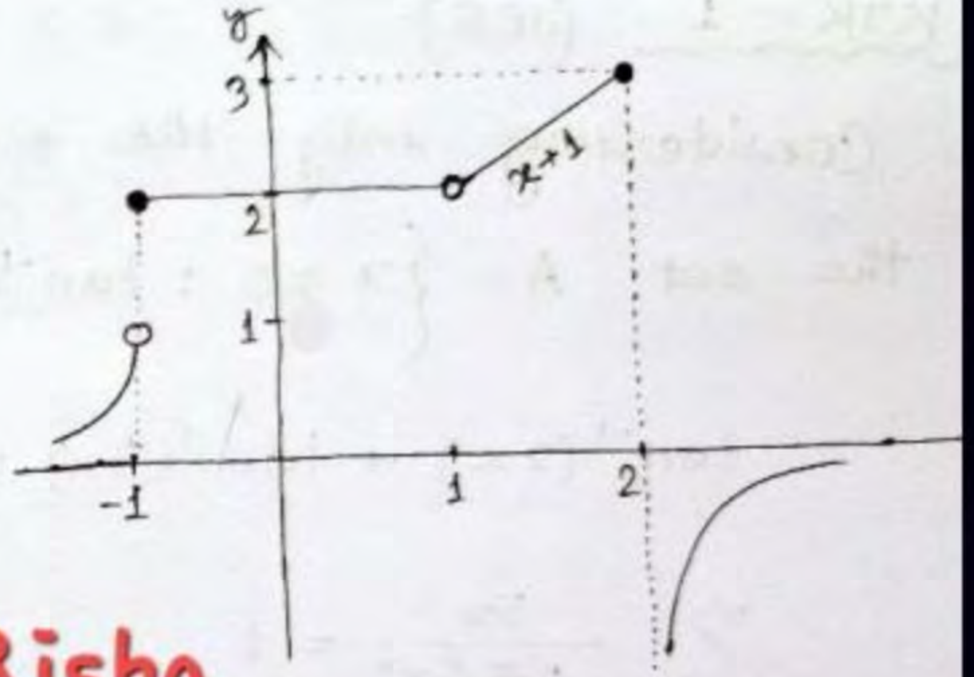
(i) $\lim_{x\to 2} f(x)$ (j) $\lim_{x\to -3} f(x)$

(k) $\lim_{x\to 5} f(x)$ (l) $\lim_{x\to 1.5} f(x)$



TAH — 12(b)

a) $\lim_{x \to -1^+} f(x) =$ 2 b) $\lim_{x \to -1^-} f(x) = 1$



c) $\lim_{x \to -1} f(x) =$ D.N.E

d) $\lim_{x \to 1^+} f(x) = 2$ e) $\lim_{x \to 1^-} f(x) = 2$

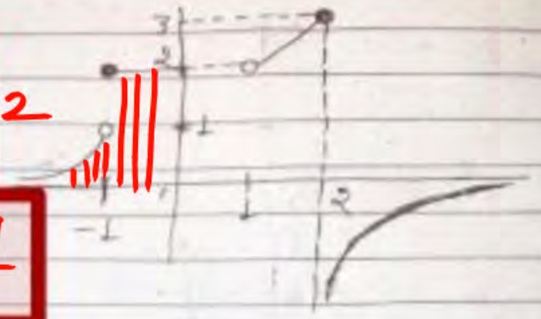
Risha
From WB

f) $\lim_{x \to 1} f(x) = 2$

g) $\lim_{x \to 2^+} f(x) = \infty$ h) $\lim_{x \to 2^-} f(x) = 3$ i) $\lim_{x \to 2} f(x) =$ D.N.E

j) $\lim_{x \to -3} f(x) =$ N.D k) $\lim_{x \to 5} f(x) =$ N.D l) $\lim_{x \to 1.5} f(x) = 2.5$

TAH-12(B)



(a) $\lim_{x \to -1^+} f(x) =$ ~~2~~ 2

(b) $\lim_{x \to -1^-} f(x) =$ ~~2~~ 1

(c) $\lim_{x \to -1} f(x) =$ DNE

(d) $\lim_{x \to 1^+} f(x) = 2$

(e) $\lim_{x \to 1^-} f(x) = 2$

(f) $\lim_{x \to 1} f(x) = 2$

(g) $\lim_{x \to 2^+} f(x) =$ ~~2~~ $-\infty$

(h) $\lim_{x \to 2^-} f(x) =$ ~~2~~

(i) $\lim_{x \to 2} f(x) =$ DNE

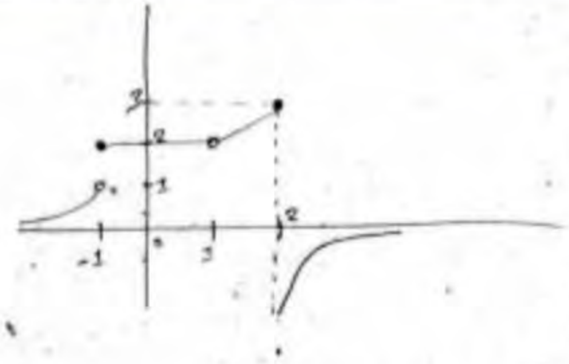
(j) $\lim_{x \to 2^-} f(x) = 0$

(k) $\lim_{x \to 5} f(x) = 0$

(l) $\lim_{x \to 2.5} f(x) = 2.5$

Name Sanjeet kumar
From - Varanasi (U.P)

TAH 12(b)



(a) $\lim_{\theta \to 1^+} f(\theta) = 2$

(b) $\lim_{\theta \to -1^-} f(\theta) = +1$

(c) $\lim_{\theta \to -1} f(\theta) =$ DNE

(d) $\lim_{\theta \to 5^+} f(\theta) = 2$

(e) $\lim_{\theta \to 5^-} f(\theta) = 2$

(f) $\lim_{\theta \to 1} f(\theta) = 2$

(g) $\lim_{\theta \to 2^+} f(\theta) = -\infty$

(h) $\lim_{\theta \to 2^-} f(\theta) = 3$

(i) $\lim_{\theta \to 2} f(\theta) =$ DNE

(j) $\lim_{\theta \to 3} f(\theta) =$ N.D.

(k) $\lim_{\theta \to 5} f(\theta) =$ N.D.

(l) $\lim_{\theta \to 2.5} f(\theta) =$

Sanjeev

QUESTION

TAH 12c

Which of the following statements about the function y = f(x) graphed here are true, and which are false?

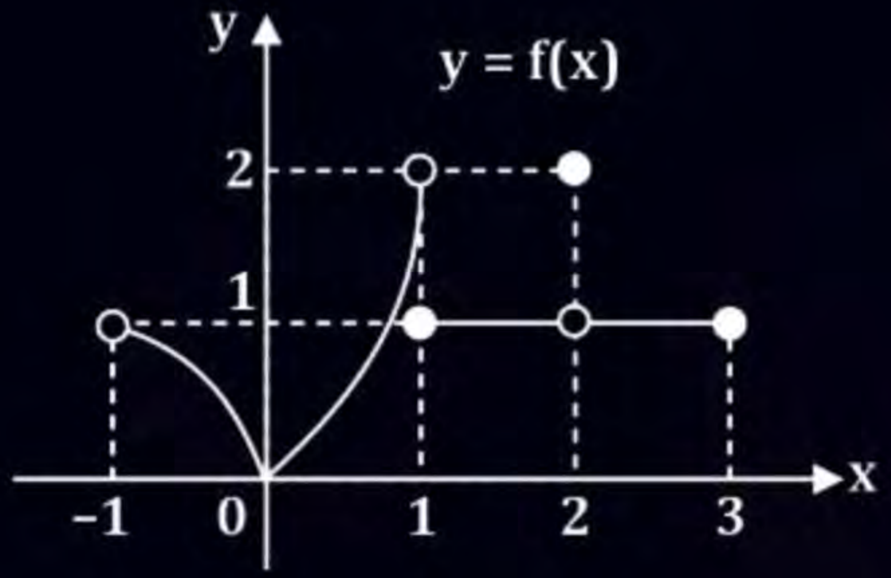
(f) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

(g) $\lim_{x\to c} f(x)$ exists at every $c \in (-1, 1)$

(h) $\lim_{x\to c} f(x)$ does not exist.for some cε(-1, 1)

(i) $\lim_{x\to 1^-} f(x) = 0$

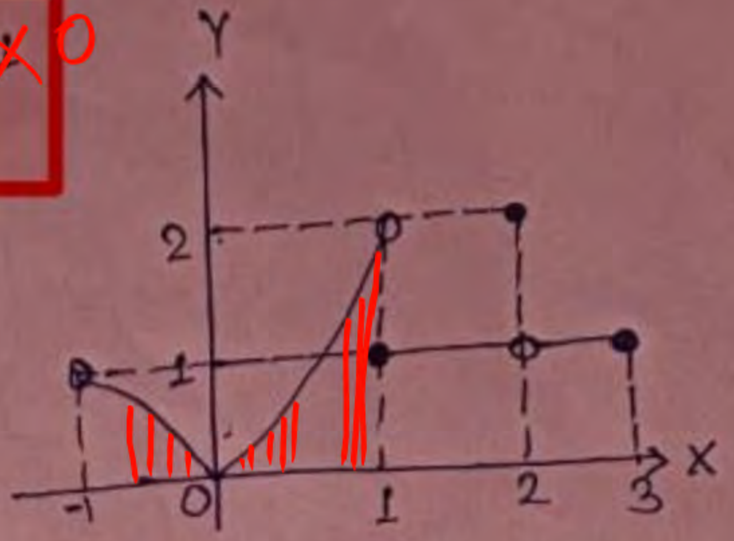
(j) $\lim_{x\to 3^+} f(x)$ does not exist.



Tab-12C

(a) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0$

= (True)



(b) $\lim_{x \to c} f(x)$ exist at every $c \in (-1, 1)$

$\Rightarrow$ (True)

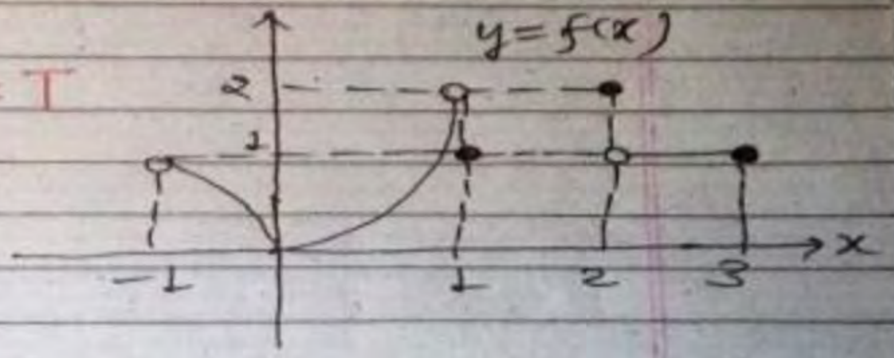
(c) $\lim_{x \to c} f(x)$ does not exist (False)

(d) $\lim_{x \to 1^-} f(x) = 2$ → (True)

(e) $\lim_{x \to 3^+} f(x)$ does not exist. → (True)

ROHINI SOLANKI
FROM - U.P.

TAH-12 C



(f) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ = T

(g) $\lim_{x \to c} f(x)$ exists at every $c \in (-1, 1)$ = T

(h) $\lim_{x \to c} f(x)$ does not exist = F

(i) $\lim_{x \to 1^-} f(x) = 0$ = F

(j) $\lim_{x \to 3^+} f(x)$ does not exist = T

Name Sanjeet Kumar
From – Varanasi (U.P)

QUESTION

KTK 9

(i) If $\tan^{-1} 4 + \tan^{-1} 5 = \cot^{-1}(\lambda)$, then find λ.

(ii) Which is greater $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$ or $\cot^{-1}(-1)$

$x = 4$, $y = 5$, $xy = 20 > 1$

$\tan^{-1}4 + \tan^{-1}5 = \pi + \tan^{-1}\left(\frac{4+5}{1-20}\right)$

$= \pi + \tan^{-1}\left(-\frac{9}{19}\right)$

$\pi - \tan^{-1}\frac{9}{19} = \cot^{-1}\lambda$

$\pi - \cot^{-1}\lambda = \tan^{-1}\frac{9}{19}$

$\cot^{-1}(-\lambda) = \cot^{-1}\left(\frac{19}{9}\right)$

$-\lambda = 19/9 \Rightarrow \lambda = -19/9.$

KTK -9

(1) If $\tan^{-1}4 + \tan^{-1}5 = \cot^{-1}(\lambda)$ then λ will be

$\pi + \tan^{-1}\left(\frac{9}{-19}\right) = \cot^{-1}(\lambda)$

$\Rightarrow \pi + \cot^{-1}\left(\frac{19}{-9}\right) - \pi = \cot^{-1}(\lambda)$

$\Rightarrow \boxed{\lambda = \frac{19}{9}}$ Ans.

$\pi + \tan^{-1}\left(\frac{-9}{19}\right) = \cot^{-1}\lambda$

$\pi + \cot^{-1}\left(\frac{-19}{9}\right) - \pi = \cot^{-1}\lambda$

$\lambda = -\frac{19}{9}$

(2) Which is greater $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$ or $\cot^{-1}(-1)$.

$\Rightarrow \cos^{-1}\left[\frac{7}{25}\times\frac{3}{5} - \sqrt{1-\frac{49}{625}}\sqrt{1-\frac{9}{25}}\right]$

$\cos^{-1}\left[\frac{21}{125} - \frac{24}{25}\times\frac{4}{5}\right]$

$\Rightarrow \cos^{-1}\left[\frac{21-96}{125}\right]$

$\Rightarrow \cos^{-1}\left[-\frac{75}{125}\right]$

$\Rightarrow \boxed{\pi - \cos^{-1}\left(\frac{3}{5}\right)}$

ROHINI SOLANKI FROM - U.P.

Now, $\cot^{-1}(-1) = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

then, $\cot^{-1}(-1)$ is greater than $\left[\cos^{-1}\frac{7}{5} + \cos^{-1}\frac{3}{5}\right]$

$\Rightarrow \cot^{-1}(-1) > \left[\cos^{-1}\frac{7}{5} + \cos^{-1}\frac{3}{5}\right]$ Ans.

KTK 9

$\tan^{-1}(4) + \tan^{-1}(5) = \cot^{-1}(\lambda)$

$\tan^{-1}\left[\frac{9}{1-20}\right] = \tan^{-1}\left(\frac{1}{\lambda}\right)$

$\therefore \lambda \rightarrow \frac{-19}{9}$ Ans

kamran Ashraf
Muzaffarpur

(ii) $\cos^{-1}\frac{7}{25} + \cos^{-1}\frac{3}{5}$

$\tan^{-1}\frac{24}{7} + \tan^{-1}\frac{4}{3} \rightarrow \tan^{-1}\frac{28}{1-21}$

$\rightarrow \tan^{-1}\frac{28}{-20} \rightarrow \cot^{-1}\left(\frac{-20}{28}\right) \gg \cot^{-1}(-1)$ Ans

OPPO A59 5G

LIMITS KE KARAN ARJUN

$(n \notin N,\ |x|<1)$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \infty$$

(i) $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x-a} = na^{n-1}$

(x^n: (variable)^n, a^n: (const)^n)

(ii) $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x^m - a^m} = \dfrac{n}{m}a^{n-m}$



$$\lim_{x\to a}\frac{x^n-a^n}{x-a} = \lim_{t\to 0}\frac{(a+t)^n-a^n}{a+t-a} = \lim_{t\to 0}\frac{a^n\left(1+\frac{t}{a}\right)^n - a^n}{t} \quad \left(\frac{t}{a}\to 0\right)$$

Put $x = a+t$

as $x \to a$, $t \to$

$$= \lim_{t\to 0}\frac{a^n\left(1+\frac{nt}{a}+\frac{n(n-1)}{2!}\frac{t^2}{a^2}+\cdots\right)-a^n}{t}$$

$$= \lim_{t\to 0}\frac{a^n\cdot\frac{n}{a}t\left(1+\frac{(n-1)}{2!}\frac{t}{a}+\cdots\right)}{t} = n\cdot a^{n-1}$$ Ans.

$$n \in N \quad (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$$

$$= 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \frac{n(n-1)\cdots(n-(n-1))}{n!}x^n$$

$$+ \frac{n(n-1)(n-2)\cdots(n-(n))}{(n+1)!}x^{n+1}$$

$$(1+x)^3 = 1 + 3x + \frac{3(3-1)}{2!}x^2 + \frac{3(3-1)(3-2)}{3!}x^3 + \underbrace{\frac{3(3-1)(3-2)(3-3)}{4!}x^4}_{0} + \underbrace{\frac{3(3-1)(3-2)(3-3)(3-4)}{5!}x^5}_{0} + \cdots \infty$$

$$+ \frac{n(n-1)\cdots(n-n)(n-(n+1))}{(n+2)!}x^{n+2}$$

Ex: $(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}x^4 + \cdots \infty$

$-1 < x < 1$

(ii) $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m}$

$$= \lim_{x \to a} \frac{\frac{x^n - a^n}{x-a}}{\frac{x^m - a^m}{x-a}}$$

$$= \frac{\lim_{x \to a} \frac{x^n - a^n}{x-a}}{\lim_{x \to a} \frac{x^m - a^m}{x-a}} = \frac{n \cdot a^{n-1}}{m \cdot a^{m-1}} = \frac{n}{m} a^{n-m}.$$

Law of LOVE :

We Love to create ∞ in Den & 0 in numerator

6'oz $\frac{1}{\infty} \to 0$, $\frac{\text{st}^{\text{n}} \text{ tending to } 0}{\text{non zero quantity}} \to 0$

Ex: $\lim\limits_{x\to\infty} \frac{1\cdot x^3+x^2+5}{3x^3+7x+8}$

$= \lim\limits_{x\to\infty} \frac{x^3\left(1+\frac{1}{x}+\frac{5}{x^3}\right)}{x^3\left(3+\frac{7}{x^2}+\frac{8}{x^3}\right)} = \frac{1}{3}$

Ex: $\lim\limits_{x\to\infty} \frac{5x^4+6x^3+7}{2x^4+5x+6} = \frac{5}{2}$

$l = \lim\limits_{x\to\infty,-\infty} \frac{P(x)}{Q(x)}$ > both polynomial of same degree 'n'

then $l = \frac{\text{coeff of } x^n \text{ in } P(x)}{\text{coeff of } x^n \text{ in } Q(x)}$

if $\ell = \lim_{x \to \infty, -\infty} \frac{P(x)}{Q(x)}$ > polynomials in x where degree of $P(x)$ < degree of $g(x)$

then $\ell = 0$

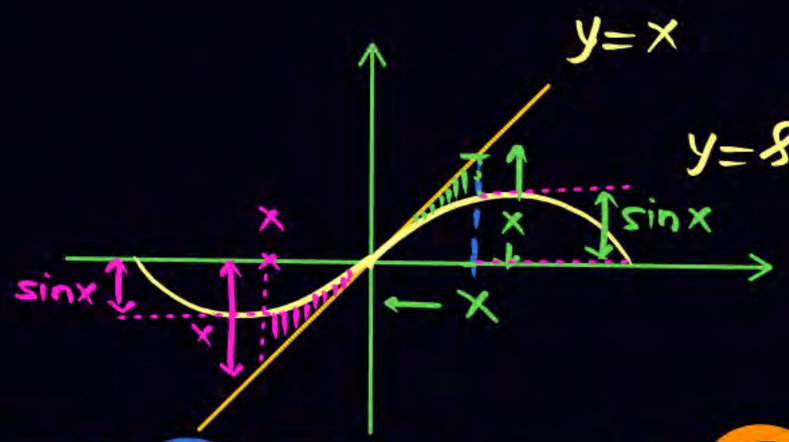
Ex: $\lim_{x \to -\infty} \frac{5x^2 + 6x + 2}{6x^3 + 7x + 8} = \lim_{x \to -\infty} \frac{x^2\left(5 + \frac{6}{x} + \frac{2}{x^2}\right)}{x^3\left(6 + \frac{7}{x^2} + \frac{8}{x^3}\right)}$

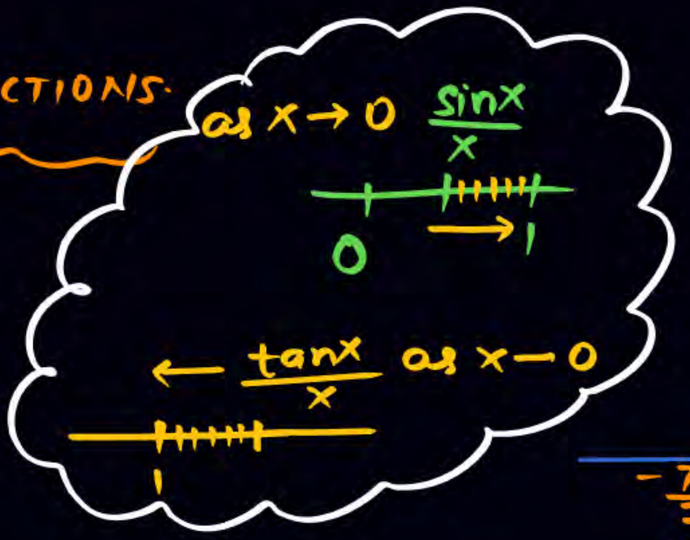
$= \lim_{x \to -\infty} \frac{1}{x} \cdot \frac{(5 + 6/x + 2/x^2)}{6 + 7/x^2 + 8/x^3} = 0 \cdot \frac{5}{6} = 0$.

If $\ell = \lim_{x \to \infty} \frac{P(x)}{Q(x)}$ > polynomials in x where degree of $P(x)$ > degree $Q(x)$

then ℓ = Infinite

Limit of Trigonometric Functions







★ x should be in radian

★★ In nbd of x = 0

$0 < \frac{\sin x}{x} < 1$

★★★ In nbd of x = 0 $\frac{\tan x}{x} > 1$

★ $\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{x}{\sin x} = \lim_{x\to 0} x \operatorname{cosec} x = \lim_{x\to 0} \frac{\sin^{-1} x}{x} = \lim_{x\to 0} \frac{x}{\sin^{-1} x}$

★ $\lim_{x\to 0} \frac{\tan x}{x} = 1 = \lim_{x\to 0} \frac{x}{\tan x} = \lim_{x\to 0} x \cot x = \lim_{x\to 0} \frac{\tan^{-1} x}{x} = \lim_{x\to 0} \frac{x}{\tan^{-1} x}$

$\lim_{x\to 0}\frac{\sin^{-1}x}{x}=1$ b'coz $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$

put $x=\sin\theta$ $\quad x\to 0,\ \theta\to 0$

$\lim_{\theta\to 0}\frac{\sin^{-1}(\sin\theta)}{\sin\theta}$

$\lim_{\theta\to 0}\frac{\theta}{\sin\theta}=1$

[·] G.I.F

a) $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]=0$

b) $\lim_{x\to 0}\left[\frac{\tan x}{x}\right]=1$

c) $\left[\lim_{x\to 0}\frac{\sin x}{x}\right]=[1]=1$

d) $\left[\lim_{x\to 0}\frac{\tan x}{x}\right]=[1]=1$

e) $\lim_{x\to 0}\left[\frac{2\sin 2x}{x}\right]=\lim_{x\to 0}\left[2^2\frac{\sin 2x}{2x}\right]=3=2^2-1$ (1⁻ → 4⁻)

(f) $\lim_{x\to 0}\left[\frac{4\sin 3x}{x}\right]=\lim_{x\to 0}\left[12\frac{\sin 3x}{3x}\right]=11.$ (1⁻ → 12⁻)

as $x \to a$ ☁ $\to 0$

★ $\lim\limits_{x \to a} \dfrac{\sin(\text{☁})}{\text{☁}} = 1$

★ $\lim\limits_{x \to a} \dfrac{\tan(\text{☁})}{\text{☁}} = 1$

Ex: $\lim\limits_{x \to 0} \dfrac{\sin(\tan x)}{\tan x} \to \dfrac{0}{0} = 1$

Ex: $\lim\limits_{x \to \pi/2} \dfrac{\tan(\cos x)}{\cos x} \to \dfrac{0}{0} = 1$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

as $x-a \to 0$

$$\lim_{x-a}\frac{1-\cos(\square)}{\square^2}=\frac{1}{2}$$

Proof: $\lim_{x\to 0}\frac{1-\cos x\cdot(1+\cos x)}{(1+\cos x)\cdot x^2}$

$\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$

$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\frac{1}{1+\cos x}$

$= 1^2\cdot\frac{1}{1+1}=1/2$

Aao Machay Dhmaal Deh Swaal pe Deh Swaal

QUESTION

If $\lim_{x\to 2}\left(\frac{x^{n+1}-2^{n+1}}{x-2}\right)=80$ and $n\in N$, find n.

$$\lim_{x-a}\frac{x^n-a^n}{x-a}=n\,a^{n-1}$$

$$(n+1)\,2^{n+1-1}=80$$

$$(n+1)\cdot 2^n=16\cdot 5=(4+1)\cdot 2^4$$

$$n=4$$

QUESTION [JEE Mains 2020]

ASRQ

If $\lim\limits_{x\to 1}\dfrac{x+x^2+x^3+.....+x^n-n}{x-1}=820, (n\in N)$ then the value of n is equal to

$\lim\limits_{x\to 1}\dfrac{x+x^2+x^3+\text{---}+x^n-n}{x-1}$ Sab mai eak eak baat do

$\lim\limits_{x\to 1}\dfrac{x-1+x^2-1+x^3-1+--+x^n-1}{x-1}$

$\lim\limits_{x\to 1}\dfrac{x-1}{x-1}+\dfrac{x^2-1^2}{x-1}+\dfrac{x^3-1^3}{x-1}+--+\dfrac{x^n-1^n}{x-1}$

$=1+2\cdot 1^{2-1}+3\cdot 1^{3-1}+--+n\cdot 1^{n-1}$

$=1+2+3+--+n$

$=\dfrac{n(n+1)}{2}$

$\dfrac{n(n+1)}{2}=820$

$n(n+1)=2\times 41\times 20$

$n(n+1)=40\times 41$

$n=40$

QUESTION

$$\lim_{x\to 0}\frac{(\cos x)^{\frac{1}{3}}-(\cos x)^{\frac{1}{2}}}{\sin^2 x}$$ equals

(A) 1/12

(B) 1/6

(C) 1/3

(D) 1/2

$\cos x = t$

as $x \to 0$

$\lim_{x\to a}\frac{x^n-a^n}{x-a}$

$$\lim_{t\to 1}\frac{t^{\frac{1}{3}}-t^{\frac{1}{2}}}{1-t^2}=\lim_{t\to 1}\frac{t^{1/3}-1^{\frac{1}{3}}-(t^{\frac{1}{2}}-1^{\frac{1}{2}})}{1-t^2}$$

$$=-\lim_{t\to 1}\frac{t^{1/3}-1^{1/3}-(t^{1/2}-1^{1/2})}{(t-1)(t+1)}$$

$$=-\lim_{t\to 1}\left(\frac{t^{1/3}-1^{1/3}}{t-1}-\frac{t^{1/2}-1^{1/2}}{t-1}\right)\frac{1}{t+1}$$

$$=-\left(\frac{1}{3}\cdot 1^{1/3-1}-\frac{1}{2}\cdot 1^{1/2-1}\right)\cdot\frac{1}{2}=-(1/3-1/2)\cdot 1/2=1/12$$

Ans. A


Using Law of LOVE

QUESTION

$$\lim_{n\to\infty} \frac{(3(n+1))!}{(n+1)^3(3n)!} \text{ equals}$$

A) 3

B) 9

C) 27

D) 36

$$= \lim_{n\to\infty} \frac{(3n+3)!}{(n+1)^3 \cdot (3n)!}$$

$n! = n(n-1)!$
$= n(n-1)(n-2)!$
$= n(n-1)(n-2)(n-3)!$

$$= \lim_{n\to\infty} \frac{(3n+3)(3n+2)(3n+1)(3n)!}{(n+1)^3 \cdot (3n)!}$$

$$= \lim_{n\to\infty} \frac{(3n+3)(3n+2)(3n+1)}{(n+1)^3}$$ degree 3

$$= \frac{27}{1} = 27$$

QUESTION

$$\lim_{x\to\infty} \frac{(x-3)^{40}\cdot(5x+1)^{10}}{(3x^2-2)^{25}}$$ > degree 50

$$\lim_{x\to-\infty} \frac{x^{40}\left(1-\frac{3}{x}\right)^{40}\cdot x^{10}\left(5+\frac{1}{x}\right)^{10}}{x^{50}\left(3-\frac{2}{x^2}\right)^{25}} = \frac{1^{40}\cdot 5^{10}}{3^{25}} = \frac{5^{10}}{3^{25}}.$$

QUESTION

100 brackets

$$\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\ldots\ldots+(x+100)^{10}}{x^{10}+10^{10}}$$

$$\lim_{x\to\infty}\frac{x^{10}\left(1+\frac{1}{x}\right)^{10}+x^{10}\left(1+\frac{2}{x}\right)^{10}+\ldots\ldots+x^{10}\left(1+\frac{100}{x}\right)^{10}}{x^{10}\left(1+\frac{10^{10}}{x^{10}}\right)}=\frac{1+1+\ldots\ldots+1}{1}=100$$

QUESTION [JEE Mains 2023 (31 Jan)]

$$\lim_{x\to\infty} \frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6} x^3$$

$$\frac{(\sqrt{3}+\sqrt{3})^6+(\sqrt{3}-\sqrt{3})^6}{(1+1)^6+(1-1)^6} = \frac{(2\sqrt{3})^6}{2^6} = \sqrt{3}^6 = 27$$

(A) is equal to 9

(B) is equal to 27/2

(C) does not exist

(D) is equal to 27

$$\lim_{x\to\infty} \frac{\left(\sqrt{x}\sqrt{3+\frac{1}{x}}+\sqrt{x}\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{x}\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6}{\left(x+x\sqrt{1-1/x^2}\right)^6+\left(x-x\sqrt{1-1/x^2}\right)^6} x^3$$

$$\lim_{x\to\infty} \frac{(\sqrt{x})^6\left[\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-1/x}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-1/x}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-1/x^2}\right)^6+\left(1-\sqrt{1-1/x^2}\right)^6\right]} x^3$$

$$= \frac{(\sqrt{3}+\sqrt{3})^6+0}{2^6} = \frac{(2\sqrt{3})^6}{2^6} = \sqrt{3}^6 = 3^3 = 27$$

QUESTION

Tan 1

$$\lim_{n\to\infty} \frac{(1+2+3+\ldots\ldots+n)(1^3+2^3+3^3+\ldots.+n^3)}{(1^2+2^2+3^2+\ldots..+n^2)^2}$$

Ans. 9/8

QUESTION

$$\lim_{n\to\infty} \frac{\sqrt{n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{n^6 - 6n^5 + 2} - \sqrt[5]{n^7 + 3n^3 + 1}}$$

$n^{3/2}$ $n^{4/3}$ $n^{3/2}$ $n^{7/5}$

$\frac{7}{5} \quad \frac{-3}{2}$

$= 14 - 15$

$$\lim_{n\to\infty} \frac{n^{3/2}\sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + n^{\frac{4}{3}}\sqrt[3]{1 + \frac{1}{n^4}}}{n^{3/2}\sqrt[4]{1 - \frac{6}{n} + \frac{2}{n^6}} - n^{7/5}\sqrt[5]{1 + \frac{3}{n^2} + \frac{1}{n^5}}}$$

1.5 1.33... 1.4

$$\lim_{n\to\infty} \frac{\left(\sqrt{1 - \frac{2}{n} + \frac{1}{n^3}} + \frac{1}{n^{1/6}}\sqrt[3]{1 + \frac{1}{n^4}}\right) n^{3/2}}{\left(\sqrt[4]{1 - 6/n + 2/n^6} - \frac{1}{n^{1/10}}\sqrt[5]{1 + 3/n^2 + 1/n^5}\right) n^{3/2}} = 1$$

QUESTION

$$\lim_{n\to\infty} \frac{\sqrt{16n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{81n^6 - 6n^5 + 2} - \sqrt[5]{n^7 + 3n^3 + 1}}$$

$n^{3/2}$ $n^{4/3}$ $n^{3/2}$ $n^{7/5}$

$$= \frac{\sqrt{16}}{\sqrt[4]{81}} = \frac{4}{3}$$

QUESTION

$\{x\} = x - [x]$

$[x] = x - \{x\}$

$$\lim_{n\to\infty} \frac{[1^2x]+[2^2x]+[3^2x]+ \ldots\ldots + [n^2x]}{n^3}$$ where [.] greatest integer function.

$$\lim_{n\to\infty} \frac{1^2x - \{1^2x\} + 2^2 - \{2^2x\} + 3^2x - \{3^2x\} + - - + n^2x - \{n^2x\}}{n^3}$$

$$\lim_{n\to\infty} \frac{(1^2+2^2+3^2+\cdots+n^2)x}{n^3} - \frac{(\{1^2x\}+\{2^2x\}+\{3^2x\}+ - - +\{n^2x\})}{n^3}$$

0

$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)x}{6n^3} - \frac{(\{1^2x\}+\{2^2x\}+ - - +\{n^2x\})}{n^3}$$

$$= \frac{2x}{6} = x/3.$$

Ans. x/3

QUESTION

ASRQ

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x-6} = \frac{1}{3}$$

Gadho/Gadhiyoo aisaa naa karo

M①

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x-6}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}{3x-6}$$

$$\lim_{x\to-\infty}\frac{|x|\sqrt{1+1/x^2}}{3x-6}$$

$$\lim_{x\to\infty}\frac{-x\sqrt{1+1/x^2}}{x(3-6/x)} = -1/3 \text{ Ans}$$

M② put $x=-t$ as $x\to-\infty$, $t\to\infty$

$$\lim_{t\to\infty}\frac{\sqrt{(-t)^2+1}}{3(-t)-6}$$

$$-\lim_{t\to\infty}\frac{\sqrt{t^2+1}}{3t+6}$$

$$= -\frac{1}{3}$$

QUESTION

ASRQ

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x-6} = \frac{1}{3}$$

Gadho/Gadhiyoo aisaa naa karo

M③ Put $x=\frac{1}{y}$

as $x\to-\infty$
$y\to 0^-$

$$\lim_{y\to0^-}\frac{\sqrt{\frac{1}{y^2}+1}}{\frac{3}{y}-6} = \lim_{y\to0^-}\frac{\sqrt{1+y^2}}{\frac{\sqrt{y^2}}{3-6y}} = \lim_{y\to0^-}\frac{\sqrt{1+y^2}}{3-6y}\frac{y}{|y|}$$

$$= \lim_{y\to0^-}\frac{\sqrt{1+y^2}}{3-6y}\frac{y}{-y}$$

$$= -1/3.$$

QUESTION

ASRQ

$$\lim_{x\to 0}\left(1^{\operatorname{cosec}^2 x}+2^{\operatorname{cosec}^2 x}+3^{\operatorname{cosec}^2 x}+\ \ldots\ldots+100^{\operatorname{cosec}^2 x}\right)^{\sin^2 x}$$

$\operatorname{cosec}^2 x \to \infty$ as $x \to 0$

$\operatorname{cosec}^2 x = \frac{1}{\sin^2 x}$

(∞^0)

(Ise badad) $\to \infty$ no:

$$\lim_{x\to 0}\left(100^{\operatorname{cosec}^2 x}\right)^{\sin^2 x}\left(\left(\frac{1}{100}\right)^{\operatorname{cosec}^2 x}+\left(\frac{2}{100}\right)^{\operatorname{cosec}^2 x}+\cdots+\left(\frac{99}{100}\right)^{\operatorname{cosec}^2 x}+1\right)^{\sin^2 x}$$

$$= 100\cdot(0+0+\cdots+0+1)^0$$

$$= 100$$

(0 se 1 ke brech ka no:) $\to 0$

$$\lim_{n\to\infty}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{n+3}}$$ ASRQ

$$\lim_{n\to\infty}\frac{5^n\cdot 5+3^n-4^n}{5^n+2^n+3^n\cdot 27}=\lim_{n\to\infty}\frac{5+(3/5)^n-(4/5)^n}{1+(2/5)^n+(3/5)^n\cdot 27}=5$$

QUESTION [JEE Advanced 2012]

Tan 2

If $\lim\limits_{x\to-\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

put $x=-t$ as $x\to-\infty$, $t\to\infty$

$\lim\limits_{t\to\infty}$

(A) $a=1, b=4$

(B) $a=1,\ b=-4$

(C) $a=2, b=-3$

(D) $a=2, b=3$

Ans. B

QUESTION [JEE Mains 2022 (25 July)]

Tan 3

If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to:

A 4

B -8

C -4

D 8

Ans. C

QUESTION ASRQ

$$\lim_{x\to\infty}\left(\sqrt[3]{(x+a)(x+b)(x+c)}-x\right)=$$

$(a-b)(a^2+b^2+ab)=a^3-b^3$

(A) $\sqrt{abc}$

(B) $\dfrac{a+b+c}{3}$

(C) abc

(D) $(abc)^{1/3}$

$$\lim_{x\to\infty}\left(\sqrt[3]{(x+a)(x+b)(x+c)}-x\right) \quad (\infty-\infty)$$

$$\lim_{x\to\infty}\frac{\left(\sqrt[3]{(x+a)(x+b)(x+c)}\right)^3-x^3}{\left((x+a)(x+b)(x+c)\right)^{2/3}+x^2+x\cdot\left((x+a)(x+b)(x+c)\right)^{1/3}}$$

$$\lim_{x\to\infty}\frac{(x+a)(x+b)(x+c)-x^3}{(x^3)^{\frac{2}{3}}\left((1+a/x)(1+b/x)(1+c/x)\right)^{2/3}+x^2+x\cdot(x^3)^{1/3}\left((1+a/x)(1+\frac{b}{x})(1+\frac{c}{x})\right)^{1/3}}$$

$$(x+a)(x+b)(x+c)-x^3=x^3+(a+b+c)x^2+(ab+bc+ca)x+abc-x^3$$

$$\frac{(a+b+c)}{3}=\frac{a+b+c}{1^{2/3}+1+1^{1/3}}=$$

$$\lim_{x\to\infty}\frac{\left((a+b+c)+\frac{ab+bc+ca}{x}+\frac{abc}{x^2}\right)x^2}{x^2\left[\left((1+a/x)(1+b/x)(1+c/x)\right)^{2/3}+1+\left((1+a/x)(1+b/x)(1+c/x)\right)^{1/3}\right]}$$

$$\lim_{x \to \infty} \left(\sqrt[n]{(x+a_1)(x+a_2)(x+a_3)\cdots(x+a_n)} - x \right) = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

QUESTION

$$\lim_{n\to\infty}\frac{1^2n+2^2(n-1)+3^2(n-2)+\ldots.+n^2\cdot1}{1^3+2^3+3^3+\ldots\ldots+n^3}$$ is equal to :

A $\frac{1}{3}$

B $\frac{2}{3}$

C $\frac{1}{2}$

D $\frac{1}{6}$

M① $\lim_{n\to\infty}\dfrac{1^2\cdot n+2^2(n-1)+3^2(n-2)+--+n^2\cdot1}{1^3+2^3+---+n^3}$

$$\lim_{n\to\infty}\frac{\sum_{r=1}^{n} r^2\,(n-(r-1))}{\left(\frac{n(n+1)}{2}\right)^2}=\lim_{n\to\infty}\frac{\sum_{r=1}^{n} r^2((n+1)-r)}{\frac{n^2(n+1)^2}{4}}$$

$$=\lim_{n\to\infty}\frac{\sum_{r=1}^{n}\left((n+1)r^2-r^3\right)}{\frac{n^2(n+1)^2}{4}}=\lim_{n\to\infty}\frac{(n+1)\frac{n(n+1)(2n+1)}{6}-\left(\frac{n(n+1)}{2}\right)^2}{\frac{n^2(n+1)^2}{4}}$$

$$=\frac{\frac{2}{6}-1/4}{1/4}=\frac{1/3-1/4}{1/4}=\frac{1}{3}$$

M②

$$\lim_{n\to\infty} \frac{1^2\cdot n + 2^2(n-1) + 3^2(n-2) + \cdots + n^2\cdot 1}{1^3+2^3+3^3+\cdots+n^3} + 1 - 1$$

$$\lim_{n\to\infty} \frac{1^2 n + 2^2\cdot(n-1) + 3^2(n-2) + \cdots + n^2\cdot 1 + 1^3+2^3+3^3+\cdots+n^3}{1^3+2^3+\cdots+n^3} - 1$$

$$\lim_{n\to\infty} \frac{1^2(n+1) + 2^2(n-1+2) + 3^2(n-2+3) + \cdots + n^2(n+1)}{\frac{n^2(n+1)^2}{4}} - 1$$

$$\lim_{n\to\infty} \frac{(n+1)(1^2+2^2+\cdots+n^2)}{\left(\frac{n(n+1)}{2}\right)^2} - 1 = \lim_{n\to\infty} \frac{(n+1)\frac{n(n+1)(2n+1)}{6}}{\frac{n^2(n+1)^2}{4}} - 1 = \frac{2/6}{1/4} - 1 = \frac{4}{3} - 1 = 1/3$$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Homework From Module

INVERSE TRIGONOMETRIC FUNCTIONS: Complete

LCD

(Solution to TAH)

QUESTION

If $\lim_{x \to 0} \frac{f(x)}{x^2} = 1$, find

(i) $\lim_{x \to 0} f(x)$

(ii) $\lim_{x \to 0} \frac{f(x)}{x}$

Archita Dwivedi
(From UP)

IAH-1 @

Given: If $\lim_{x \to 0} \frac{f(x)}{x^2} = 1$ find.

(i) $\lim_{x \to 0} \frac{f(x)}{x} = ?$

Soln:- As we know

$$\lim_{x \to 0} x^2 = 0.$$

and $\lim_{x \to 0} (x) = 0$

Now; $\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} \left[x \cdot \frac{f(x)}{x^2} \right]$

$= \lim_{x \to 0} x \cdot \lim_{x \to 0} \frac{f(x)}{x^2}$

$= 0 \cdot 1.$

$= 0.$

Ans: $\lim_{x \to 0} \left(\frac{f(f(x))}{x} \right) = 0$

QUESTION

If $\lim_{x \to a}[f(x) + g(x)] = 2$ and $\lim_{x \to a}[f(x) - g(x)] = 1$, then $\lim_{x \to a} f(x)\, g(x)$

A need not exist

B exist and is $\frac{3}{4}$

C exists and is $\frac{-3}{4}$

D exists and is $\frac{4}{3}$

Ans: B

Tab-1 (b)

Archita Dwivedi
(From UP)

Q. If $\lim_{x \to a} [f(x) + g(x)] = 2$ & $\lim_{x \to a} [f(x) - g(x)] = 2$

then $\lim_{x \to a} f(x) \cdot g(x) = ?$

Soln $2 + 1 = \lim_{x \to a} [f(x) + g(x)] + [f(x) - g(x)]$

$3 = \lim_{x \to a} 2f(x)$

$\lim_{x \to a} f(x) = \frac{3}{2}$

$2 = \lim_{x \to a} [f(x) + g(x)] - [f(x) - g(x)]$

$1 = \lim_{x \to a} 2g(x)$

$\lim_{x \to a} g(x) = \frac{1}{2}$

$\lim_{x \to a} f(x) \cdot g(x) = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

Ans. (B) exist in $\frac{3}{4}$.

Q If $\lim_{x\to a}[f(x)+g(x)]=2$ and $\lim_{x\to a}[f(x)-g(x)]=1$ then $\lim_{x\to a} f(x)g(x)$

(A) need not exist
(B) exist and is 3/4
(C) exists and is $-3/4$
(D) exists and is $-1/3$

$\Rightarrow \lim_{x\to a}[f(x)+g(x)]=2$ ——— (1)

and $\lim_{x\to a}[f(x)-g(x)]=1$ ——— (2)

(1)+(2)

$$\lim_{x\to a}[f(x)+g(x)]+\lim_{x\to a}[f(x)-g(x)]=2+1$$

$$\lim_{x\to a}[f(x)+g(x)+f(x)-g(x)]=3$$

$$\lim_{x\to a}2f(x)=3$$

$$\lim_{x\to a}f(x)=3/2$$

TAN - 1(b)
Shubhajit Roy
From W.B.

(1) − (2)

$$\lim_{x\to a}[f(x)+g(x)]-\lim_{x\to a}[f(x)-g(x)]=2-1$$

$$\lim_{x\to a}[f(x)+g(x)-f(x)+g(x)]=1$$

$$\lim_{x\to a}g(x)=1/2$$

Now,

$$\lim_{x\to a}f(x)g(x)=\lim_{x\to a}f(x)\lim_{x\to a}g(x)$$

$$=\frac{3}{2}\cdot\frac{1}{2}$$

$$=\frac{3}{4}$$

(B) exist and is 3/4 [B] [Ans.]

QUESTION

Evaluate: $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x^2 - 5x + 4} \right)$ Ans. 1/3

Evaluate: $\lim_{x \to 1} \left(\frac{1 - x^{-\frac{1}{3}}}{1 - x^{-\frac{2}{3}}} \right)$ Ans. 1/2

Evaluate: $\lim_{x \to 2} \left(\frac{x^3 + 7x^2 - 36}{x^2 + 2x - 8} \right)$ Ans. 28/5

TAH 2

Q (i) $\lim_{x \to 1} \left(\frac{x^2-3x+2}{x^2-5x+4} \right)$

Sol^n $\lim_{x \to 1} \frac{(x-1)(x-2)}{(x-1)(x-4)} \Rightarrow \lim_{x \to 1} \frac{x-2}{x-4} = \frac{1}{3}$.

(ii) $\lim_{x \to 2} \left(\frac{x^3+7x^2-36}{x^2+2x-8} \right)$

Deepak from Batala punjab

Sol^n $\lim_{x \to 2} \left(\frac{x^2(x-2)+9x(x-2)+18(x-2)}{(x+4)(x-2)} \right)$

$\lim_{x \to 2} \frac{x^2+9x+18}{x+4} = \frac{4+18+18}{6} = \frac{40}{6} = \frac{20}{3}$

Tab-2 Evaluate $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-5x+4}\right)$ $\left(\frac{0}{0}\right)$

Solve $\lim_{x\to 1}\frac{x^2-3x+2}{x^2-5x+4}$

$= \lim_{x\to 1}\frac{(x-2)(x-1)}{(x-4)(x-1)}$

$= \frac{1}{3}$

(ii) $\lim_{x\to 2}\frac{x^3+7x^2-36}{x^2+2x-8}$

$\lim_{x\to 2}\frac{(x-2)(x^2+9x+18)}{(x-2)(x+4)}$

$= \frac{40}{6} \quad \frac{30}{3}$

Anshika From MP

QUESTION

For $a > 0$, $\lim_{x \to a^2} \frac{\sqrt{x} + x - a - a^2}{x\sqrt{x} + x\sqrt{a} - a^2\sqrt{x} - a^2\sqrt{a}}$ equals

A $\frac{2a + 1}{2a(a+\sqrt{a})}$

B $\frac{2a + 1}{4a^{3/2}}$

C $\frac{a + 2}{a^{3/2}}$

D None

Tab-3 for $a>0$, $\lim_{x\to a^2} \frac{\sqrt{x}+x-a-a^2}{x\sqrt{x}+x\sqrt{a}-a^2\sqrt{x}-a^2\sqrt{a}}$

Sol: $\lim_{x\to a^2} \frac{\sqrt{x}+x-a-a^2}{x\sqrt{x}+x\sqrt{a}-a^2\sqrt{x}-a^2\sqrt{a}}$

$\lim_{x\to a^2} \frac{(\sqrt{x}-a)(1+\sqrt{x}+a)}{(\sqrt{x}+\sqrt{a})(\sqrt{x}-a)(\sqrt{x}+a)}$

$\lim_{x\to a^2} \frac{1+\sqrt{x}+a}{(\sqrt{x}+\sqrt{a})(\sqrt{x}+a)}$

$= \frac{2a+1}{2a(a+\sqrt{a})}$

Anshika From MP

QUESTION

$$\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$$

TAH 4A

Q $\lim_{x\to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$

Sol^n Put $2^x = t$

as $x \to 2$, $t \to 4$

$$\lim_{t\to 4} \frac{t + \frac{8}{t} - 6}{\frac{1}{\sqrt{t}} - \frac{2}{t}}$$

Deepak from Batala punjab

$$\lim_{t\to 4} \frac{t^2 - 6t + 8}{\sqrt{t} - 2} \Rightarrow \lim_{t\to 4} \frac{(t-2)(t-4)}{\sqrt{t}-2}$$

$$\lim_{t\to 4} \frac{(\sqrt{t}-2)(\sqrt{t}+2)(t-2)}{(\sqrt{t}-2)}$$

$$\lim_{t\to 4} (\sqrt{t}+2)(t-2) = 2 \cdot 4 = 8$$

Tah-4 $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ ~~$2^x = t$~~

Sol^n Let $2^x = t$

$$= \lim_{t \to 4} \frac{t + \frac{2^3}{t} - 6}{\sqrt{\frac{1}{t}} - \frac{2}{t}}$$

$$= \lim_{t \to 4} \frac{t^2 + 8 - 6t}{\sqrt{t} - 2}$$

[rationalised]

$$= \lim_{t \to 4} \frac{(t-4)(t-2)(\sqrt{t}+2)}{t-4}$$

$$= \lim_{t \to 4} (t-2)(\sqrt{t}+2)$$

$$= 8$$

Anshika From MP

QUESTION

(i) $\lim_{x \to 0} \frac{x}{\sqrt{5-x}-\sqrt{5+x}}$

ASRQ

(ii) $\lim_{x \to \frac{\pi}{2}} \tan^2 x \left\{\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2}\right\}$

TAH 4B

$$\lim_{x\to 0}\frac{x}{\sqrt{5-x}-\sqrt{5+x}}$$

$$\lim_{x\to 0}\frac{x\left(\sqrt{5-x}+\sqrt{5+x}\right)}{(5-x)(5+x)} \Rightarrow \lim_{x\to 0}\frac{\sqrt{5-x}+\sqrt{5+x}}{-2}$$

$$\Rightarrow \frac{\sqrt{5}+\sqrt{5}}{-2} = -\sqrt{5}$$

Deepak from Batala
punjab

QUESTION

Evaluate: $\lim_{x\to 0} \frac{x}{\sqrt{a+x}-\sqrt{a-x}}$ Ans $\sqrt{a}$

Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$ Ans. $1/2$

Evaluate: $\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ Ans. $2/3\sqrt{3}$

Evaluate: $\lim_{x\to -1} \frac{\sqrt[3]{7-x}-2}{(x+1)}$ Ans. $-\frac{1}{12}$

Evaluate: $\lim_{x\to 1} \left(\frac{\sqrt{x^2+8}-\sqrt{10-x^2}}{\sqrt{x^2+3}-\sqrt{5-x^2}}\right)$ Ans. $\frac{2}{3\sqrt{3}}$

Evaluate: $\lim_{x\to 0} \left(\frac{\sqrt{1+x}-1}{x}\right)$ Ans. $1/2$

Evaluate: $\lim_{x\to 0} \frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$ Ans. $1/3$

TAH 5

Q(i) $\lim_{x\to 0} \frac{x}{\sqrt{a+x}-\sqrt{a-x}}$

$\lim_{x\to 0} \frac{x(\sqrt{a+x}+\sqrt{a-x})}{a+x-(a-x)} \Rightarrow \lim_{x\to 0} \frac{\sqrt{a+x}+\sqrt{a-x}}{2} = \frac{2\sqrt{a}}{2} = \sqrt{a}$

(ii) $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

$\lim_{x\to 0} \frac{1+x-(1-x)}{2x(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+\sqrt{1-x}} = \frac{1}{2}$

(iii) $\lim_{x\to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$\lim_{x\to a} \frac{(a+2x-3x)}{(3a+x-4x)} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$\lim_{x\to a} \frac{1}{3} \frac{\sqrt{3a+x}+2\sqrt{x}}{(\sqrt{a+2x}+\sqrt{3x})} = \frac{1}{3} \frac{2\sqrt{a}+2\sqrt{a}}{(\sqrt{3a}+\sqrt{3a})}$

$= \frac{4}{3(2\sqrt{3})} = \frac{2}{3\sqrt{3}}$

(iv) $\lim_{x\to -1} \frac{3\sqrt{7-x}-2}{(x+1)}$ $\left[(a-b)(a^2+b^2+ab) = a^3-b^3\right]$

$\lim_{x\to -1} \frac{(3\sqrt{7-x})^3-(3\sqrt{8})^3}{(x+1)\left((7-x)^{2/3}+(8)^{2/3}+(8(7-x))^{1/3}\right)}$

$\lim_{x\to -1} \frac{-(1+x)}{(x+1)\left((7-x)^{2/3}+8^{2/3}+8(7-x)^{1/3}\right)}$

Deepak from Batala punjab

$= \frac{-1}{8^{2/3}+8^{2/3}+8^{2/3}} = \frac{-1}{3(4)} = \frac{-1}{12}$

(v) $\lim_{x\to 1} \frac{\sqrt{x^2+8}-\sqrt{10-x^2}}{\sqrt{x^2+3}-\sqrt{5-x^2}}$

$\lim_{x\to 1} \frac{(x^2+8-10+x^2)}{(x^2+3)-(5-x^2)} \frac{\sqrt{x^2+3}+\sqrt{5-x^2}}{\sqrt{x^2+8}+\sqrt{10-x^2}}$

$\lim_{x\to 1} \frac{(2x^2-2)(\sqrt{x^2+3}+\sqrt{5-x^2})}{(2x^2-2)(\sqrt{x^2+8}+\sqrt{10-x^2})}$

$\lim_{x\to 1} \frac{\sqrt{x^2+3}+\sqrt{5-x^2}}{\sqrt{x^2+8}+\sqrt{10-x^2}} = \frac{2+2}{3+3} = \frac{2}{3}$

(vi) $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

$\lim_{x\to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$

(vii) $\lim_{x\to 0} \frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$

$\lim_{x\to 0} \frac{1+4x-(5+2x)}{(x-2)\sqrt{1+4x}+\sqrt{5+2x}}$

Deepak from Batala punjab

$\lim_{x\to 0} \frac{2(x-2)}{(x-2)(\sqrt{1+4x}+\sqrt{5+2x})}$

$\lim_{x\to 0} \frac{2}{\sqrt{1+4x}+\sqrt{5+2x}} = \frac{2}{1+\sqrt{5}}$ Ans.

Tah-S ① $\lim_{x\to 0} \frac{x}{\sqrt{a+x}-\sqrt{a-x}}$

Solve $= \lim_{x\to 0} \frac{x(\sqrt{a+x}+\sqrt{a-x})}{a+x-(a-x)}$

$= \lim_{x\to 0} \frac{x(\sqrt{a+x}+\sqrt{a-x})}{2x}$

$= \sqrt{a}$

② $\lim_{x\to 0} \left(\frac{\sqrt{1+x}-1}{x}\right)$

$= \lim_{x\to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1}$

$= \frac{1}{2}$

Anshika
From MP

③ $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

$\lim_{x\to 0} \frac{1+x-(1-x)}{2x(\sqrt{1+x}+\sqrt{1-x})}$

$= \frac{1}{2}$

④ $\lim_{x\to 0} \frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$

$\lim_{x\to 0} \frac{-4+2x}{(x-2)(\sqrt{1+4x}+\sqrt{5+2x})}$

$= \frac{2}{1+\sqrt{5}}$

(5) $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$\lim_{x \to a} \frac{a+2x-3x}{3a+x-4x} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$\lim_{x \to a} \frac{a-x}{3(a-x)} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$

$\lim_{x \to 0} \frac{\sqrt{3a} + 2\sqrt{a}}{3(\sqrt{3a} + \sqrt{3a})}$

$\frac{2}{3\sqrt{3}}$

(6) $\lim_{x \to -1} \frac{\sqrt[3]{7-x} - 2}{(x+1)}$

$\lim_{x \to -1} \frac{(\sqrt[3]{7-x})^3 - (2)^3}{(x+1)((7-x)^{2/3} + 4 + 2\sqrt[3]{7-x})}$

$\lim_{x \to -1} \frac{7-x-8}{(x+1)((7-x)^{2/3} + 4 + 2\sqrt[3]{7-x})}$

$= \frac{-1}{12}$

(7) $\lim_{x \to 1} \frac{\sqrt{x^2+8} - \sqrt{10-x^2}}{\sqrt{x^2+3} - \sqrt{5-x^2}}$

$\lim_{x \to 1} \frac{x^2+8-(10-x^2)}{(x^2+3)-(5-x^2)} \times \frac{\sqrt{x^2+3} + \sqrt{5-x^2}}{\sqrt{x^2+8} + \sqrt{10-x^2}}$

$\lim_{x \to 1} \frac{2x^2-2}{2x^2-2} \times \frac{\sqrt{x^2+3} + \sqrt{5-x^2}}{\sqrt{x^2+8} + \sqrt{10-x^2}}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Anshika
From MP

THANK YOU



PRAYAS

JEE 2025

Lecture-04 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Question Practice on Methods to Evaluate Limits & Trigonometric Limits
2. Exponential & Logarithmic Limits

Recap:

① $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$

② $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x} = \lim_{x \to 0} x \operatorname{cosec} x = \lim_{x \to 0} \frac{\sin^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\sin^{-1} x}$

③ $\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x} = \lim_{x \to 0} x \cot x = \lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\tan^{-1} x}$

$\lim_{x \to 0} \left[\frac{\sin x}{x}\right] = 0$

$\lim_{x \to 0} \left[\frac{\tan x}{x}\right] = 1$

$\frac{\sin x}{x} \to$ as $x \to 0$



$\leftarrow \frac{\tan x}{x}$ as $x \to 0$



Homework Discussion

QUESTION [JEE Advanced 2012]

TAH 2

If $\lim\limits_{x\to-\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) $a=1, b=4$

(B) $a=1, b=-4$

(C) $a=2, b=-3$

(D) $a=2, b=3$

$\lim\limits_{x\to-\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)$

$\lim\limits_{x\to-\infty}\frac{(x^2+x+1)-(ax+b)(x+1)}{x+1}$

$\lim\limits_{x\to-\infty}\frac{x^2+x+1-(ax^2+ax+bx+b)}{x+1}$

$\lim\limits_{x\to-\infty}\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}$

$x=-t$

as $x\to-\infty$, $t\to\infty$.

$\lim\limits_{t\to\infty}\frac{t^2(1-a)-t(1-a-b)+1-b}{1-t}$

for limit to exist $1-a=0$

$a=1$

Ans. B

$$\lim_{t \to \infty} \frac{-t(-b) + 1 - b}{1 - t} = \frac{b}{-1} = 4 \text{ (given)}$$

$b = -4$

QUESTION [JEE Mains 2022 (25 July)] TAH 3

If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to:

A 4

B -8

C -4

D 8

$$\lim_{n\to\infty}\sqrt{n^2-n-1}+(n\alpha+\beta)$$

$$\lim_{n\to\infty}\frac{n^2-n-1-(n\alpha+\beta)^2}{\sqrt{n^2-n-1}-(n\alpha+\beta)}=\lim_{n\to\infty}\frac{n^2-n-1-(n^2\alpha^2+\beta^2+2n\alpha\beta)}{\sqrt{n^2-n-1}-(n\alpha+\beta)}$$

$$=\lim_{n\to\infty}\frac{n^2(1-\alpha^2)-n(1+2\alpha\beta)-1-\beta^2}{n\left(\sqrt{1-\frac{1}{n}-1/n^2}-(\alpha+\frac{\beta}{n})\right)}$$

$1-\alpha^2=0$ (0/0) $\Rightarrow \alpha=-1,1$

$\frac{-1-2\alpha\beta}{1-\alpha}=0$.

Clearly $\alpha\neq 1$ $\quad -1-2(-1)\beta=0$

$\beta=1/2$.

$8(\alpha+\beta)=8(-1+1/2)=-4$

$$=\lim_{n\to\infty}\frac{-n(1+2\alpha\beta)-1-\beta^2}{n\left[\sqrt{1-1/n-1/n^2}-(\alpha+\beta/n)\right]}=\lim_{n\to\infty}\frac{-n\left(1+2\beta\alpha-\frac{(1+\beta^2)}{n}\right)}{n(\quad}$$

Ans. C

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION [JEE Mains 2024 (1 Feb)]

ASRQ
Hybrid Function+Limits

Let $f(x)=\begin{cases}x-1, & x \text{ is even},\\ 2x, & x \text{ is odd},\end{cases}$ $x \in N$.

If for some $a \in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where $[t]$ denotes the greatest integer less than or equal to t, is equal to :

(A) 169

(B) 121

(C) 225

(D) 144

$\left[\frac{12-h}{12}\right]=\left[1-\frac{h}{12}\right]$

$f(x)=\begin{cases}x-1 & x \text{ is even}\\ 2x & x=\text{odd}\end{cases}$ (x-1: odd; 2x: even)

even → f → odd

odd → f → even

$f(f(f(a)))=21$ (f(f(f(a))): even; 21: odd)

$f(f(a))-1=21$

$f(f(a))=22 \Rightarrow 2f(a)=22$ (f(f(a)): odd)

$f(a)=11 \Rightarrow a-1=11 \Rightarrow a=12$. (f(a): even)

$\lim\limits_{x\to 12^-}\left(\frac{|x|^3}{12}-\left[\frac{x}{12}\right]\right)$ ($x\to 12^-$)

$=\lim\limits_{x\to 12^-}\left(\frac{x^3}{12}-0\right)=\frac{12^3}{12}=144$.

Ans. D

QUESTION [JEE Mains 2023 (25 Jan)] ASRQ

Tan 1

The value of $\lim\limits_{n\to\infty} \dfrac{1+2-3+4+5-6+\ldots.+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(A) $\dfrac{3}{2\sqrt{2}}$

(B) $\dfrac{3}{2}(\sqrt{2}+1)$

(C) $3(\sqrt{2}+1)$

(D) $\dfrac{\sqrt{2}+1}{2}$

$$\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots+3n-2+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

M①

$$\lim_{n\to\infty} \frac{(1+4+7+\cdots+3n-2)+(2+5+8+\cdots+(3n-1))-(3+6+9+\cdots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

(n terms) (n terms) (n terms.)

M②

$$\lim_{n\to\infty} \frac{(1+2+3+4+5+6+\cdots+(3n-2)+(3n-1)+3n)-2(3+6+9+\cdots+3n)}{\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

QUESTION [JEE Mains 2024 (6 April)]

Tan2

ASRQ
Hybrid (Seq+ Limits)

$$\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots\cdots+n^2)}$$ is equal to:

A $\frac{2}{3}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D $\frac{1}{3}$

Ans. D

QUESTION [JEE Mains 2023 (6 April)]

Tan 3

ASRQ
Hybrid (Seq+ Limits)

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right) \text{ is}$$

A $1/\sqrt{d}$

B 1

C 0

D $\sqrt{d}$

QUESTION [JEE Mains 2024 (4 April)]

ASRQ

Tan4

If $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m,n)=1$, then $8m+12n$ is equal to

Ans. 100

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 5

Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in R - \{1, 0\}$, $\lim\limits_{x \to 0} \left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to :

(A) 1

(B) 2

(C) 0

(D) 1/2

Ans. B

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 6

$$\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}},\ (a\neq 0) \text{ is equal to:}$$

A $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

B $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

C $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

D $\left(\frac{2}{9}\right)^{\frac{4}{3}}$

Ans. B

QUESTION [JEE Mains 2022 (27 June)]

TAH 7

Let a be an integer such that $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer $\leq$ t. Then a is equal to:

$[x+m]=[x]+m$, $m\in I$

$\lim_{x\to 7}\frac{18-(1+[-x])}{[x]-3a}$

$\lim_{x\to 7}\frac{17-[-x]}{[x]-3a}$

(A) -6

(B) -2

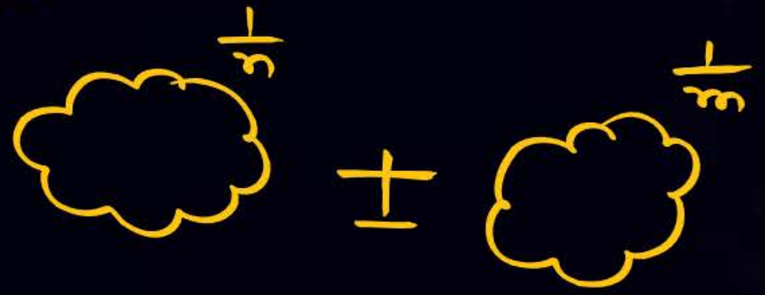
(C) 2

(D) 6

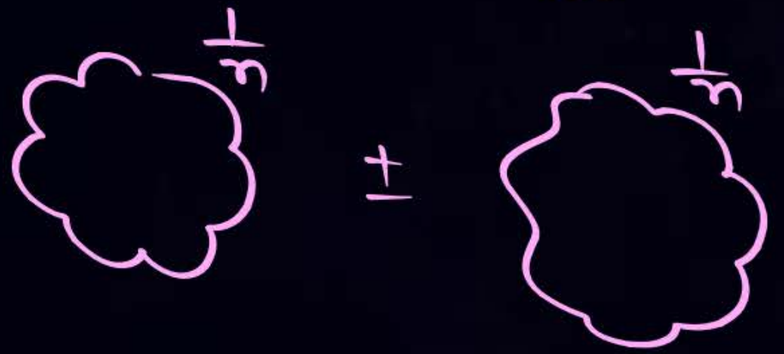
Ans. A

Using Binomial Theorem

🤔🤔Kaha Use Karnaa hai???



$n, m \in N$
$n \neq m$.



$n \in N$
$n \geq 4$

QUESTION

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$$

M① Rationalization (already done)

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}$$

$$\lim_{x\to 0}\frac{\left(1+\frac{1}{3}x+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}x^2+\cdots-\infty\right)-\left(1+\frac{1}{3}(-x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-x)^2+\cdots-\infty\right)}{x}$$

$$\lim_{x\to 0}\frac{2/3\,x+2\cdot\left(\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3\right)+\cdots-\infty}{x}=2/3$$ Ans

QUESTION

$$\lim_{x\to\infty} \sqrt[3]{x^3+3x^2} - \sqrt{x^2-2x} \qquad (\infty-\infty)$$

$$\lim_{x\to\infty} (x^3)^{\frac{1}{3}}\left(1+\frac{3}{x}\right)^{\frac{1}{3}} - (x^2)^{\frac{1}{2}}\left(1-\frac{2}{x}\right)^{\frac{1}{2}}$$

$$\lim_{x\to\infty} x\left[\left(1+\frac{3}{x}\right)^{1/3} - \left(1-\frac{2}{x}\right)^{1/2}\right]$$

$$= \lim_{x\to\infty} x\left[\left(1+\frac{1}{3}\cdot\frac{3}{x}+\frac{1/3(1/3-1)}{2!}\frac{9}{x^2}+\cdots\infty\right)-\left(1+\frac{1}{2}\left(\frac{-2}{x}\right)+\frac{1/2\cdot(1/2-1)}{2!}(-2/x)^2+\cdots\infty\right)\right]$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1/3(1/3-1)}{2!}\frac{9}{x}+\cdots\infty\right)-\left(-1+\frac{1/2(1/2-1)}{2!}\left(\frac{4}{x}\right)+\cdots\infty\right)\right]$$

$$= 1-(-1)=2$$

QUESTION [JEE Advanced 2009]

Let $L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

A $a = 2$

B $a = 1$

C $L = \frac{1}{64}$

D $L = \frac{1}{32}$

$$L = \lim_{x\to 0} \frac{a - \sqrt{a^2-x^2} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{a - a\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{a - a\left(1 + \frac{1}{2}\left(-\frac{x^2}{a^2}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{x^2}{a^2}\right)^2 + \cdots - \infty\right) - x^2/4}{x^4}$$

$$= \lim_{x\to 0} \frac{x^2\left(\frac{1}{2a} - \frac{1}{4}\right) - \frac{1/2(1/2-1)}{2!}\left(\frac{x^4}{a^3}\right) + \frac{1/2(1/2-1)(1/2-2)}{3!}\frac{x^6}{a^5} + \cdots - \infty}{x^4}$$

for limit to exist

$$\frac{1}{2a} - \frac{1}{4} = 0$$

$$a = 2$$

$$\lim_{x \to 0} \frac{-\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{x^4}{a^3}\right) + \frac{1}{3!}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{x^6}{a^5} - \cdots - \infty}{x^4}$$

$$\lim_{x \to 0} \frac{-1/2 \cdot (-1/2)}{2!} \cdot \frac{1}{a^3} + \frac{1/2\,(-1/2)\,(-3/2)}{3!}\frac{x^2}{a^5} + \cdots - \infty$$

$$= \frac{1}{8} \cdot \frac{1}{a^3} = \frac{1}{64}$$

QUESTION

Lilliput Swaal

$$\lim_{x\to 0}\frac{1-\cos 5x}{3x^2}$$

$$\lim_{x\to 0}\frac{1-\cos 5x}{(5x)^2}\cdot\frac{25x^2}{3x^2}$$

$$\frac{1}{2}\cdot\frac{25}{3}=\frac{25}{6}.$$

$$\lim_{x\to a}\frac{1-\cos\square}{\square^2}=\frac{1}{2}$$

as $x-a\to 0$

QUESTION

Lilliput Swaal

$$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$$

$$\lim_{x\to 0}\frac{1+\tan x-(1+\sin x)}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{x^3\left(\quad\right)_2}=\lim_{x\to 0}\underset{\Gamma^1}{\frac{\sin x}{x}}\underset{\Gamma^{\frac{1}{2}}}{\frac{(1-\cos x)}{x^2}}\underset{\Gamma^1}{\frac{1}{\cos x}}\underset{\Gamma^{1/2}}{\frac{1}{(\quad)^2}}$$

$$=1/4$$

QUESTION [JEE Mains 2019] Lilliput Swaal

$$\lim_{x\to 0} \frac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$$ is equal to:

A 1

B 4

C 0

D 2

$$\lim_{x\to 0} \frac{4x}{\tan 4x} \left(\frac{\tan 2x}{2x}\right)^2 \cdot x \left(\frac{x}{\sin x}\right)^2 \frac{4x^2}{4x} \cdot \frac{1}{x^2}$$

$\hookrightarrow 1$ $\hookrightarrow 1$ $\hookrightarrow 1$

$= 1$

ASNC

Whenever we have limit $x \to a$ where 'a' is a non zero number, in such problems it is better to put $x = a+t$ because maximum formula we have in limits are at $x \to 0$



Khud ki Buddhi Kaa Istmaal naa karnaa Ghatak Saabit Ho Saktaa Hai (Conditions Apply: Honi bhi Chahiyay)

QUESTION [JEE Mains 2017] **Lilliput Swaal**

$$\lim_{x\to\pi/2} \frac{\cot x-\cos x}{(\pi-2x)^3} \text{ equals :}$$

(A) $\frac{1}{4}$

(B) $\frac{1}{24}$

(C) $\frac{1}{16}$

(D) $\frac{1}{8}$

Kyaa apne yahi kiyaa ??

$\lim_{x\to\pi/2} \frac{\sin x}{x} = 1$

Toh aap —— hai!!

$\lim_{x\to\pi/2} \frac{\sin x}{x} = \frac{2}{\pi}$ ← $\pi/2$

$\lim_{x\to\pi/2} \frac{\cot x-\cos x}{(\pi-2x)^3}$ $\left(\frac{0}{0}\right)$

$x = \pi/2 + t$ as $x \to \pi/2$, $t \to 0$

$\lim_{t\to 0} \frac{\cot(\pi/2+t)-\cos(\pi/2+t)}{(\pi-2(\pi/2+t))^3}$

$\lim_{t\to 0} \frac{-\tan t+\sin t}{-8t^3} = \lim_{t\to 0} \frac{\sin t\left(\frac{1}{\cos t}-1\right)}{8t^3} = \lim_{t\to 0} \frac{\sin t}{t}\frac{(1-\cos t)}{t^2}\cdot\frac{1}{8\cos t}$

$= 1\cdot\frac{1}{2}\cdot\frac{1}{8} = \frac{1}{16}.$

QUESTION [JEE Mains 2019]

Brain Lagaoo

$$\lim_{x\to\pi/4}\frac{\cot^3 x-\tan x}{\cos\left(x+\pi/4\right)}\text{ is :}$$

(A) $4\sqrt{2}$

(B) 8

(C) 4

(D) $8\sqrt{2}$

$$= \lim_{x\to\pi/4}\frac{\frac{1}{\tan^3 x}-\tan x}{\cos(x+\pi/4)}$$

$$= \lim_{x\to\pi/4}\frac{1-\tan^4 x}{\cos(x+\pi/4)\cdot\tan^3 x}$$

masoom

$$= \lim_{x\to\pi/4}\frac{(1-\tan x)(1+\tan x)(1+\tan^2 x)}{\cos(x+\pi/4)\cdot\tan^3 x}$$

Dorinde. masoom

$$= \lim_{x\to\pi/4}\frac{\cos x-\sin x}{\cos x\cdot\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})^2(\sqrt{2})^2}{(\sqrt{2})}$$

$$= \lim_{x\to\pi/4}\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)}{\cos x\cdot\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})(\sqrt{2})}{(\sqrt{2})}$$

$$= \lim_{x\to\pi/4}\frac{\sqrt{2}\cos(x+\pi/4)}{\cos x\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})(\sqrt{2})}{(\sqrt{2})}$$

$$\frac{\sqrt{2}\cdot 2\cdot 2}{\frac{1}{\sqrt{2}}\cdot 1}=8.$$

QUESTION

The value of $\lim_{x \to \pi/6} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$ is

Tan9

$x = \frac{\pi}{6} + t$

A 1/12

B 1/24

C 1/36

D 1/48

QUESTION

(ASRQ)

The limiting value of the function $f(x) = \dfrac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ when $x \to \dfrac{\pi}{4}$ is

A $\sqrt{2}$

B $1/\sqrt{2}$

C $3\sqrt{2}$

D $3/\sqrt{2}$

$x = \pi/4 + t$

$$\lim_{t \to 0} \frac{2\sqrt{2} - (\cos(\pi/4 + t) + \sin(\pi/4 + t))^3}{1 - \sin 2(\pi/4 + t)}$$

$\sin(\pi/2 + 2t)$

$$\lim_{t \to 0} \frac{2\sqrt{2} - \left(\frac{1}{\sqrt{2}}\cos t - \frac{\sin t}{\sqrt{2}} + \frac{\sin t}{\sqrt{2}} + \frac{\cos t}{\sqrt{2}}\right)^3}{1 - \cos 2t}$$

$$\lim_{t \to 0} \frac{2\sqrt{2} - (\sqrt{2}\cos t)^3}{1 - \cos 2t}$$

$$\lim_{t \to 0} \frac{2\sqrt{2}(1 - \cos^3 t)}{1 - \cos 2t} = \lim_{t \to 0} \frac{2\sqrt{2}(1 - \cos t)(1 + \cos^2 t + \cos t)}{(1 - \cos 2t)}$$

$$\lim_{t \to 0} \frac{2\sqrt{2}\,(1-\cos t)}{t^2} \cdot \frac{4t^2}{(1-\cos 2t)} \cdot \frac{1}{4} \qquad (1+\cos^2 t+\cos t)$$

$$= \cancel{2\sqrt{2}} \cdot \cancel{\frac{1}{2}} \cdot \frac{1}{1/2} \cdot \frac{1}{4} \cdot 3$$

$$= 2\sqrt{2} \cdot \frac{3}{4}$$

$$= \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}.$$

ASNC

Machli Jal ki Rani Hai Jeevan uska Paani Hai Haath lagaoge Dar Jayegi bahar nikalogay Mar jayaygi Issi Tarah....Limit x ki power ki r Rani hai jyaada doge mar jayaygi kum doge chal jayaygi



Khud ki Buddhi Kaa Istmaal naa karnaa Ghatak Saabit Ho Saktaa Hai (Conditions Apply: Honi bhi Chahiyay)

QUESTION

$$\lim_{x\to 0}\frac{3\sin x - x^2 + x^3}{\tan x + 2\sin^2 x + 5x^4}$$

Divide Nr & Den by x

$$\lim_{x\to 0}\frac{3\frac{\sin x}{x} - x + x^2}{\frac{\tan x}{x} + 2\frac{\sin x}{x}\cdot\sin x + 5x^3}$$

$$= \frac{3\cdot 1 - 0 + 0}{1 + 0 + 0} = \underline{3} \text{ Ans}$$

Ex: $\lim_{x\to 0}\frac{\sin x}{x} = 1$

Ex: $\lim_{x\to 0}\frac{\sin x}{x^2} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{x}$ (1) D.N.E

Ex: $\lim_{x\to 0}\frac{1-\cos x}{x^4}$

$= \lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1}{x^2}$ DNE

Ex: $\lim_{x\to 0}\frac{\sin x}{x^{1/3}} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{x}{x^{1/3}}$

$= \lim_{x\to 0}\frac{\sin x}{x}\,x^{2/3} = 0$

QUESTION

$$\lim_{x \to 0} \frac{\tan^3 x + \tan^3 2x + \tan^2 3x}{\tan^3 4x + \tan^2 5x + \tan^3 6x}$$



ASNC

Never Apply Limit Partially
Jab Bhi Question may limit Rakhi
Jati Hai to Puray Question Mai eak
Saath Limit rakhte Hai



Agar Aisaa nahin kiya to Chamarkaari Parinaam Parpt Hone ki Puri Puri Sambhavna Hai !!!

QUESTION

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{\tan^2 x}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x\cdot\cos 2x}{\tan^2 x\,(1+\cos x\cdot\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x(2\cos^2 x-1)}{\tan^2 x\ (\ )_2} = \lim_{x\to 0}\frac{-2\cos^4 x+\cos^2 x+1}{\tan^2 x\ (\ )_2}$$

$$= \lim_{x\to 0}\frac{-2\cos^4 x+2\cos^2 x-\cos^2 x+1}{\tan^2 x\ (\ )_2}$$

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{\sin^4 x} = \lim_{x\to 0}\frac{1-\cos\left(2\sin^2\frac{x}{2}\right)}{\left(2\sin^2\frac{x}{2}\right)^2}\cdot\frac{\left(2\sin^2\frac{x}{2}\right)^2}{\sin^4 x}$$

$$= \lim_{x\to 0}\frac{1-\cos(2\sin^2 x/2)}{(2\sin^2 x/2)^2}\cdot\frac{4\sin^4 x/2}{(2\sin x/2\cos x/2)^4}$$

$$= 1/2\cdot\frac{1}{4} = 1/8$$

$$\lim_{x\to 0}\frac{2\cos^2 x(1-\cos^2 x)+1-\cos^2 x}{\tan^2 x\ (\ )}$$

$$\lim_{x\to 0}\frac{(1-\cos^2 x)(1+2\cos^2 x)}{\tan^2 x\ (\ )}$$

$$\lim_{x\to 0}\frac{\sin^2 x(1+2\cos^2 x)\cos^2 x}{\sin^2 x\ (\ )_2}$$

$$= 3/2$$

Don't Apply Limit Partially. Lallu naa Bano!!!

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{1-\cos\left(\frac{1-\cos x}{x^2}\cdot x^2\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{1-\cos\left(x^2/2\right)}{x^4} \qquad \text{as } \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$\lim_{x\to 0}\frac{1-\cos\left(x^2/2\right)}{x^4/4\cdot 4}=\frac{1}{8}$ is wrong although the answer may be correct.

QUESTION [JEE Mains 2021 (August)]

$$\lim_{x\to 0}\frac{\sin^2(\pi\cos^4 x)}{x^4}$$ is equal to

(A) π^2

(B) $2\pi^2$

(C) $4\pi^2$

(D) 4π

Kyaa appnae bhi aisaa Sochaa!!

$\lim_{x\to 0}\frac{\sin^2(\pi\cos^4 x)}{(\pi\cos^4 x)^2}\cdot\frac{\pi\cos^4 x}{x^4}$

→ Toh app ——— hai

$\sin(\pi-\theta)=\sin\theta$

$\lim_{x\to 0}\frac{\sin^2(\pi-\pi\cos^4 x)}{x^4}$

$\lim_{x\to 0}\left(\frac{\sin(\pi(1-\cos^4 x))}{\pi(1-\cos^4 x)}\right)^2$ ($\pi(1-\cos^4 x)\to 0$)

$\left(\frac{1-\cos x}{x^2}\right)^2$

$\frac{\pi^2(1-\cos x)^2(1+\cos x)^2(1+\cos^2 x)^2}{x^4\times(1/2)^2}$

$=1\cdot\pi^2\cdot\frac{1}{4}\cdot 4\cdot 4$

QUESTION [JEE Mains 2021 (March)]

Tah 11

The value of the limit $\lim\limits_{\theta \to 0} \frac{\tan(\pi \cos^2\theta)}{\sin(2\pi \sin^2\theta)}$ is equal to:

A $-1/2$

B $-1/4$

C 0

D $1/4$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Homework From Module

LCD

PRARAMBH : Q1 TO Q11,Q16 TO Q27,Q35 TO Q39

(Solution to TAH)

QUESTION

$$\lim_{n\to\infty} \frac{(1 + 2 + 3 + \ldots\ldots + n)(1^3 + 2^3 + 3^3 + \ldots. + n^3)}{(1^2 + 2^2 + 3^2 + \ldots.. + n^2)^2}$$

Ans. 9/8

$$\lim_{n\to\infty} \frac{(1+2+3+\cdots+n)(1^3+2^3+3^3+\cdots+n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$$

$$\lim_{n\to\infty} \frac{\left(\frac{n(n+1)}{2}\right)\left(\frac{n^2(n+1)^2}{4}\right)}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2} = \lim_{n\to\infty} \frac{n^2\left(\frac{1+1/n}{2}\right)n^4\left(\frac{(1+1/n)^2}{4}\right)}{n^6\left(\frac{(1+1/n)(2+1/n)}{6}\right)^2}$$

TAH 1
ABHIJEET KUMAR PATRA
ODISHA

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}{\left(\frac{2}{6}\right)^2} = \frac{\frac{1}{8}}{\frac{4}{36}}$$

$$= \frac{1}{8} \times \frac{36^9}{4} = \frac{9}{8} \quad (\text{Ans})$$

TAH-1

Q. $\lim_{n\to\infty} \dfrac{(1+2+3+4+\ldots+n)(1^3+2^3+3^3+\ldots+n^3)}{(1^2+2^2+3^2+\ldots+n^2)^2}$

$\lim_{n\to\infty} \dfrac{\left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2}\right)^2}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2}$

$\lim_{n\to\infty} \dfrac{n^8(n+1)(n+1)^2}{8} \Big/ \dfrac{n^4(n+1)^2(2n+1)^2}{36}$

$\lim_{n\to\infty} \dfrac{9}{2} \times \dfrac{n(n+1)}{(2n+1)^2}$

$\lim_{n\to\infty} \dfrac{9}{2} \dfrac{n^2(1+1/n)}{n^2(2+\frac{1}{n})^2}$

$\lim_{n\to\infty} \dfrac{9}{2} \dfrac{(1+1/n)}{(2+\frac{1}{n})^2} = \dfrac{9}{8}$

Rajkanya From Bihar

TAH 1 Q $\lim_{n\to\infty} \frac{(1+2+3+\cdots+n)(1^3+2^3+3^3+\cdots+n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$

Ans $\lim_{n\to\infty} \frac{(1+2+3+\cdots n)(1^3+2^3+3^3+\cdots n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$

$$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2} \times \left(\frac{n(n+1)}{2}\right)^2}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2}$$

$$\lim_{n\to\infty} \frac{\{n(n+1)\}^3}{8} \times \frac{36}{\{n(n+1)(2n+1)\}^2}$$

$$\frac{36}{8\times 4} = \frac{9}{8}$$

TAH 1

SHIVAM HARSHWARDHAN
FROM HARIDWAR

QUESTION [JEE Advanced 2012]

If $\lim_{x \to -\infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then

A $a = 1, b = 4$

B $a = 1, b = -4$

C $a = 2, b = -3$

D $a = 2, b = 3$

Ans. B

If $\lim_{x \to -\infty} \left(\frac{x^2+x+1}{x+1} - ax - b \right) = 4$, then (Adv 2012)

(A) $a=1, b=4$
(B) $a=1, b=-4$
(C) $a=2, b=-3$
(D) $a=2, b=3$

$$\lim_{x \to -\infty} \left(\frac{x^2+x+1}{x+1} - (ax+b) \right) = 4$$

$$\lim_{x \to -\infty} \left(\frac{x^2+x+1-(x+1)(ax+b)}{x+1} \right) = 4$$

TAH 2
ABHIJEET KUMAR
PATRA
ODISHA

$$\lim_{x \to -\infty} \frac{x^2+x+1-(ax^2+(b+a)x+b)}{x+1} = 4$$

Should be 0

$$\lim_{x \to -\infty} \frac{x^2(1-a)+x(1-a-b)+1-b}{x+1} = 4$$

$\Rightarrow 1-a=0$

$a=1$

$$\lim_{x \to -\infty} \frac{x(-b)+1-b}{x+1} = 4 \Rightarrow \frac{-b}{1}$$

$$= 4$$

$\therefore a, b = -4$

TAH - 2.

Q. If $\lim_{x \to -\infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1}{1 - t} + at - b \right) = 4$ | Put $x = -t$, as $x \to -\infty$, $t \to \infty$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1 + (at - b)(1 - t)}{1 - t} \right) = 4$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1 + at - at^2 - b + bt}{(1 - t)} \right) = 4$

$\lim_{t \to \infty} \left(\frac{(1 - a)t^2 + (a + b - 1)t + 1 - b}{1 - t} \right) = 4$

here, $1 - a = 0$

$a = 1$ [degree of N^r = degree of D^r]

$\lim_{t \to \infty} \left(\frac{(a + b - 1)t + 1 - b}{1 - t} \right) = 4$

$\lim_{t \to \infty} \left(\frac{t((a + b - 1) + 1/t - b/t)}{t(\frac{1}{t} - 1)} \right) = 4$

$\frac{a + b - 1}{-1} = 4$

$a + b - 1 = -4$

$a + b = -3$

as, $a = 1$, $b = -4$

Rajkanya From Bihar

QUESTION [JEE Mains 2022 (25 July)]

If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to:

A 4

B -8

C -4

D 8

Ans. C

TAH 3

$\lim_{n\to\infty} (\sqrt{n^2-n+1} - n\alpha + \beta) = 0$

On Rationalising,

$\lim_{n\to\infty} \left(\frac{n^2-n+1 - (n\alpha+\beta)^2}{\sqrt{n^2-n+1} - (n\alpha+\beta)} \right) = 0$

$\lim_{n\to\infty} \left(\frac{n^2-n+1 - n^2\alpha^2 - 2n\alpha\beta - \beta^2}{\sqrt{n^2-n+1} - (n\alpha) - \beta} \right) = 0$

$\lim_{n\to\infty} \left(\frac{n^2(1 - 1/n + 1/n^2 - \alpha^2 - 2\alpha\beta/n - \beta^2/n^2)}{n[\sqrt{1 - 1/n + 1/n^2} - \alpha - \beta/n]} \right) = 0$

$\lim_{n\to\infty} \left(\frac{(1-\alpha^2)n - (1 + 2\alpha\beta) - 1/n - \beta/n}{(\sqrt{1 - 1/n + 1/n^2} - \alpha - \beta/n)} \right) = 0$

for lim to be defined $\rightarrow 1 - \alpha^2 = 0$

$\alpha = \pm 1$

$\therefore \frac{-(2\alpha\beta+1)}{1-\alpha} \Rightarrow \alpha \neq 1$

Hence, $\alpha = (-1)$

$\therefore (2\alpha\beta + 1) = 0$

$\alpha\beta = +1/2$

$\beta = 1/2$ $\quad \alpha = -1$

$8(\alpha+\beta) = 8(-1+1/2)$

$= -8/2$

$\therefore 8(\alpha+\beta) = -4$ Ans

Rupan karmakar ❤️from Port Blair

Q) If $\lim_{n\to\infty} (\sqrt{n^2-n-1} + n\alpha + \beta) = 0$, then $8(\alpha+\beta)$ is equal to:

$\lim_{n\to\infty} (n(\sqrt{1 - 1/n - 1/n^2} + \alpha + \beta/n) = 0$

$\lim_{n\to\infty} \frac{n^2 - n - 1 - (n\alpha+\beta)^2}{\sqrt{n^2-n-1} - (n\alpha+\beta)}$

SHRADDHA TAMRAKAR
JABALPUR M.P

$\lim_{n\to\infty} \frac{n^2(1-\alpha^2) + n(-1 - 2\alpha\beta) + (-1-\beta^2)}{\sqrt{n^2-n-1} - (n\alpha+\beta)} = 0$

only possibility if $n^2 = 0$

$1 - \alpha^2 = 0$

$\pm 1 = \alpha$

$\lim_{n\to\infty} \frac{n[(-1-2\alpha\beta) + (-\frac{1-\beta^2}{n})]}{n(\sqrt{1 - 1/n - 1/n^2} - (\alpha + \beta/n))}$

$\frac{-1-2\alpha\beta}{1-\alpha} = 0 \rightarrow \alpha \neq 1$

$\alpha = -1$

$-1 - 2(-1)\beta = 0$

$\beta = 1/2$

$\therefore 8(\alpha+\beta) = 8(-1/2)$

$= -4$ Ans

TAH 3

QUESTION

(RPP 3)

In a kite ABCD, $AB = AD$ and $CB = CD$. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$ can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

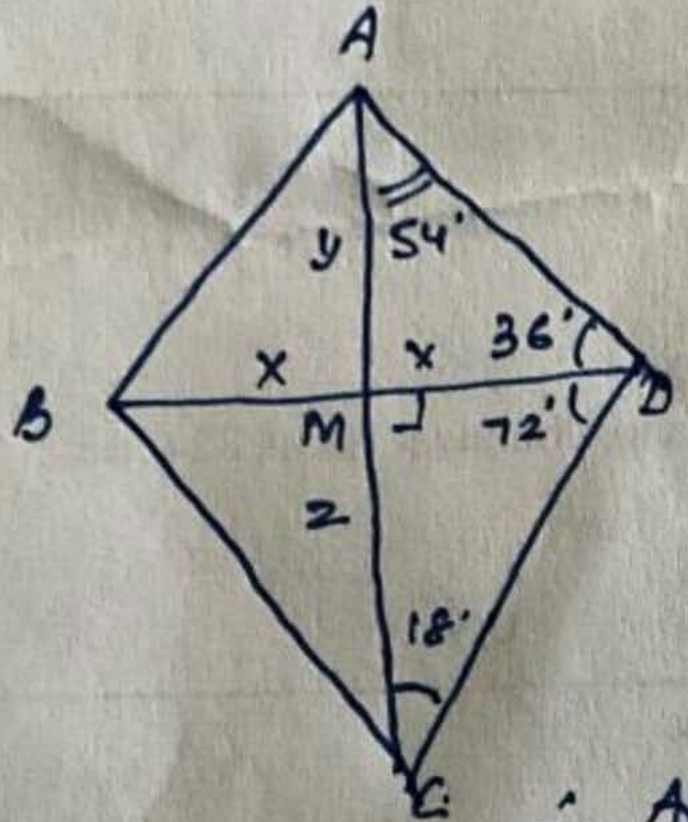
A (1, 2, 1)

B (2, 1, 1)

C (4, 3, 2)

D (1, 1, 2)

Ans. D



$$\frac{Ar(\triangle ABD)}{Ar(\triangle CBD)} = \frac{\frac{1}{2}\cdot 2x\cdot y}{\frac{1}{2}\cdot 2x\cdot z} = \frac{y}{z}$$

Now $y = x\tan 36°$

$z = x\tan 72°$

$$\therefore \frac{Ar\triangle ABD}{Ar(\triangle CBD)} = \frac{x\tan 36°}{x\tan 72°}$$

$$= \frac{\tan 36°}{\frac{2\tan 36°}{1-\tan^2 36°}}$$

$$= \frac{1-\tan^2 36°}{2} \Rightarrow (a,b,c) = (1,1,2)$$ Ans.

Bhoolnaa Mat Yeh Batayaa thaa

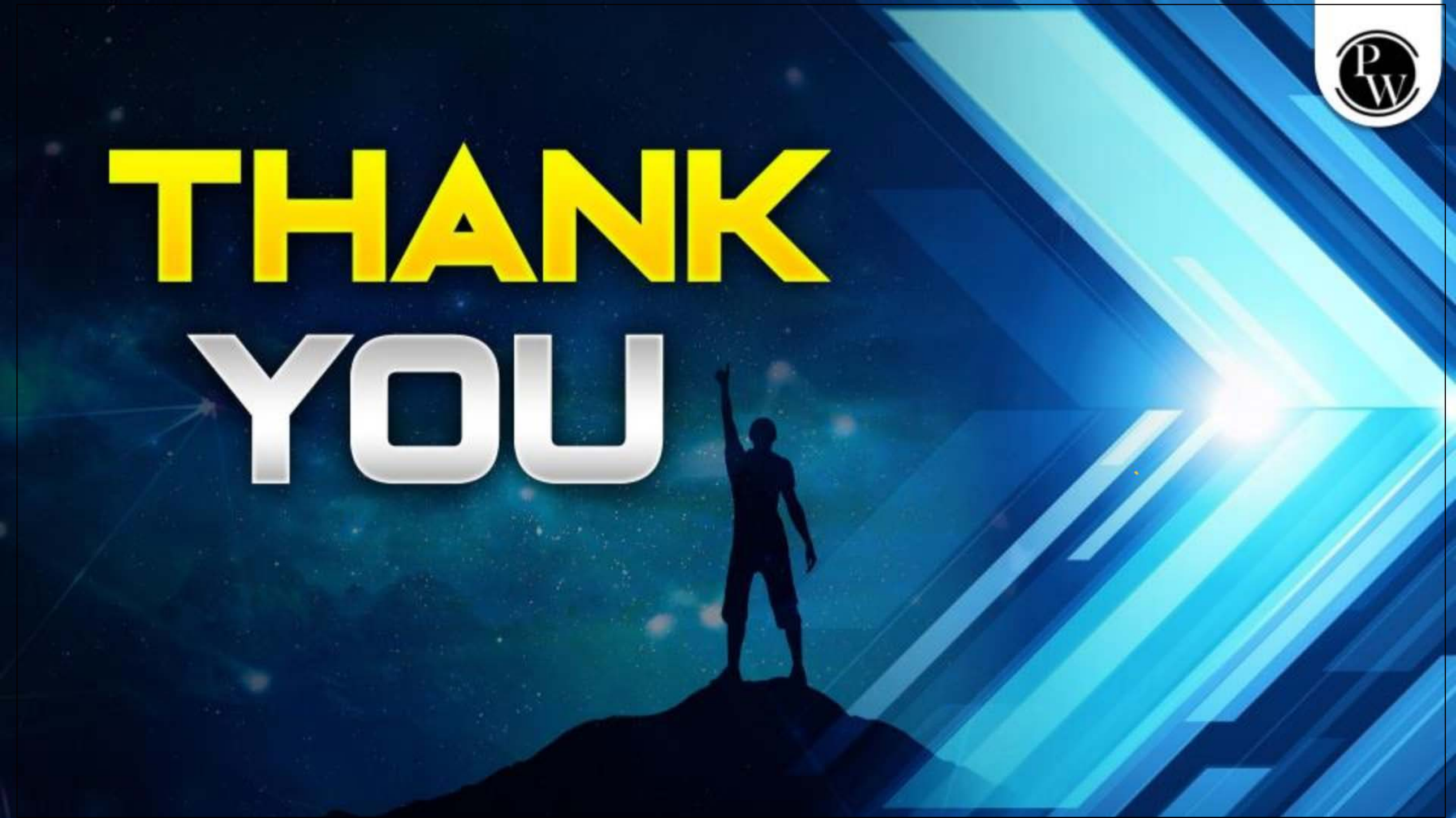
For a square matrix A the Trace of product $A.A^T$ is the sum of squares of all the entries of A

Mann ki Baat Ashish Sir ke Saath

How to reduce silly mistakes in exams??

1. Take enough sleep before exam.
2. Analyze the pattern of mistakes you make!!
3. Daily : Mobile – 15 min Alarm. (4 Questions)




THANK
YOU

PRAYAS

JEE 2025

Lecture-04 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Question Practice on Methods to Evaluate Limits & Trigonometric Limits
2. Exponential & Logarithmic Limits

Recap:

① $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$

② $\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} \frac{x}{\sin x} = \lim_{x \to 0} x \operatorname{cosec} x = \lim_{x \to 0} \frac{\sin^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\sin^{-1} x}$

③ $\lim_{x \to 0} \frac{\tan x}{x} = 1 = \lim_{x \to 0} \frac{x}{\tan x} = \lim_{x \to 0} x \cot x = \lim_{x \to 0} \frac{\tan^{-1} x}{x} = \lim_{x \to 0} \frac{x}{\tan^{-1} x}$

$\lim_{x \to 0} \left[\frac{\sin x}{x}\right] = 0$

$\lim_{x \to 0} \left[\frac{\tan x}{x}\right] = 1$

$\frac{\sin x}{x} \to$ as $x \to 0$



$\leftarrow \frac{\tan x}{x}$ as $x \to 0$



Homework Discussion

QUESTION [JEE Advanced 2012]

TAH 2

If $\lim\limits_{x\to-\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) $a=1, b=4$

(B) $a=1, b=-4$

(C) $a=2, b=-3$

(D) $a=2, b=3$

$\lim\limits_{x\to-\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)$

$\lim\limits_{x\to-\infty}\frac{(x^2+x+1)-(ax+b)(x+1)}{x+1}$

$\lim\limits_{x\to-\infty}\frac{x^2+x+1-(ax^2+ax+bx+b)}{x+1}$

$\lim\limits_{x\to-\infty}\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}$

$x=-t$

as $x\to-\infty$, $t\to\infty$.

$\lim\limits_{t\to\infty}\frac{t^2(1-a)-t(1-a-b)+1-b}{1-t}$

for limit to exist $1-a=0$

$a=1$

Ans. B

$$\lim_{t \to \infty} \frac{-t(-b) + 1 - b}{1 - t} = \frac{b}{-1} = 4 \text{ (given)}$$

$b = -4$

QUESTION [JEE Mains 2022 (25 July)] TAH 3

If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to:

A 4

B -8

C -4

D 8

$$\lim_{n\to\infty}\sqrt{n^2-n-1}+(n\alpha+\beta)$$

$$\lim_{n\to\infty}\frac{n^2-n-1-(n\alpha+\beta)^2}{\sqrt{n^2-n-1}-(n\alpha+\beta)}=\lim_{n\to\infty}\frac{n^2-n-1-(n^2\alpha^2+\beta^2+2n\alpha\beta)}{\sqrt{n^2-n-1}-(n\alpha+\beta)}$$

$$=\lim_{n\to\infty}\frac{n^2(1-\alpha^2)-n(1+2\alpha\beta)-1-\beta^2}{n\left(\sqrt{1-\frac{1}{n}-1/n^2}-(\alpha+\frac{\beta}{n})\right)}$$

$1-\alpha^2=0$ (0/0) $\Rightarrow \alpha=-1,1$

$\frac{-1-2\alpha\beta}{1-\alpha}=0$.

Clearly $\alpha\neq 1$ $\quad -1-2(-1)\beta=0$

$\beta=1/2$.

$8(\alpha+\beta)=8(-1+1/2)=-4$

$$=\lim_{n\to\infty}\frac{-n(1+2\alpha\beta)-1-\beta^2}{n\left[\sqrt{1-1/n-1/n^2}-(\alpha+\beta/n)\right]}=\lim_{n\to\infty}\frac{-n\left(1+2\beta\alpha-\frac{(1+\beta^2)}{n}\right)}{n(\quad}$$

Ans. C

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION [JEE Mains 2024 (1 Feb)]

ASRQ
Hybrid Function+Limits

Let $f(x)=\begin{cases}x-1, & x \text{ is even},\\ 2x, & x \text{ is odd},\end{cases}$ $x \in N$.

If for some $a \in N$, $f(f(f(a)))=21$, then $\lim\limits_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where $[t]$ denotes the greatest integer less than or equal to t, is equal to :

(A) 169

(B) 121

(C) 225

(D) 144

$\left[\frac{12-h}{12}\right]=\left[1-\frac{h}{12}\right]$

$f(x)=\begin{cases}x-1 & x \text{ is even}\\ 2x & x=\text{odd}\end{cases}$ (x-1: odd; 2x: even)

even → f → odd

odd → f → even

$f(f(f(a)))=21$ (f(f(f(a))): even; 21: odd)

$f(f(a))-1=21$

$f(f(a))=22 \Rightarrow 2f(a)=22$ (f(f(a)): odd)

$f(a)=11 \Rightarrow a-1=11 \Rightarrow a=12$. (f(a): even)

$\lim\limits_{x\to 12^-}\left(\frac{|x|^3}{12}-\left[\frac{x}{12}\right]\right)$ ($x\to 12^-$)

$=\lim\limits_{x\to 12^-}\left(\frac{x^3}{12}-0\right)=\frac{12^3}{12}=144$.

Ans. D

QUESTION [JEE Mains 2023 (25 Jan)] ASRQ

Tan 1

The value of $\lim\limits_{n\to\infty} \dfrac{1+2-3+4+5-6+\ldots.+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(A) $\dfrac{3}{2\sqrt{2}}$

(B) $\dfrac{3}{2}(\sqrt{2}+1)$

(C) $3(\sqrt{2}+1)$

(D) $\dfrac{\sqrt{2}+1}{2}$

$$\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots+3n-2+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

M①

$$\lim_{n\to\infty} \frac{(1+4+7+\cdots+3n-2)+(2+5+8+\cdots+(3n-1))-(3+6+9+\cdots+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

(n terms) (n terms) (n terms.)

M②

$$\lim_{n\to\infty} \frac{(1+2+3+4+5+6+\cdots+(3n-2)+(3n-1)+3n)-2(3+6+9+\cdots+3n)}{\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

QUESTION [JEE Mains 2024 (6 April)]

Tan2

ASRQ
Hybrid (Seq+ Limits)

$$\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots\cdots+n^2)}$$ is equal to:

A $\frac{2}{3}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D $\frac{1}{3}$

Ans. D

QUESTION [JEE Mains 2023 (6 April)]

Tan 3

ASRQ
Hybrid (Seq+ Limits)

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right) \text{ is}$$

A $1/\sqrt{d}$

B 1

C 0

D $\sqrt{d}$

QUESTION [JEE Mains 2024 (4 April)]

ASRQ

Tan4

If $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m,n)=1$, then $8m+12n$ is equal to

Ans. 100

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 5

Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in R - \{1, 0\}$, $\lim\limits_{x \to 0} \left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to :

(A) 1

(B) 2

(C) 0

(D) 1/2

Ans. B

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 6

$$\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}},\ (a\neq 0) \text{ is equal to:}$$

A $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

B $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

C $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

D $\left(\frac{2}{9}\right)^{\frac{4}{3}}$

Ans. B

QUESTION [JEE Mains 2022 (27 June)]

TAH 7

Let a be an integer such that $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer $\leq$ t. Then a is equal to:

$[x+m]=[x]+m$, $m\in I$

$\lim_{x\to 7}\frac{18-(1+[-x])}{[x]-3a}$

$\lim_{x\to 7}\frac{17-[-x]}{[x]-3a}$

(A) -6

(B) -2

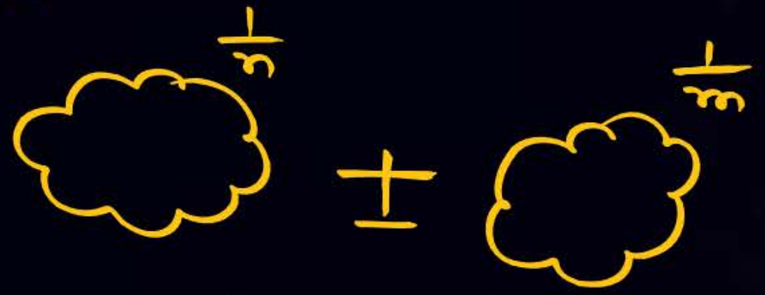
(C) 2

(D) 6

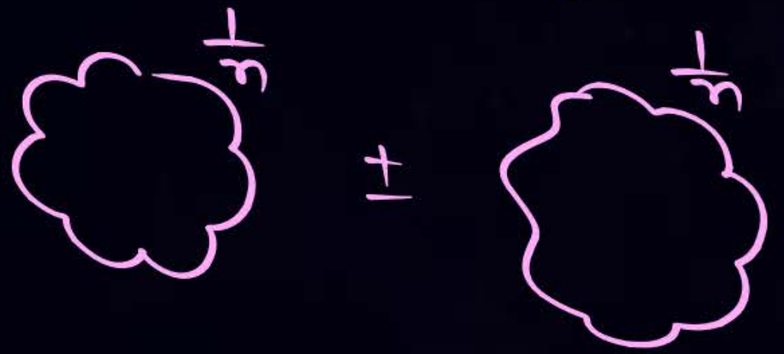
Ans. A

Using Binomial Theorem

🤔🤔Kaha Use Karnaa hai???



$n, m \in N$
$n \neq m$.



$n \in N$
$n \geq 4$

QUESTION

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$$

M① Rationalization (already done)

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{3}}-(1-x)^{\frac{1}{3}}}{x}$$

$$\lim_{x\to 0}\frac{\left(1+\frac{1}{3}x+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}x^2+\cdots-\infty\right)-\left(1+\frac{1}{3}(-x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-x)^2+\cdots-\infty\right)}{x}$$

$$\lim_{x\to 0}\frac{2/3\,x+2\cdot\left(\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}x^3\right)+\cdots-\infty}{x}=2/3$$ Ans

QUESTION

$$\lim_{x\to\infty} \sqrt[3]{x^3+3x^2} - \sqrt{x^2-2x} \qquad (\infty-\infty)$$

$$\lim_{x\to\infty} (x^3)^{\frac{1}{3}}\left(1+\frac{3}{x}\right)^{\frac{1}{3}} - (x^2)^{\frac{1}{2}}\left(1-\frac{2}{x}\right)^{\frac{1}{2}}$$

$$\lim_{x\to\infty} x\left[\left(1+\frac{3}{x}\right)^{1/3} - \left(1-\frac{2}{x}\right)^{1/2}\right]$$

$$= \lim_{x\to\infty} x\left[\left(1+\frac{1}{3}\cdot\frac{3}{x}+\frac{1/3(1/3-1)}{2!}\frac{9}{x^2}+\cdots\infty\right)-\left(1+\frac{1}{2}\left(\frac{-2}{x}\right)+\frac{1/2\cdot(1/2-1)}{2!}(-2/x)^2+\cdots\infty\right)\right]$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1/3(1/3-1)}{2!}\frac{9}{x}+\cdots\infty\right)-\left(-1+\frac{1/2(1/2-1)}{2!}\left(\frac{4}{x}\right)+\cdots\infty\right)\right]$$

$$= 1-(-1)=2$$

QUESTION [JEE Advanced 2009]

Let $L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

A $a = 2$

B $a = 1$

C $L = \frac{1}{64}$

D $L = \frac{1}{32}$

$$L = \lim_{x\to 0} \frac{a - \sqrt{a^2-x^2} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{a - a\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x\to 0} \frac{a - a\left(1 + \frac{1}{2}\left(-\frac{x^2}{a^2}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{x^2}{a^2}\right)^2 + \cdots - \infty\right) - x^2/4}{x^4}$$

$$= \lim_{x\to 0} \frac{x^2\left(\frac{1}{2a} - \frac{1}{4}\right) - \frac{1/2(1/2-1)}{2!}\left(\frac{x^4}{a^3}\right) + \frac{1/2(1/2-1)(1/2-2)}{3!}\frac{x^6}{a^5} + \cdots - \infty}{x^4}$$

for limit to exist

$$\frac{1}{2a} - \frac{1}{4} = 0$$

$$a = 2$$

$$\lim_{x \to 0} \frac{-\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{x^4}{a^3}\right) + \frac{1}{3!}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{x^6}{a^5} - \cdots - \infty}{x^4}$$

$$\lim_{x \to 0} \frac{-1/2 \cdot (-1/2)}{2!} \cdot \frac{1}{a^3} + \frac{1/2\,(-1/2)\,(-3/2)}{3!}\frac{x^2}{a^5} + \cdots - \infty$$

$$= \frac{1}{8} \cdot \frac{1}{a^3} = \frac{1}{64}$$

QUESTION

Lilliput Swaal

$$\lim_{x\to 0}\frac{1-\cos 5x}{3x^2}$$

$$\lim_{x\to 0}\frac{1-\cos 5x}{(5x)^2}\cdot\frac{25x^2}{3x^2}$$

$$\frac{1}{2}\cdot\frac{25}{3}=\frac{25}{6}.$$

$$\lim_{x\to a}\frac{1-\cos\square}{\square^2}=\frac{1}{2}$$

as $x-a\to 0$

QUESTION

Lilliput Swaal

$$\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$$

$$\lim_{x\to 0}\frac{1+\tan x-(1+\sin x)}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to 0}\frac{\sin x\left(\frac{1}{\cos x}-1\right)}{x^3\left(\quad\right)_2}=\lim_{x\to 0}\underset{\Gamma^1}{\frac{\sin x}{x}}\underset{\Gamma^{\frac{1}{2}}}{\frac{(1-\cos x)}{x^2}}\underset{\Gamma^1}{\frac{1}{\cos x}}\underset{\Gamma^{1/2}}{\frac{1}{(\quad)^2}}$$

$$=1/4$$

QUESTION [JEE Mains 2019] Lilliput Swaal

$$\lim_{x\to 0} \frac{x\cot(4x)}{\sin^2 x \cot^2(2x)}$$ is equal to:

A 1

B 4

C 0

D 2

$$\lim_{x\to 0} \frac{4x}{\tan 4x} \left(\frac{\tan 2x}{2x}\right)^2 \cdot x \left(\frac{x}{\sin x}\right)^2 \frac{4x^2}{4x} \cdot \frac{1}{x^2}$$

$\hookrightarrow 1$ $\hookrightarrow 1$ $\hookrightarrow 1$

$= 1$

ASNC

Whenever we have limit $x \to a$ where 'a' is a non zero number, in such problems it is better to put $x = a+t$ because maximum formula we have in limits are at $x \to 0$



Khud ki Buddhi Kaa Istmaal naa karnaa Ghatak Saabit Ho Saktaa Hai (Conditions Apply: Honi bhi Chahiyay)

QUESTION [JEE Mains 2017] **Lilliput Swaal**

$$\lim_{x\to\pi/2} \frac{\cot x-\cos x}{(\pi-2x)^3} \text{ equals :}$$

(A) $\frac{1}{4}$

(B) $\frac{1}{24}$

(C) $\frac{1}{16}$

(D) $\frac{1}{8}$

Kyaa apne yahi kiyaa ??

$\lim_{x\to\pi/2} \frac{\sin x}{x} = 1$

Toh aap —— hai!!

$\lim_{x\to\pi/2} \frac{\sin x}{x} = \frac{2}{\pi}$ ← $\pi/2$

$\lim_{x\to\pi/2} \frac{\cot x-\cos x}{(\pi-2x)^3}$ $\left(\frac{0}{0}\right)$

$x = \pi/2 + t$ as $x \to \pi/2$, $t \to 0$

$\lim_{t\to 0} \frac{\cot(\pi/2+t)-\cos(\pi/2+t)}{(\pi-2(\pi/2+t))^3}$

$\lim_{t\to 0} \frac{-\tan t+\sin t}{-8t^3} = \lim_{t\to 0} \frac{\sin t\left(\frac{1}{\cos t}-1\right)}{8t^3} = \lim_{t\to 0} \frac{\sin t}{t}\frac{(1-\cos t)}{t^2}\cdot\frac{1}{8\cos t}$

$= 1\cdot\frac{1}{2}\cdot\frac{1}{8} = \frac{1}{16}.$

QUESTION [JEE Mains 2019]

Brain Lagaoo

$$\lim_{x\to\pi/4}\frac{\cot^3 x-\tan x}{\cos\left(x+\pi/4\right)}\text{ is :}$$

(A) $4\sqrt{2}$

(B) 8

(C) 4

(D) $8\sqrt{2}$

$$= \lim_{x\to\pi/4}\frac{\frac{1}{\tan^3 x}-\tan x}{\cos(x+\pi/4)}$$

$$= \lim_{x\to\pi/4}\frac{1-\tan^4 x}{\cos(x+\pi/4)\cdot\tan^3 x}$$

masoom

$$= \lim_{x\to\pi/4}\frac{(1-\tan x)(1+\tan x)(1+\tan^2 x)}{\cos(x+\pi/4)\cdot\tan^3 x}$$

Dorinde. masoom

$$= \lim_{x\to\pi/4}\frac{\cos x-\sin x}{\cos x\cdot\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})^2(\sqrt{2})^2}{(\sqrt{2})}$$

$$= \lim_{x\to\pi/4}\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)}{\cos x\cdot\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})(\sqrt{2})}{(\sqrt{2})}$$

$$= \lim_{x\to\pi/4}\frac{\sqrt{2}\cos(x+\pi/4)}{\cos x\cos(x+\pi/4)}\cdot\frac{(\sqrt{2})(\sqrt{2})}{(\sqrt{2})}$$

$$\frac{\sqrt{2}\cdot 2\cdot 2}{\frac{1}{\sqrt{2}}\cdot 1}=8.$$

QUESTION

The value of $\lim_{x \to \pi/6} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$ is

Tan9

$x = \frac{\pi}{6} + t$

A 1/12

B 1/24

C 1/36

D 1/48

QUESTION

(ASRQ)

The limiting value of the function $f(x) = \dfrac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ when $x \to \dfrac{\pi}{4}$ is

A $\sqrt{2}$

B $1/\sqrt{2}$

C $3\sqrt{2}$

D $3/\sqrt{2}$

$x = \pi/4 + t$

$$\lim_{t \to 0} \frac{2\sqrt{2} - (\cos(\pi/4 + t) + \sin(\pi/4 + t))^3}{1 - \sin 2(\pi/4 + t)}$$

$\sin(\pi/2 + 2t)$

$$\lim_{t \to 0} \frac{2\sqrt{2} - \left(\frac{1}{\sqrt{2}}\cos t - \frac{\sin t}{\sqrt{2}} + \frac{\sin t}{\sqrt{2}} + \frac{\cos t}{\sqrt{2}}\right)^3}{1 - \cos 2t}$$

$$\lim_{t \to 0} \frac{2\sqrt{2} - (\sqrt{2}\cos t)^3}{1 - \cos 2t}$$

$$\lim_{t \to 0} \frac{2\sqrt{2}(1 - \cos^3 t)}{1 - \cos 2t} = \lim_{t \to 0} \frac{2\sqrt{2}(1 - \cos t)(1 + \cos^2 t + \cos t)}{(1 - \cos 2t)}$$

$$\lim_{t \to 0} \frac{2\sqrt{2}\,(1-\cos t)}{t^2} \cdot \frac{4t^2}{(1-\cos 2t)} \cdot \frac{1}{4} \qquad (1+\cos^2 t+\cos t)$$

$$= \cancel{2\sqrt{2}} \cdot \cancel{\frac{1}{2}} \cdot \frac{1}{1/2} \cdot \frac{1}{4} \cdot 3$$

$$= 2\sqrt{2} \cdot \frac{3}{4}$$

$$= \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}}.$$

ASNC

Machli Jal ki Rani Hai Jeevan uska Paani Hai Haath lagaoge Dar Jayegi bahar nikalogay Mar jayaygi Issi Tarah....Limit x ki power ki r Rani hai jyaada doge mar jayaygi kum doge chal jayaygi



Khud ki Buddhi Kaa Istmaal naa karnaa Ghatak Saabit Ho Saktaa Hai (Conditions Apply: Honi bhi Chahiyay)

QUESTION

$$\lim_{x\to 0}\frac{3\sin x - x^2 + x^3}{\tan x + 2\sin^2 x + 5x^4}$$

Divide Nr & Den by x

$$\lim_{x\to 0}\frac{3\frac{\sin x}{x} - x + x^2}{\frac{\tan x}{x} + 2\frac{\sin x}{x}\cdot\sin x + 5x^3}$$

$$= \frac{3\cdot 1 - 0 + 0}{1 + 0 + 0} = \underline{3} \text{ Ans}$$

Ex: $\lim_{x\to 0}\frac{\sin x}{x} = 1$

Ex: $\lim_{x\to 0}\frac{\sin x}{x^2} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{x}$ (1) D.N.E

Ex: $\lim_{x\to 0}\frac{1-\cos x}{x^4}$

$= \lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1}{x^2}$ DNE

Ex: $\lim_{x\to 0}\frac{\sin x}{x^{1/3}} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{x}{x^{1/3}}$

$= \lim_{x\to 0}\frac{\sin x}{x}\,x^{2/3} = 0$

QUESTION

$$\lim_{x \to 0} \frac{\tan^3 x + \tan^3 2x + \tan^2 3x}{\tan^3 4x + \tan^2 5x + \tan^3 6x}$$



ASNC

Never Apply Limit Partially
Jab Bhi Question may limit Rakhi
Jati Hai to Puray Question Mai eak
Saath Limit rakhte Hai



Agar Aisaa nahin kiya to Chamarkaari Parinaam Parpt Hone ki Puri Puri Sambhavna Hai !!!

QUESTION

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{\tan^2 x}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x\cdot\cos 2x}{\tan^2 x\,(1+\cos x\cdot\sqrt{\cos 2x})}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x(2\cos^2 x-1)}{\tan^2 x\ (\ )_2} = \lim_{x\to 0}\frac{-2\cos^4 x+\cos^2 x+1}{\tan^2 x\ (\ )_2}$$

$$= \lim_{x\to 0}\frac{-2\cos^4 x+2\cos^2 x-\cos^2 x+1}{\tan^2 x\ (\ )_2}$$

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{\sin^4 x} = \lim_{x\to 0}\frac{1-\cos\left(2\sin^2\frac{x}{2}\right)}{\left(2\sin^2\frac{x}{2}\right)^2}\cdot\frac{\left(2\sin^2\frac{x}{2}\right)^2}{\sin^4 x}$$

$$= \lim_{x\to 0}\frac{1-\cos(2\sin^2 x/2)}{(2\sin^2 x/2)^2}\cdot\frac{4\sin^4 x/2}{(2\sin x/2\cos x/2)^4}$$

$$= 1/2\cdot\frac{1}{4} = 1/8$$

$$\lim_{x\to 0}\frac{2\cos^2 x(1-\cos^2 x)+1-\cos^2 x}{\tan^2 x\ (\ )}$$

$$\lim_{x\to 0}\frac{(1-\cos^2 x)(1+2\cos^2 x)}{\tan^2 x\ (\ )}$$

$$\lim_{x\to 0}\frac{\sin^2 x(1+2\cos^2 x)\cos^2 x}{\sin^2 x\ (\ )_2}$$

$$= 3/2$$

Don't Apply Limit Partially. Lallu naa Bano!!!

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{1-\cos\left(\frac{1-\cos x}{x^2}\cdot x^2\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{1-\cos\left(x^2/2\right)}{x^4} \qquad \text{as } \lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

$\lim_{x\to 0}\frac{1-\cos\left(x^2/2\right)}{x^4/4\cdot 4}=\frac{1}{8}$ is wrong although the answer may be correct.

QUESTION [JEE Mains 2021 (August)]

$$\lim_{x\to 0}\frac{\sin^2(\pi\cos^4 x)}{x^4}$$ is equal to

(A) π^2

(B) $2\pi^2$

(C) $4\pi^2$

(D) 4π

Kyaa appnae bhi aisaa Sochaa!!

$\lim_{x\to 0}\frac{\sin^2(\pi\cos^4 x)}{(\pi\cos^4 x)^2}\cdot\frac{\pi\cos^4 x}{x^4}$

→ Toh app ——— hai

$\sin(\pi-\theta)=\sin\theta$

$\lim_{x\to 0}\frac{\sin^2(\pi-\pi\cos^4 x)}{x^4}$

$\lim_{x\to 0}\left(\frac{\sin(\pi(1-\cos^4 x))}{\pi(1-\cos^4 x)}\right)^2$ ($\pi(1-\cos^4 x)\to 0$)

$\left(\frac{1-\cos x}{x^2}\right)^2$

$\frac{\pi^2(1-\cos x)^2(1+\cos x)^2(1+\cos^2 x)^2}{x^4\times(1/2)^2}$

$=1\cdot\pi^2\cdot\frac{1}{4}\cdot 4\cdot 4$

QUESTION [JEE Mains 2021 (March)]

Tah 11

The value of the limit $\lim\limits_{\theta \to 0} \frac{\tan(\pi \cos^2\theta)}{\sin(2\pi \sin^2\theta)}$ is equal to:

A $-1/2$

B $-1/4$

C 0

D $1/4$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Homework From Module

LCD

PRARAMBH : Q1 TO Q11,Q16 TO Q27,Q35 TO Q39

(Solution to TAH)

QUESTION

$$\lim_{n\to\infty} \frac{(1 + 2 + 3 + \ldots\ldots + n)(1^3 + 2^3 + 3^3 + \ldots. + n^3)}{(1^2 + 2^2 + 3^2 + \ldots.. + n^2)^2}$$

Ans. 9/8

$$\lim_{n\to\infty} \frac{(1+2+3+\cdots+n)(1^3+2^3+3^3+\cdots+n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$$

$$\lim_{n\to\infty} \frac{\left(\frac{n(n+1)}{2}\right)\left(\frac{n^2(n+1)^2}{4}\right)}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2} = \lim_{n\to\infty} \frac{n^2\left(\frac{1+1/n}{2}\right)n^4\left(\frac{(1+1/n)^2}{4}\right)}{n^6\left(\frac{(1+1/n)(2+1/n)}{6}\right)^2}$$

TAH 1
ABHIJEET KUMAR PATRA
ODISHA

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}{\left(\frac{2}{6}\right)^2} = \frac{\frac{1}{8}}{\frac{4}{36}}$$

$$= \frac{1}{8} \times \frac{36^9}{4} = \frac{9}{8} \quad (\text{Ans})$$

TAH-1

Q. $\lim_{n\to\infty} \dfrac{(1+2+3+4+\ldots+n)(1^3+2^3+3^3+\ldots+n^3)}{(1^2+2^2+3^2+\ldots+n^2)^2}$

$\lim_{n\to\infty} \dfrac{\left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2}\right)^2}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2}$

$\lim_{n\to\infty} \dfrac{n^8(n+1)(n+1)^2}{8} \Big/ \dfrac{n^4(n+1)^2(2n+1)^2}{36}$

$\lim_{n\to\infty} \dfrac{9}{2} \times \dfrac{n(n+1)}{(2n+1)^2}$

$\lim_{n\to\infty} \dfrac{9}{2} \dfrac{n^2(1+1/n)}{n^2(2+\frac{1}{n})^2}$

$\lim_{n\to\infty} \dfrac{9}{2} \dfrac{(1+1/n)}{(2+\frac{1}{n})^2} = \dfrac{9}{8}$

Rajkanya From Bihar

TAH 1 Q $\lim_{n\to\infty} \frac{(1+2+3+\cdots+n)(1^3+2^3+3^3+\cdots+n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$

Ans $\lim_{n\to\infty} \frac{(1+2+3+\cdots n)(1^3+2^3+3^3+\cdots n^3)}{(1^2+2^2+3^2+\cdots+n^2)^2}$

$$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2} \times \left(\frac{n(n+1)}{2}\right)^2}{\left(\frac{n(n+1)(2n+1)}{6}\right)^2}$$

$$\lim_{n\to\infty} \frac{\{n(n+1)\}^3}{8} \times \frac{36}{\{n(n+1)(2n+1)\}^2}$$

$$\frac{36}{8\times 4} = \frac{9}{8}$$

TAH 1

SHIVAM HARSHWARDHAN
FROM HARIDWAR

QUESTION [JEE Advanced 2012]

If $\lim_{x \to -\infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then

A $a = 1, b = 4$

B $a = 1, b = -4$

C $a = 2, b = -3$

D $a = 2, b = 3$

Ans. B

If $\lim_{x \to -\infty} \left(\frac{x^2+x+1}{x+1} - ax - b \right) = 4$, then (Adv 2012)

(A) $a=1, b=4$
(B) $a=1, b=-4$
(C) $a=2, b=-3$
(D) $a=2, b=3$

$$\lim_{x \to -\infty} \left(\frac{x^2+x+1}{x+1} - (ax+b) \right) = 4$$

$$\lim_{x \to -\infty} \left(\frac{x^2+x+1-(x+1)(ax+b)}{x+1} \right) = 4$$

TAH 2
ABHIJEET KUMAR
PATRA
ODISHA

$$\lim_{x \to -\infty} \frac{x^2+x+1-(ax^2+(b+a)x+b)}{x+1} = 4$$

Should be 0

$$\lim_{x \to -\infty} \frac{x^2(1-a)+x(1-a-b)+1-b}{x+1} = 4$$

$\Rightarrow 1-a=0$

$a=1$

$$\lim_{x \to -\infty} \frac{x(-b)+1-b}{x+1} = 4 \Rightarrow \frac{-b}{1}$$

$$= 4$$

$\therefore a, b = -4$

TAH - 2.

Q. If $\lim_{x \to -\infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1}{1 - t} + at - b \right) = 4$ | Put $x = -t$, as $x \to -\infty$, $t \to \infty$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1 + (at - b)(1 - t)}{1 - t} \right) = 4$

$\lim_{t \to \infty} \left(\frac{t^2 - t + 1 + at - at^2 - b + bt}{(1 - t)} \right) = 4$

$\lim_{t \to \infty} \left(\frac{(1 - a)t^2 + (a + b - 1)t + 1 - b}{1 - t} \right) = 4$

here, $1 - a = 0$

$a = 1$ [degree of N^r = degree of D^r]

$\lim_{t \to \infty} \left(\frac{(a + b - 1)t + 1 - b}{1 - t} \right) = 4$

$\lim_{t \to \infty} \left(\frac{t((a + b - 1) + 1/t - b/t)}{t(\frac{1}{t} - 1)} \right) = 4$

$\frac{a + b - 1}{-1} = 4$

$a + b - 1 = -4$

$a + b = -3$

as, $a = 1$, $b = -4$

Rajkanya From Bihar

QUESTION [JEE Mains 2022 (25 July)]

If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to:

A 4

B -8

C -4

D 8

Ans. C

TAH 3

$\lim_{n\to\infty} (\sqrt{n^2-n+1} - n\alpha + \beta) = 0$

On Rationalising,

$\lim_{n\to\infty} \left(\frac{n^2-n+1 - (n\alpha+\beta)^2}{\sqrt{n^2-n+1} - (n\alpha+\beta)} \right) = 0$

$\lim_{n\to\infty} \left(\frac{n^2-n+1 - n^2\alpha^2 - 2n\alpha\beta - \beta^2}{\sqrt{n^2-n+1} - (n\alpha) - \beta} \right) = 0$

$\lim_{n\to\infty} \left(\frac{n^2(1 - 1/n + 1/n^2 - \alpha^2 - 2\alpha\beta/n - \beta^2/n^2)}{n[\sqrt{1 - 1/n + 1/n^2} - \alpha - \beta/n]} \right) = 0$

$\lim_{n\to\infty} \left(\frac{(1-\alpha^2)n - (1 + 2\alpha\beta) - 1/n - \beta/n}{(\sqrt{1 - 1/n + 1/n^2} - \alpha - \beta/n)} \right) = 0$

for lim to be defined $\rightarrow 1 - \alpha^2 = 0$

$\alpha = \pm 1$

$\therefore \frac{-(2\alpha\beta+1)}{1-\alpha} \Rightarrow \alpha \neq 1$

Hence, $\alpha = (-1)$

$\therefore (2\alpha\beta + 1) = 0$

$\alpha\beta = +1/2$

$\beta = 1/2$ $\quad \alpha = -1$

$8(\alpha+\beta) = 8(-1+1/2)$

$= -8/2$

$\therefore 8(\alpha+\beta) = -4$ Ans

Rupan karmakar ❤️from Port Blair

Q) If $\lim_{n\to\infty} (\sqrt{n^2-n-1} + n\alpha + \beta) = 0$, then $8(\alpha+\beta)$ is equal to:

$\lim_{n\to\infty} (n(\sqrt{1 - 1/n - 1/n^2} + \alpha + \beta/n) = 0$

$\lim_{n\to\infty} \frac{n^2 - n - 1 - (n\alpha+\beta)^2}{\sqrt{n^2-n-1} - (n\alpha+\beta)}$

SHRADDHA TAMRAKAR
JABALPUR M.P

$\lim_{n\to\infty} \frac{n^2(1-\alpha^2) + n(-1 - 2\alpha\beta) + (-1-\beta^2)}{\sqrt{n^2-n-1} - (n\alpha+\beta)} = 0$

only possibility if $n^2 = 0$

$1 - \alpha^2 = 0$

$\pm 1 = \alpha$

$\lim_{n\to\infty} \frac{n[(-1-2\alpha\beta) + (-\frac{1-\beta^2}{n})]}{n(\sqrt{1 - 1/n - 1/n^2} - (\alpha + \beta/n))}$

$\frac{-1-2\alpha\beta}{1-\alpha} = 0 \rightarrow \alpha \neq 1$

$\alpha = -1$

$-1 - 2(-1)\beta = 0$

$\beta = 1/2$

$\therefore 8(\alpha+\beta) = 8(-1/2)$

$= -4$ Ans

TAH 3

QUESTION

(RPP 3)

In a kite ABCD, $AB = AD$ and $CB = CD$. If $\angle A = 108^\circ$ and $\angle C = 36^\circ$ then the ratio of the area of $\triangle ABD$ to the area of $\triangle CBD$ can be written in the form $\frac{a - b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

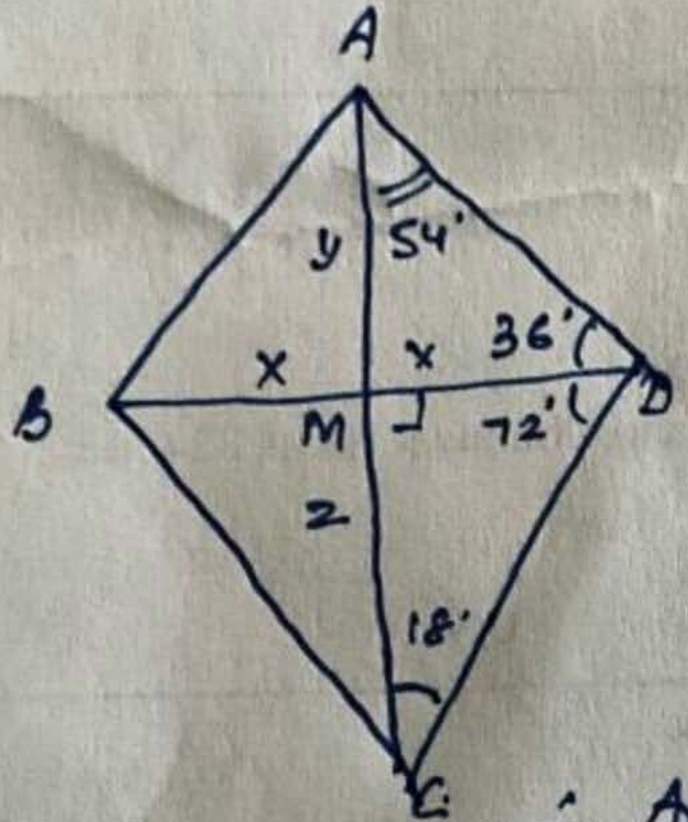
A (1, 2, 1)

B (2, 1, 1)

C (4, 3, 2)

D (1, 1, 2)

Ans. D



$$\frac{Ar(\triangle ABD)}{Ar(\triangle CBD)} = \frac{\frac{1}{2}\cdot 2x\cdot y}{\frac{1}{2}\cdot 2x\cdot z} = \frac{y}{z}$$

Now $y = x\tan 36°$

$z = x\tan 72°$

$$\therefore \frac{Ar\triangle ABD}{Ar(\triangle CBD)} = \frac{x\tan 36°}{x\tan 72°}$$

$$= \frac{\tan 36°}{\frac{2\tan 36°}{1-\tan^2 36°}}$$

$$= \frac{1-\tan^2 36°}{2} \Rightarrow (a,b,c) = (1,1,2)$$ Ans.

Bhoolnaa Mat Yeh Batayaa thaa

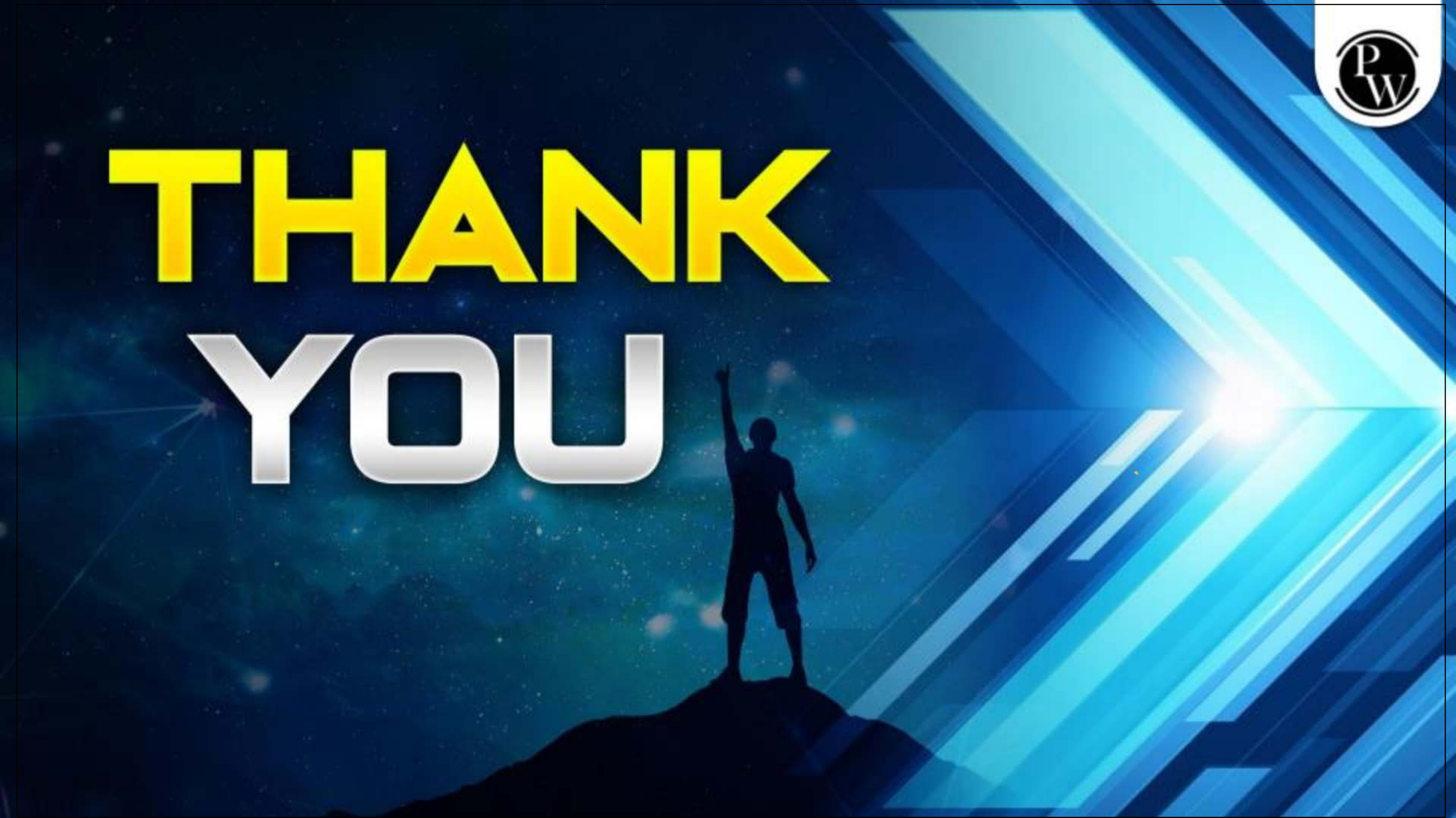
For a square matrix A the Trace of product $A.A^T$ is the sum of squares of all the entries of A

Mann ki Baat Ashish Sir ke Saath

How to reduce silly mistakes in exams??

1. Take enough sleep before exam.
2. Analyze the pattern of mistakes you make!!
3. Daily : Mobile – 15 min Alarm. (4 Questions)




THANK
YOU

PRAYAS

JEE 2025

Lecture-05 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Exponential & Logarithmic Limits
2. Generalised 1^{∞} form

Recap *of previous lecture*

[·] : GIF

1. $\lim_{x\to 0}\left[\frac{\tan x}{x}\right]+\left[2\frac{\tan 2x}{x}\right]+\left[\frac{3\tan 3x}{x}\right]$ $= \lim_{x\to 0}\left[\frac{\tan x}{x}\right]^{1^+}+\left[\frac{2^2\tan 2x}{2x}\right]^{1^+}+\left[\frac{3^2\tan 3x}{3x}\right]^{1^+}$

$= 1 + 4 + 9 = 14$

2. $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]+\left[\frac{2\sin 2x}{x}\right]+\left[\frac{3\sin 3x}{x}\right] = \lim_{x\to 0}\left[\frac{\sin x}{x}\right]^{1^-}+\left[\frac{2^2\sin 2x}{2x}\right]^{1^-}+\left[\frac{3^2\sin 3x}{3x}\right]$

$= 0 + (2^2-1) + (3^2-1) = 11$

3. $\lim_{x\to 0}\left[\frac{\sin^{-1} x}{x}\right]+\left[\frac{2\sin^{-1}2x}{x}\right]+\left[\frac{3\sin^{-1}3x}{x}\right] = \lim_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right]^{1^+}+\left[\frac{2^2\sin^{-1}2x}{2x}\right]^{1^+}+\left[\frac{3^2\sin^{-1}3x}{3x}\right]^{1^+} = 14$

4. $\lim_{x\to 0}\left[\frac{\tan^{-1} x}{x}\right]+\left[\frac{2\tan^{-1}2x}{x}\right]+\left[\frac{3\tan^{-1}3x}{x}\right] = 0+2^2-1+3^2-1=11$

M① $\lim_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right]$ $\quad x=\sin\theta$

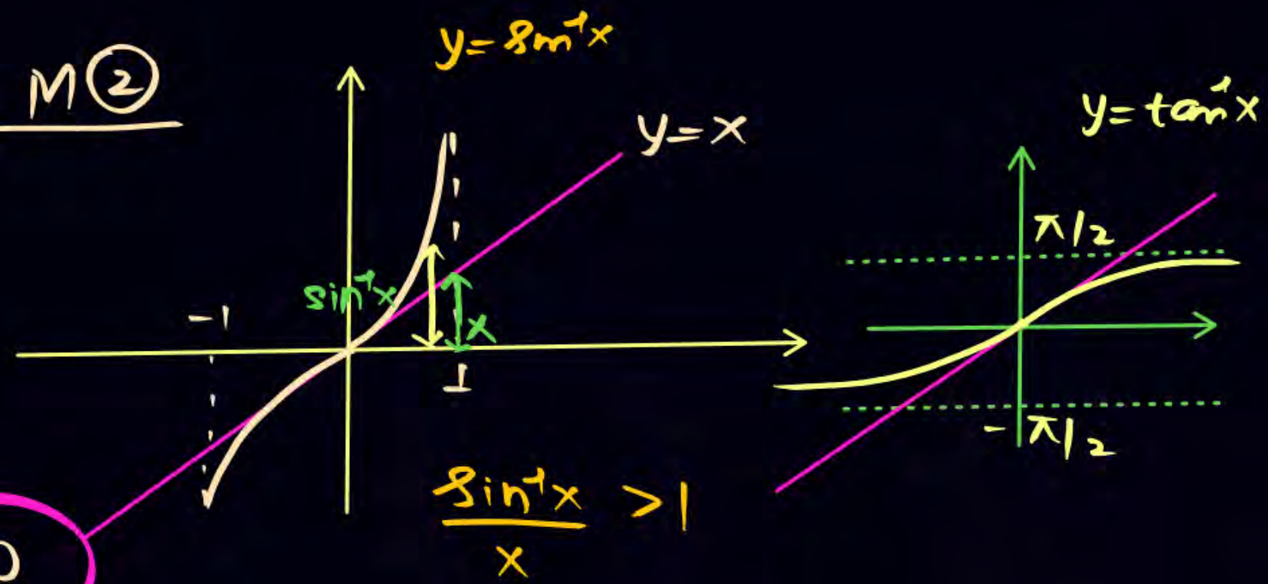
$\theta \to 0^+$

$\lim_{\theta\to 0}\left[\frac{\theta}{\sin\theta}\right] = 1$

★ $\frac{\sin^{-1}x}{x} > 1$ in viccinity of $x=0$

★ $\frac{\tan^{-1}x}{x} < 1$ in viccinity of $x=0$

M②



$\frac{\sin^{-1}x}{x} > 1$

$\lim_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right] =$

In viccinity/nbd of $x=0$

- $0 < \frac{\sin x}{x} < 1$
- $\frac{\tan x}{x} > 1$
- $0 < \frac{\tan^{-1} x}{x} < 1$
- $\frac{\sin^{-1} x}{x} > 1$

Recap *of previous lecture*

5. $\lim_{x\to\infty}\left(\left((2x+1)(4x+3)(27x+9)\right)^{\frac{1}{3}}-6x\right)=$

$\lim_{x\to\infty}(8\cdot 27)^{1/3}\left(\left(x+\frac{1}{2}\right)\left(x+\frac{3}{4}\right)\left(x+\frac{1}{3}\right)\right)^{\frac{1}{3}}-6x = 6\lim_{x\to\infty}\left(\left(x+\frac{1}{2}\right)\left(x+\frac{3}{4}\right)\left(x+\frac{1}{3}\right)\right)^{\frac{1}{3}}-x\Big)$

6. $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{1}{3}}{3} = 2\left(\frac{1}{2}+\frac{3}{4}+\frac{1}{3}\right)$

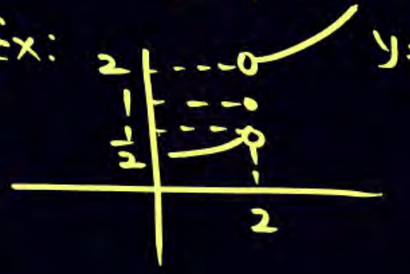
$1+\frac{3}{2}+\frac{2}{3}$

$\frac{19}{6}$ Ans

6. If $f(2)=1$ then $\lim_{h\to 0} f(2+h)=1$. (T/F) ✓

7. If $f(3)=4$ then $\lim_{h\to 0} f(3-h)=4$. (T/F) ✓

Ex:



$y=f(x)$

$\lim_{h\to 0} f(2+h)=$ DNE

RHL: $\lim_{h\to 0^+} f(2+h)=2$

LHL: $\lim_{h\to 0^-} f(2+h)=\frac{1}{2}$

QUESTION

Tah 1

Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2 \sin 2x}{x}\right] + ... + \left[\frac{10 \sin 10x}{x}\right]$

(where [y] is the largest integer $\leq y$). The value of $\lim_{x \to 0} f(x)$ equals

A 55

B 164

C 165

D 375

QUESTION

Tan 2

If $\lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+\ldots+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)=100$, then the value of n, is

Note: [k] denotes the greatest integer less than or equal to k

A 2

B 3

C 4

D 5

Homework Discussion

QUESTION [JEE Mains 2024 (6 April)]

ASRQ
Hybrid (Seq+ Limits)

TAH 2

$$\lim_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots+n^2)}$$ is equal to:

Re-do

A $\frac{2}{3}$

B $\frac{1}{2}$

C $\frac{3}{4}$

D $\frac{1}{3}$

$$T_r = (r^2-r)(n-r) = r^2(n-r)-r(n-r)$$
$$= nr^2-r^3-nr+r^2$$
$$= (n+1)r^2-r^3-nr.$$

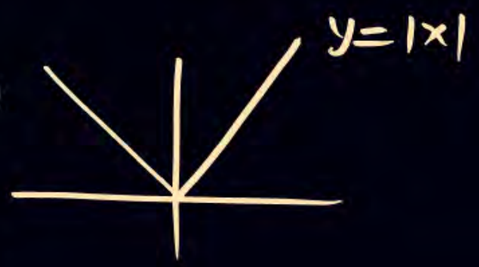
$$S=\sum_{r=1}^{n-1}T_r = (n+1)\sum_{r=1}^{n-1}r^2-\sum_{r=1}^{n-1}r^3-n\sum_{r=1}^{n-1}r$$

Ans. D

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 5

Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in R - \{1, 0\}$, $\lim\limits_{x \to 0} \left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to :



(A) 1

(B) 2

(C) 0

(D) 1/2

RHL $\lim\limits_{x \to 0^+} \left|\frac{1-x+x}{\lambda-x+0}\right| = \lim\limits_{x \to 0^+} \left|\frac{1}{\lambda-x}\right| = \frac{1}{|\lambda|}$

LHL $\lim\limits_{x \to 0^-} \left|\frac{1-x-x}{\lambda-x-1}\right| = \lim\limits_{x \to 0^-} \left|\frac{1-2x}{\lambda-1-x}\right| = \frac{1}{|\lambda-1|}$

$\frac{1}{|\lambda|} = \frac{1}{|\lambda-1|}$

$|\lambda| = |\lambda-1|^2$

$\lambda^2 = \lambda^2 + 1 - 2\lambda$

$\lambda = 1/2$

limit = 2.

Ans. B

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION [AIEEE 2011]

(ASRQ) + KCLS

$$\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$$

(A) equals $-\sqrt{2}$

(B) equals $\frac{1}{\sqrt{2}}$

(C) does not exist

(D) equals $\sqrt{2}$

$x = 2+t$

as $x-2, t\to 0$

$$\lim_{t\to 0}\frac{\sqrt{1-\cos 2t}}{t}$$

$$\lim_{t\to 0}\frac{\sqrt{2\sin^2 t}}{t}$$

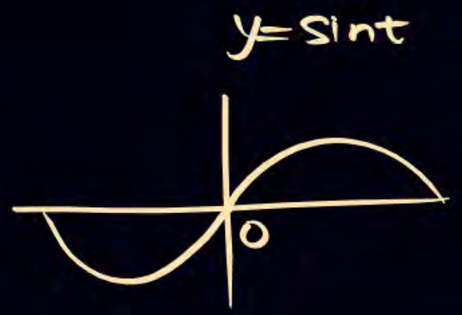
$$\lim_{t\to 0}\frac{\sqrt{2}\,|\sin t|}{t}$$

$$\lim_{t\to 0^+}\sqrt{2}\,\frac{|\sin t|}{t} = \lim_{t\to 0^+}\sqrt{2}\,\frac{\sin t}{t} = \sqrt{2}$$

$$\lim_{t\to 0^-}\frac{\sqrt{2}\,|\sin t|}{t} = \lim_{t\to 0^-}-\sqrt{2}\,\frac{\sin t}{t} = -\sqrt{2}$$

$\Downarrow$

Limit D.N.E



QUESTION [JEE Mains 2023 (8 April)]

Tan3

If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim\limits_{x \to \frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{1/2} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$, then k is equal to

(A) $\beta 2$

(B) β

(C) α

(D) 2α

$ax^2+bx+1=0 \begin{cases} \alpha \\ \beta \end{cases} > \alpha\beta = \frac{1}{a}$

$\downarrow x - \frac{1}{x}$

$\frac{a}{x^2} + \frac{b}{x} + 1 = 0 \Rightarrow \frac{x^2+bx+a=0}{x^2}$

$x^2+bx+a=0 \begin{cases} 1/\alpha \\ 1/\beta \end{cases}$.

$x^2+bx+a = 1\cdot\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right) = \frac{(\alpha x-1)(\beta x-1)}{\alpha\beta} = a(\alpha x-1)(\beta x-1)$

QUESTION [JEE Mains 2020]

TAH 4

If $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is

$(h>0)$

$$\lim_{h\to 0} [0+h] = 0$$

$$\lim_{h\to 0} [0-h] = \lim_{h\to 0} (-1) = -1$$

QUESTION [JEE Mains 2019]

Let [x] denote the greatest integer less than or equal to x. Then

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))}{x^2}$$

(A) does not exist

(B) equal to π

(C) equals 0

(D) equals $\pi+1$

RHL $\lim\limits_{x\to 0^+}\dfrac{\tan(\pi\sin^2 x)+(|x|-\sin(x\cdot[x]))}{x^2}$

$\lim\limits_{x\to 0^+}\dfrac{\tan(\pi\sin^2 x)+(x-\sin 0)}{x^2}$

$\lim\limits_{x\to 0^+}\left(\dfrac{\tan(\pi\sin^2 x)}{x^2}+\dfrac{1}{x}\right)$

$\lim\limits_{x\to 0^+}\left(\dfrac{\tan(\pi\sin^2 x)}{\pi\sin^2 x}\cdot\pi\left(\dfrac{\sin x}{x}\right)^2+\dfrac{1}{x}\right)$ ($\to 0$, $\to 1$, $\to 1$, $\to \infty$) $=\infty$ D.N.E

QUESTION [JEE Mains 2019]

(ASRQ) + KCLS

$$\lim_{x\to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$$

(A) $\sqrt{\pi}$

(B) $\sqrt{\pi/2}$

(C) $\sqrt{2/\pi}$

(D) $1/\sqrt{2\pi}$

$$\lim_{x\to 0} \frac{\sin^{-1}x}{x} = 1$$

$$\lim_{x\to 1^-} \frac{\pi - 2\sin^{-1}x}{\sqrt{1-x}\,(\sqrt{\pi} + \sqrt{2\sin^{-1}x})}$$

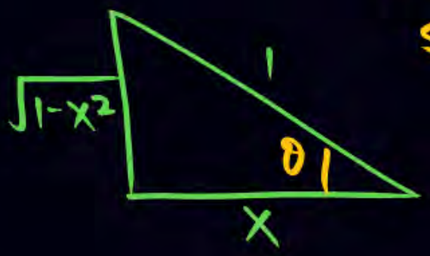
$$= \lim_{x\to 1^-} \frac{2(\pi/2 - \sin^{-1}x)}{\sqrt{1-x}\,(\sqrt{\pi} + \sqrt{2\sin^{-1}x})}$$

$\cos^{-1}x = \theta$

$\cos\theta = x$

$\sin\theta = \sqrt{1-x^2}$

$\theta = \sin^{-1}\sqrt{1-x^2}$



$$= \lim_{x\to 1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}\,(\sqrt{\pi} + \sqrt{2\sin^{-1}x})}$$

$$= \lim_{x\to 1^-} \frac{2\sin^{-1}\sqrt{1-x^2}}{\sqrt{1-x}\,(\sqrt{\pi} + \sqrt{2\sin^{-1}x})} = \lim_{x\to 1^-} \frac{2\sin^{-1}\sqrt{1-x^2}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}}\,(\sqrt{\pi} + \sqrt{2\sin^{-1}x})$$

$$= \lim_{x\to 1^-} 2\left(\frac{\sin^{-1}\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)^{1} \frac{\sqrt{1+x}}{\sqrt{\pi} + \sqrt{2\sin^{-1}x}} = \frac{2\sqrt{2}}{\sqrt{\pi} + \sqrt{2}\cdot\frac{\pi}{2}} = \frac{2\sqrt{2}}{\sqrt{\pi} + \sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

QUESTION [JEE Mains 2019]

(ASRQ) + KCLS

$$\lim_{x\to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$$

(A) $\sqrt{\pi}$

(B) $\sqrt{\pi/2}$

(C) $\sqrt{2/\pi}$

(D) $1/\sqrt{2\pi}$

M(2)

$$\lim_{x\to 1^-} \frac{\pi - 2\sin^{-1}x}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{2\sin^{-1}x}\right)}$$

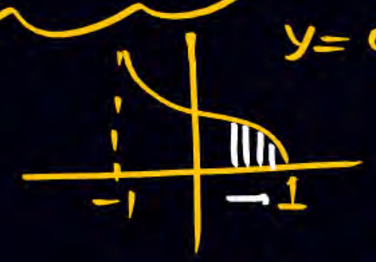
$$= \lim_{x\to 1^-} \frac{2\left(\pi/2 - \sin^{-1}x\right)}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{2\sin^{-1}x}\right)}$$

$$= \lim_{x\to 1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}\left(\sqrt{\pi} + \sqrt{2\sin^{-1}x}\right)}$$

$$= \lim_{\theta\to 0^+} \frac{2\theta}{\sqrt{1-\cos\theta}\left(\sqrt{\pi} + \sqrt{2\sin^{-1}(\cos\theta)}\right)}$$

$$= \lim_{\theta\to 0^+} \frac{2\theta}{\sqrt{1-\cos^2\theta}} \frac{\sqrt{1+\cos\theta}}{\sqrt{\pi} + \sqrt{2\sin^{-1}(\cos\theta)}}$$

$$\lim_{x\to 0} \frac{\sin^{-1}x}{x} = 1$$



$\cos^{-1}x = \theta \in [0, \pi]$

$x = \cos\theta$

as $x \to 1^- \Rightarrow \theta \to 0^+$

$$= \lim_{\theta \to 0^+} \frac{2\theta}{|\sin\theta|} \frac{\sqrt{1+\cos\theta}}{\sqrt{\pi} + \sqrt{2}\sin^{-1}(\cos\theta)}$$

$$= \lim_{\theta \to 0^+} \frac{2\theta}{\sin\theta} \cdot \frac{\sqrt{1+\cos\theta}}{\sqrt{\pi} + \sqrt{2}\sin^{-1}(\cos\theta)}$$

$$= \frac{2 \cdot 1 \cdot \sqrt{2}}{\sqrt{\pi} + \sqrt{2}\sin^{-1}1}$$

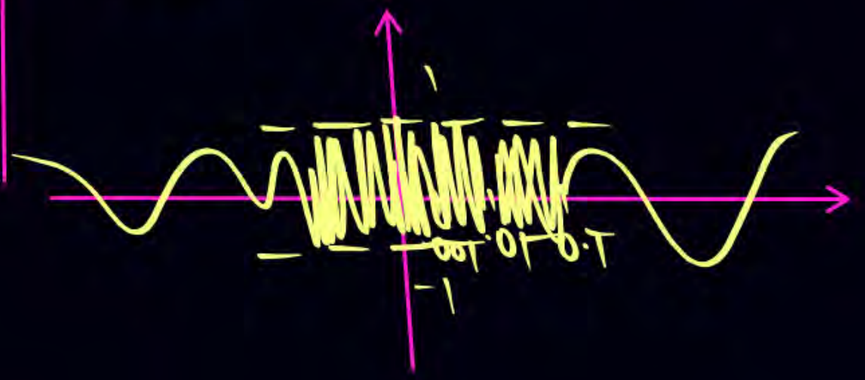
$$= \sqrt{\frac{2}{\pi}}$$

$f(x) = \sin\frac{1}{x}$

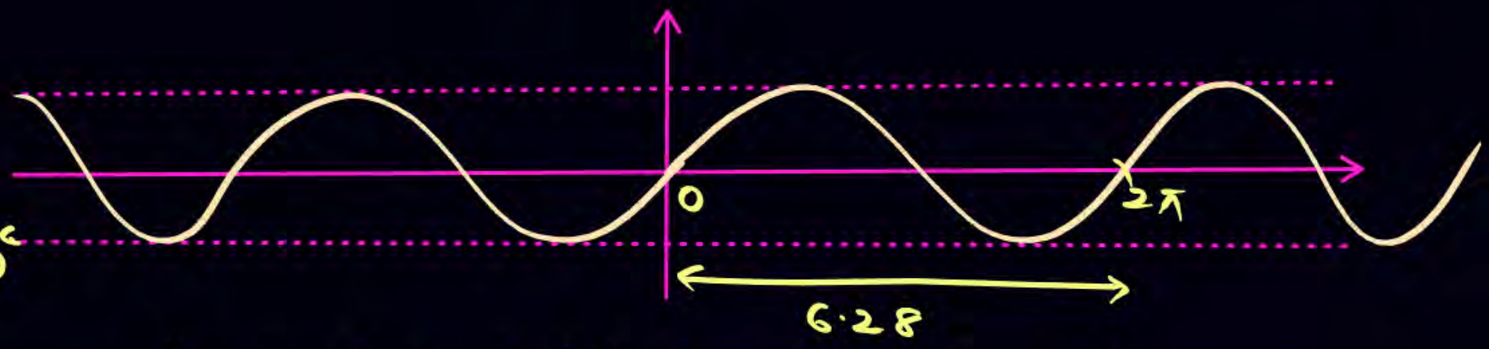
x	$\frac{1}{x}$
0·1	10
0·01	100
·001	1000

Angle diff $= 90^c$

Angle diff 900^c



$g(x) = \sin x$



In a Angle diff of approx 6 radion

sine goes one time up & 1 time down

$$\lim_{x\to 0} \underbrace{x}_{0} \cdot \underbrace{\sin\frac{1}{x}}_{\text{oscillates b/w } -1 \text{ & } 1} = 0.$$

QUESTION

Let $f(x) = x \sin\frac{1}{x}$, then the correct statement is(are)

A $\lim_{x\to 0} f(x) = 0$

B $\lim_{x\to \infty} f(x) = 1$

C $\lim_{x\to \infty} f(x) = -1$

D $f(0) = 0$

(A) $\lim_{x\to 0} x \sin\frac{1}{x} = 0$ ·

(B) $\lim_{x\to \infty} x \sin\frac{1}{x} = \lim_{x\to \infty} \frac{\sin 1/x}{1/x} = 1$ ($x \to 0$)

(D) $f(0) = 0 \sin\left(\frac{1}{0}\right)$ = not defined (not defined)

$\sin\frac{1}{x}$ is manjurikaa in आस पड़ोस of $x=0$ only

QUESTION

(a) $\lim_{x\to\pi/2} \dfrac{\tan x}{\tan 3x}$

(b) $\lim_{x\to\pi/2} \dfrac{\tan x}{\tan 5x}$

Tan 5

(c) $\lim_{x\to-\infty} \dfrac{x^4\sin(1/x)+x^2}{1+|x|^2}$

$x=-t$

$as\ x\to-\infty,\ t\to\infty$

$$\lim_{t\to\infty} \frac{-t^4\sin\frac{1}{t}+t^2}{1+|-t|^2}$$

$$=\lim_{t\to\infty} \frac{-t^4\sin 1/t+t^2}{1+t^2} = \lim_{t\to\infty} \frac{-t^3\frac{\sin 1/t}{1/t}+t^2}{1+t^2} = \lim_{t\to\infty} \frac{t^3\left(-\frac{t\sin 1/t}{1/t}+1\right)}{t^2(1/t^2+1)} = -\infty = \text{DNE}$$

$$\lim_{x\to-\infty} \frac{x^3\sin\frac{1}{x}+x^2}{1+|x|^2} \qquad x=-t \ as\ x\to-\infty,\ t\to\infty$$

$$\lim_{t\to\infty} \frac{t^3\sin\frac{1}{t}+t^2}{1+|-t|^2}$$

$$=\lim_{t\to\infty} \frac{t^2\cdot\frac{\sin 1/t}{1/t}+t^2}{1+t^2} = \lim_{t\to\infty} \frac{\frac{\sin 1/t}{1/t}+1}{\frac{1}{t^2}+1} = 2$$

$e^{\ln p} = p$

Limit of Exponential fns. & Logarithmic fn.

★ $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = 1$

$$\lim_{x\to 0} \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty\right) - 1}{x} = \lim_{x\to 0} \frac{x\left(1 + \frac{1}{2!}x + \frac{1}{3!}x^2 \cdots\right)}{x} = 1$$

★ $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x} = 1$ $\lim\limits_{x\to 0} \dfrac{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots - \infty}{x} = \lim\limits_{x\to 0} \dfrac{x\left(1 - \frac{1}{2}x + \frac{1}{3}x^2 - \frac{1}{4}x^3 \cdots\right)}{x} = 1$

★ $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \ln a$

$$\lim_{x\to 0} \frac{e^{\ln a^x} - 1}{x} = \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x}$$

$$= \lim_{x\to 0} \frac{e^{x\ln a} - 1}{x\ln a} \cdot \ln a$$

$$= 1 \cdot \ln a = \ln a.$$

$\lim\limits_{x\to a} \dfrac{e^{\square} - 1}{\square} = 1$ as $x \to a$, $\square \to 0$

$\lim\limits_{x\to a} \dfrac{\ln(1+\square)}{\square} = 1$

★ $\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$.

1^{∞}

$\lim_{x\to a}\left(1+\square\right)^{\frac{1}{\square}} = e$ as $x-a \to 0$

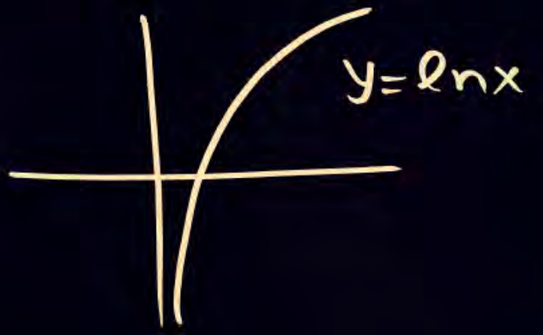
Proof: Let $\ell = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$

$$\ln \ell = \ln\left(\lim_{x\to 0}(1+x)^{\frac{1}{x}}\right) \Rightarrow \ln \ell = \lim_{x\to 0} \frac{1}{x}\ln(1+x)$$

$$= \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

$\ln \ell = 1$

$\ell = e$



Generalized 1^∞ form

$x^{n\cdot m} = (x^n)^m$

★ if $\lim_{x\to a} f(x) = 1$ & $\lim_{x\to a} g(x) = \infty$

then $\lim_{x\to a} (f(x))^{g(x)} = e^{\lim_{x\to a} g(x)(f(x)-1)}$ (1^∞)

Proof: $\lim_{x\to a} (f(x))^{g(x)} = \lim_{x\to a} (1+(f(x)-1))^{g(x)} = \lim_{x\to a} \left(1+(f(x)-1)\right)^{\frac{1}{f(x)-1}\cdot(f(x)-1)\cdot g(x)}$ ($f(x)-1 \to 0$)

$= \lim_{x\to a} \left(\left(1+(f(x)-1)\right)^{\frac{1}{f(x)-1}}\right)^{(f(x)-1)\,g(x)}$ ($f(x)-1 \to 0$)

$= e^{\lim_{x\to a} g(x)(f(x)-1)}$

Maclaurin's Infinite Series

If f is differentiable ∞ many times and some more conditions.

$$f(x) = f(0) + x\frac{f'(0)}{1!} + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots \infty$$

Ex: $e^x = e^0 + \frac{x}{1!}\cdot\frac{d}{dx}e^x\Big|_{x=0} + \frac{x^2}{2!}\frac{d^2(e^x)}{dx^2}\Big|_{x=0} + \frac{x^3}{3!}\frac{d^3}{dx^3}e^x\Big|_{x=0} + \cdots \infty$

$$= 1 + \frac{x}{1}\cdot e^0 + \frac{x^2}{2!}e^0 + \frac{x^3}{3!}e^0 + \cdots \infty$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty .$$

Limit using Series Expansion

Expansion of function like binomial expansion, exponential & logarithmic expansion, expansion of sin x, cos x, tan x should be remembered by heart which are given below :

(a) $a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + \ldots\ldots \; a > 0$

(b) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

(c) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots. \text{ for } -1 < x \leq 1$

(d) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$ (Odd Powers alternate +, −)

Limit using Series Expansion

(Even powers alternate +,-)

(e) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$

(f) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + ...$

(g) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$

(h) $\sin^{-1} x = x + \frac{1^2}{3!}x^3 + \frac{1^2 \cdot 3^2}{5!}x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!}x^7 + ...$

(i) $\sec^{-1} x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + ...$

(j) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$

(k) $(1+x)^{1/x} = e^{\frac{1}{x}\ln(1+x)} = e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 - \frac{21}{48}x^3 + ...\right]$

QUESTION

(a) $\lim_{x\to 0} \dfrac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$

(b) $\lim_{x\to \infty} \dfrac{e^{1/x^2} - 1}{2 \arctan x^2 - \pi}$

(c) $\lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

Tan 6

★ $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

★ $\cot^{-1}x = \tan^{-1}\frac{1}{x}$ $x>0$

$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x^2}} - 1}{\frac{1}{x^2}} \cdot \frac{1}{x^2} \cdot \frac{1}{2\left(\tan^{-1}x^2 = \frac{\pi}{2}\right)}$$

$$= \lim_{x\to\infty} (\quad) \cdot \frac{1}{x^2}\left(-\frac{1}{2\cot^{-1}x^2}\right)$$

$$= -\lim_{x\to\infty} (\quad) \cdot \frac{1}{x^2} \frac{1}{2\tan^{-1}\frac{1}{x^2}}$$

$$= -\lim_{x\to\infty} (\quad) \frac{\frac{1}{x^2}}{2\cdot\tan^{-1}\frac{1}{x^2}} = -1 \cdot \frac{1}{2} = -1/2$$

QUESTION

(d) $\lim_{x\to 0} \dfrac{e^{\sin 2x} - e^{\sin x}}{x}$

M①

$= \lim_{x\to 0} \dfrac{e^{\sin 2x} - 1 - (e^{\sin x} - 1)}{x}$

$= \lim_{x\to 0} \left(\dfrac{e^{\sin 2x} - 1}{x} - \dfrac{e^{\sin x} - 1}{x} \right)$

$= \lim_{x\to 0} \left(\dfrac{e^{\sin 2x} - 1}{\sin 2x} \cdot \dfrac{\sin 2x}{2x} \cdot 2 - \dfrac{e^{\sin x} - 1}{\sin x} \cdot \dfrac{\sin x}{x} \right)$

$\hookrightarrow 1 \cdot 1 \cdot 2 - 1 \cdot 1$

$= 2 - 1 = 1$

$\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$

(e) $\lim_{x\to 0} \dfrac{e^{\tan x} - e^x}{\tan x - x}$

$= \lim_{x\to 0} \dfrac{e^x (e^{\tan x - x} - 1)}{\tan x - x}$

$= e^0 \cdot 1 = 1$

M② $\lim_{x\to 0} \dfrac{e^{\sin 2x} - e^{\sin x}}{x}$ ≠ $\lim_{x\to 0} \dfrac{e^{\sin x}(e^{2\cos x} - 1)}{x}$

(Gadho/Gadhiyoo aisaa naa karo!!)

$= \lim_{x\to 0} \dfrac{e^{\sin x}(e^{\sin 2x - \sin x} - 1)}{x}$

$$= \lim_{x \to 0} e^{\sin x} \frac{\left(e^{\sin 2x - \sin x} - 1\right)}{\sin 2x - \sin x} \cdot \left(\frac{\sin 2x - \sin x}{x}\right)$$

$$= \lim_{x \to 0} e^{\sin x} \frac{\left(e^{\sin^2 x - \sin x} - 1\right)}{\sin 2x - \sin x} \cdot \left(\frac{\sin 2x}{2x} \cdot 2 - \frac{\sin x}{x}\right)$$

$$= e^0 \cdot 1 \cdot (2 - 1) = 1 \quad \underline{\text{Ans}}$$

Chmatkaar!!

$$x^8 - x^2 \neq x^2(x^4 - 1)$$

$$e^{2\sin x \cos x} - e^{\sin x}$$

$$\neq e^{\sin x}\left(e^{\frac{2\sin x \cos x}{\sin x}} - 1\right)$$

$$= e^{\sin x}\left(e^{2\cos x} - 1\right)$$

QUESTION

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$$

(1) $\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$

$x=e+t$

$=\lim\limits_{t\to 0}\dfrac{\ln(e+t)-1}{e+t-e}$

$=\lim\limits_{t-0}\dfrac{\ln(e+t)-1}{t}$

$=\lim\limits_{t-0}\dfrac{\ln(e+t)-\ln e}{t}$

OR

$\lim\limits_{t-0}\dfrac{\ln(e(1+t/e))-1}{t}$

$=\lim\limits_{t-0}\dfrac{\ln\left(\frac{e+t}{e}\right)}{t}=\lim\limits_{t-0}\dfrac{\ln(1+t/e)}{t/e\cdot e}=\dfrac{1}{e}$

OR

$\lim\limits_{t-0}\dfrac{\ln e+\ln(1+t/e)-1}{t}$

$=\lim\limits_{t-0}\dfrac{\ln(1+t/e)}{t/e}\cdot\dfrac{1}{e}=1/e.$

(2) $\lim\limits_{x\to 0}\dfrac{\ln(1+x)}{3^x-1}=\lim\limits_{x-0}\dfrac{\frac{\ln(1+x)}{x}}{\frac{3^x-1}{x}}$

$=\dfrac{1}{\ln 3}.$

(3) $\lim\limits_{x\to 1}(1-x)\log_x 2$

put $x=1+t$

$\lim\limits_{t-0}-t\log_{1+t}2=\lim\limits_{t-0}\dfrac{-t\log_e 2}{\log(1+t)}$

$=-\lim\limits_{t-0}\dfrac{t\ln 2}{\ln(1+t)}=-\ln 2\,e.$

QUESTION

$$\lim_{x\to 0}\frac{27^x-9^x-3^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to 0}\frac{9^x(3^x-1)-(3^x-1)}{2-(1+\cos x)}\cdot(\sqrt{2}+\sqrt{1+\cos x})$$

$$=\lim_{x\to 0}\frac{(9^x-1)(3^x-1)}{1-\cos x}(\quad)$$

$$=\lim_{x\to 0}\frac{\frac{9^x-1}{x}\cdot\frac{3^x-1}{x}}{\frac{1-\cos x}{x^2}}(\quad)=\frac{\ell n 9\cdot \ell n 3}{1/2}\cdot 2\sqrt{2}$$

$$=4\sqrt{2}\,\ell n 3^2\cdot \ell n 3$$

$$=4\sqrt{2}\cdot 2\ell n 3\cdot \ell n 3$$

$$=8\sqrt{2}(\ell n 3)^2$$

$$\lim_{x\to 0}(1+2x)^{5/x}=\lim_{x\to 0}\left((1+2x)^{\frac{1}{2x}}\right)^{10}$$

$$=e^{10}.$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$$

QUESTION [JEE Mains 2023 (8 April)]

Tan7

$$\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right) \text{ is equal to}$$

(A) 15

(B) 18

(C) 9

(D) 24

QUESTION

(ASRQ) + KCLS

Tan θ

The value of $\lim_{x\to 2} \frac{\sec^x\theta - \tan^x\theta - 1}{x-2}$ is equal to

$\lim_{x\to 2} \frac{\sec^x\theta - \tan^x\theta - 1}{x-2} - (\sec^2\theta - \tan^2\theta)$

(A) $\sec^2\theta \cdot \ln\sec\theta + \tan^2\theta \cdot \ln\tan\theta$

(B) $\sec^2\theta \cdot \ln\tan\theta + \tan^2\theta \cdot \ln\sec\theta$

(C) $\sec^2\theta \cdot \ln\tan\theta - \tan^2\theta \cdot \ln\sec\theta$

(D) $\sec^2\theta \cdot \ln\sec\theta - \tan^2\theta \cdot \ln\tan\theta$

QUESTION

(ASRQ) + KCLS

The value of $\lim_{x\to 0}\left((\tan x)^{\frac{1}{\sin^2 x}}+\frac{\sin 2x+2\tan^{-1}3x+3x^2}{\ln(1+3x+\sin^2 x)+xe^x}\right)$ is equal to

(A) 1

(B) 2

(C) -1

(D) -2

1^∞

$$\lim_{x\to 0}\left(( )+\frac{\frac{\sin 2x}{x}+\frac{2\tan^{-1}3x}{x}+\frac{3x^2}{x}}{\frac{\ln(1+3x+\sin^2 x)}{3x+\sin^2 x}\cdot\frac{3x+\sin^2 x}{x}+\frac{xe^x}{x}}\right)$$

$$\lim_{x\to 0}( )+\frac{\frac{2\sin 2x}{2}+\frac{6\tan^{-1}3x}{3x}+3x}{\frac{\ln(1+3x+\sin^2 x)}{3x+\sin^2 x}\left(3+\frac{\sin x}{x}\cdot\sin x\right)+ex}$$

$$=0+\frac{2+6+0}{1\cdot 3+1}=8/4=2$$

QUESTION [JEE Advanced 2022] (ASRQ) + KCLS

If $\beta = \lim\limits_{x\to 0} \dfrac{e^{x^3} - (1-x^3)^{\frac{1}{3}} + \left((1-x^2)^{\frac{1}{2}} - 1\right)\sin x}{x\sin^2 x}$

then the value of 6β is

$$= \lim_{x\to 0} \frac{e^{x^3}-1 + 1-(1-x^3)^{1/3} + \left(\sqrt{1-x^2}-1\right)\sin x}{x^3} \cdot \frac{x^2}{\sin^2 x}$$

$$= \lim_{x\to 0} \left(\frac{e^{x^3}-1}{x^3} + \frac{1-(1-x^3)^{1/3}}{x^3} + \frac{\sqrt{1-x^2}-1}{x^3}\cdot \sin x \right)( )$$

$$= \lim_{x\to 0} \left(\frac{e^{x^3}-1}{x^3} + \frac{\left(1-(1-x^3)\right)}{x^3\left(1+(1-x^3)^{2/3}+(1-x^3)^{1/3}\right)} + \frac{1-x^2-1}{x^2\left(\sqrt{1-x^2}+1\right)}\cdot \frac{\sin x}{x} \right)( )$$

$$= \left(1 + \frac{1}{3} + \left(\frac{-1}{2}\right)1\right)\cdot 1 = 5/6 = \beta \Rightarrow 6\beta = 5$$

Ans : 5

QUESTION

(1) $\lim\limits_{\substack{x\to 1\\ x\to 0\\ x\to\infty}} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

$= \lim\limits_{x\to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}}$

$= \lim\limits_{x\to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}}$

$= \left(\frac{2}{3}\right)^{\frac{1}{2}}$

, $\lim\limits_{x\to 0} \left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}} = \frac{1}{2}$

, $\lim\limits_{x\to \infty} \left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}}$

(2) $\lim\limits_{x\to\infty} \left(\frac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{x-1}}$

$\lim\limits_{x\to \infty} \left(\frac{1+2/x-1/x^2}{2-3/x-2/x^2}\right)^{\frac{2+1/x}{1-1/x}} = (1/2)^2 = 1/4$

$\lim\limits_{x\to \infty} \left(\frac{\frac{1}{x}+1}{2/x+1}\right)^{\frac{1}{1+\sqrt{x}}}$

$= 1^0 = 1$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 06

The number of solutions of equation $8[x^2 - x] + 4[x] = 13 + 12[\sin x]$ is (here [.] represents greatest integer less than or equal to 'x')

Ans. 0.00

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

#Q. The signum (or sign) function, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

. Compute the following.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$

(ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$

(iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

Ans. (i) 1; (ii) –1, (iii) does not exist; (iv) 1

QUESTION

(KTK 2)

#Q. For the function g whose graph is given, state the value of each quantity, if it exists.

(a) $\lim_{t \to 0^-} g(t)$

(b) $\lim_{t \to 0^+} g(t)$

(c) $\lim_{t \to 0} g(t)$

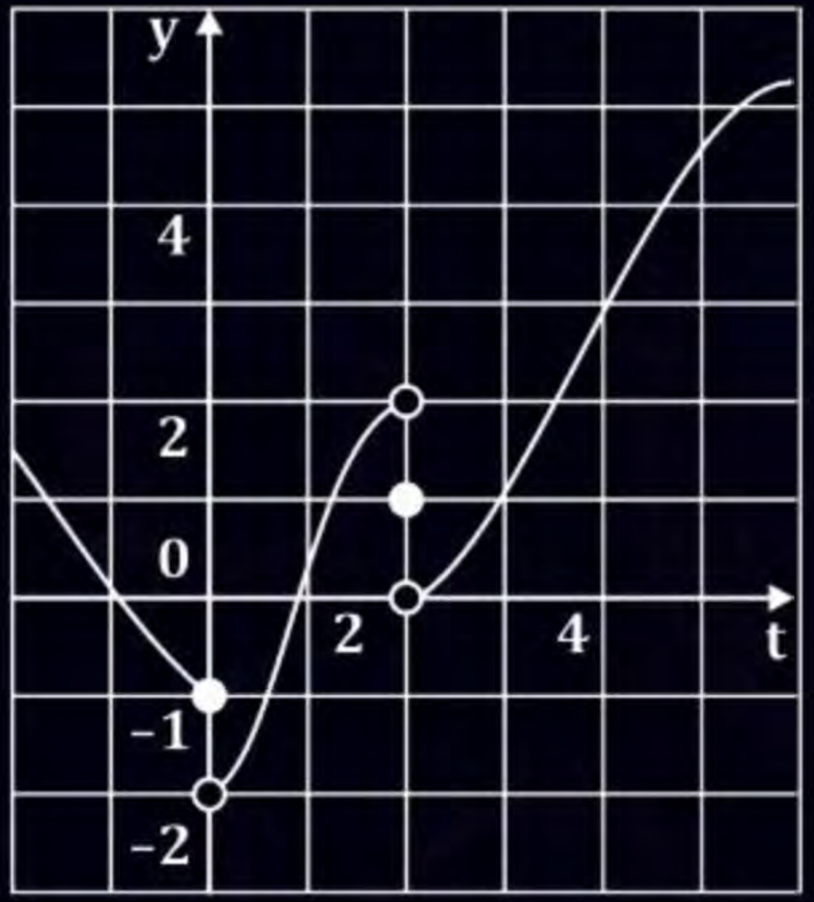
(d) $\lim_{t \to 2^-} g(t)$

(e) $\lim_{t \to 2^+} g(t)$

(f) $\lim_{t \to 2} g(t)$

(g) $g(2)$

(h) $\lim_{t \to 4} g(t)$



Ans. (a) –1; (b) –2, (c) does not exist; (d) 2,; (e) 0; (f) does not exits; (g) 1; (h) 3

QUESTION [JEE Mains 2019 (10 April)]

KTK 3

If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, then k is:

A $\frac{3}{2}$

B $\frac{8}{3}$

C $\frac{4}{3}$

D $\frac{3}{8}$

Ans. B

QUESTION [JEE Mains 2017 (8 April)]

KTK 4

$\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to:

(A) $\sqrt{3}$

(B) $\dfrac{1}{\sqrt{2}}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{1}{2\sqrt{2}}$

Ans. B

QUESTION [JEE Mains 2021 (27 August)]

KTK 5

If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is:

A $\left(1,\frac{1}{2}\right)$

B $\left(1,-\frac{1}{2}\right)$

C $\left(-1,\frac{1}{2}\right)$

D $\left(-1,-\frac{1}{2}\right)$

Ans. B

QUESTION [JEE Mains 2019 (10 April)]

KTK 6

If $\lim\limits_{x\to 1}\dfrac{x^2-ax+b}{x-1}=5$, then $a+b$ is equal to:

(A) 1

(B) -4

(C) -7

(D) 5

Ans. C

QUESTION [JEE Mains 2019 (9 Jan)]

KTK 7

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

(A) exists and equals $\frac{1}{2\sqrt{2}}$

(B) exists and equals $\frac{1}{4\sqrt{2}}$

(C) exists and equals $\frac{1}{2\sqrt{2}(1+\sqrt{2})}$

(D) does not exists

Ans. B

QUESTION

KTK 8

If $f(x) = \begin{cases} x+2; & x<1 \\ 4x-1; & 1 \le x \le 3 \\ x^2+5, & \text{where } x>3 \end{cases}$; then

(a) $\lim\limits_{x \to 1^+} f(x) = 3$

(b) $\lim\limits_{x \to 1^+} f(x) =$ does not exist

(c) $\lim\limits_{x \to 3^+} f(x) = 11$

(d) $\lim\limits_{x \to 3} f(x) =$ does not exist

QUESTION

KTK 9

Let $\lim_{x\to 1}\frac{[x]}{x} = l$ and $\lim_{x\to 1}\frac{x}{[x]} = m$ where [] denotes the greatest integer function, then

A l exist but m does not exist

B m exist but l does not exist

C l and m both exist

D neither l nor m exist

QUESTION [JEE Mains 2018 (16 April)]

KTK 10

$$\lim_{x \to 0} \frac{(27+x)^{\frac{1}{3}} - 3}{9 - (27+x)^{\frac{2}{3}}} \text{ equals}$$

A $\frac{1}{3}$

B $-\frac{1}{3}$

C $-\frac{1}{6}$

D $\frac{1}{6}$

Ans. C

(Revision Practice Problems)

QUESTION

RPP 1

If $3p\sin x-(p+\sin x)(p^2-p\sin x+\sin^2 x)=1$ has a solution for x, then the number of integral values of p are

Ans. 4.0

QUESTION

RPP2

Possible integral value(s) of m for the equation $\sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$ can be valid for some $x \in [0, 2\pi]$ is

A -1

B 0

C 1

D 2

Ans. A, B, C, D

QUESTION

RPP3

If $\sin x + \cos x + \tan x + \cot x + \sec x + \text{cosec} = 7$ and $\sin 2x = a - b\sqrt{7}$, then $a - b + 14$ is divisible by

A 2

B 3

C 4

D 7

Ans. A, C, D

Homework From Module

LCD

PRARAMBH : Q1 TO Q11,Q16 TO Q27,Q35 TO Q39

(Solution to TAH)

QUESTION [JEE Mains 2023 (25 Jan)]

ASRQ

The value of $\lim\limits_{n\to\infty} \frac{1+2-3+4+5-6+\ldots.+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$ is :

(A) $\frac{3}{2\sqrt{2}}$

(B) $\frac{3}{2}(\sqrt{2}+1)$

TAH 1

(C) $3(\sqrt{2}+1)$

(D) $\frac{\sqrt{2}+1}{2}$

The value of $\lim_{n\to\infty} \frac{1+2-3+4+5-6+\cdots+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$

(ASRQ)
(2023)

⇒ Method 1

$$\lim_{n\to\infty} \frac{(\underbrace{1+4+7+\cdots+3n-2}_{n\text{ terms}})+(\underbrace{2+5+8+\cdots+(3n-1)}_{n\text{ terms}})-(\underbrace{3+6+9+\cdots+3n}_{n\text{ terms}})}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty} \frac{\frac{n(3n-1)}{2}+\frac{n(3n+1)}{2}-\frac{n(3+3n)}{2}}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty} \frac{n^2\left[\left(\frac{3}{2}-\frac{1}{2n}\right)+\left(\frac{3}{2}+\frac{1}{2n}\right)-\left(\frac{3}{2}+\frac{3}{2n}\right)\right]}{n^2\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]}$$

$$= \frac{\left[\frac{3}{2}+\frac{3}{2}-\frac{3}{2}\right]}{\sqrt{2}-\sqrt{1}} = \frac{3}{2(\sqrt{2}-1)} = \frac{3}{2}(\sqrt{2}+1) \quad \text{(Ans)}$$

TAH 1
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ODISHA

⊛ Method 2

$$\lim_{n\to\infty} \frac{(1+2+3+\cdots+3n)-2\cdot(3+6+9+\cdots+3n)}{n^2\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]}$$

$$\lim_{n\to\infty} \frac{\frac{3n(3n+1)}{2}-\frac{3^3\cdot n(n+1)}{2}}{n^2\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]}$$

$$\lim_{n\to\infty} \frac{n^2\left[\frac{3}{2}\left(3+\frac{1}{n}\right)-3\left(1+\frac{1}{n}\right)\right]}{n^2\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]} = \frac{9/2-3}{\sqrt{2}-1} = \frac{3}{2}(\sqrt{2}+1)$$

QUESTION [JEE Mains 2024 (6 April)]

ASRQ
Hybrid (Seq+ Limits)

$$\lim_{n\to\infty} \frac{(1^2-1)(n-1)+(2^2-2)(n-2)+\cdots+((n-1)^2-(n-1))\cdot 1}{(1^3+2^3+\cdots+n^3)-(1^2+2^2+\cdots+n^2)}$$ is equal to:

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{3}$

TAH 2

Ans. D

QUESTION [JEE Mains 2023 (6 April)]

ASRQ
Hybrid (Seq+ Limits)

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right) \text{ is}$$

(A) $1/\sqrt{d}$

(B) 1

(C) 0

(D) $\sqrt{d}$

TAH 3

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} \right) \text{ is}$$

((2023), Jan)

(A) $1/\sqrt{d}$
(B) 1
(C) 0
(D) $\sqrt{d}$

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left(\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \frac{\sqrt{a_3}-\sqrt{a_4}}{a_3-a_4} + \ldots + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n} \right)$$

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \cdot \frac{1}{-d} \left[(\sqrt{a_1}-\sqrt{a_2}) + (\sqrt{a_2}-\sqrt{a_3}) + (\sqrt{a_3}-\sqrt{a_4}) + \ldots + (\sqrt{a_{n-1}}-\sqrt{a_n}) \right]$$

$$\lim_{n\to\infty} \frac{1}{-\sqrt{n}} \sqrt{d} \left[\sqrt{a_1} - \sqrt{a_n} \right]$$

TAH 3
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ODISHA

$$\lim_{n\to\infty} \frac{-1}{\sqrt{d}} \left[\frac{\sqrt{a_1}}{\sqrt{n}} - \frac{\sqrt{a_1+(n-1)d}}{\sqrt{n}} \right]$$

$$\lim_{n\to\infty} \frac{-1}{\sqrt{d}} \left[\frac{\sqrt{a_1}}{\sqrt{n}} - \sqrt{\frac{a_1}{n} + \left(\frac{n-1}{n}\right) d} \right]$$

$$\lim_{n\to\infty} \frac{-1}{\sqrt{d}} \left[\frac{\sqrt{a_1}}{\sqrt{n}} - \sqrt{\frac{a_1}{n} + \left(1-\frac{1}{n}\right) d} \right]$$

($\frac{a_1}{n} \to 0$, $\frac{1}{n} \to 0$)

$$= \frac{-1}{\sqrt{d}} (0 - \sqrt{d}) = 1$$

QUESTION [JEE Mains 2024 (4 April)]

ASRQ

If $\lim\limits_{x\to 1}\dfrac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}=\dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m,n)=1$, then $8m+12n$ is equal to

TAH 4

Ans. 100

TAH 4 – If $\lim_{x\to 1} \frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$

where gcd $(m,n) = 1$, then $8m + 12n$ is equal to ?

Sol

$$\lim_{x\to 1} \frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}}$$

TAH 4

$$\lim_{x\to 1} \frac{(5x+1-x-5)}{2x+3-x-4} \times \frac{(2x+3)^{1/2}+(x+4)^{1/2}}{(5x+1)^{2/3}+(x+5)^{2/3}+(5x+1)^{1/3}(x+5)^{1/3}}$$

$$\lim_{x\to 1} \frac{4x-4}{x-1} \times \frac{(2x+3)^{1/2}+(x+4)^{1/2}}{(5x+1)^{2/3}+(x+5)^{2/3}+(5x+1)^{1/3}(x+5)^{1/3}}$$

$$4 \times \frac{\sqrt{5}+\sqrt{5}}{6^{2/3}+6^{2/3}+6^{1/3}\times 6^{1/3}}$$

$$\frac{4 \times 2\sqrt{5}}{2(36)^{2/3}+(36)^{1/3}}$$

$$\frac{8\sqrt{5}}{3(36)^{1/3}}$$

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FROM HARIDWAR

$$\frac{8\sqrt{5}}{3(2\cdot 3)^{2/3}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$$

$\therefore m = 8$
$n = 3$

$\therefore 8m + 12n \Rightarrow 64 + 36$
$\Rightarrow 100$

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 5

Let [t] denote the greatest integer $\leq$ t. If for some $\lambda \in R - \{1, 0\}$, $\lim\limits_{x \to 0} \left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to :

(A) 1

(B) 2

(C) 0

(D) 1/2

Ans. B

Let $[t]$ denote the greatest integer $\leq t$. If for some $\lambda \in R - \{1, 0\}$, $\lim_{x \to 0} \left| \frac{1 - x + |x|}{\lambda - x + [x]} \right| = L$, then L is Equal to : (2020)

$\Rightarrow$ RHL,

$$\lim_{x \to 0^+} \left| \frac{1 - x + x}{\lambda - x + 0} \right| = \left| \frac{1}{\lambda} \right|$$

LHL,

$$\lim_{x \to 0^-} \left| \frac{1 - x - x}{\lambda - x - 1} \right| = \lim_{x \to 0^-} \left| \frac{1 - 2x}{\lambda - 1 - x} \right| = \left| \frac{1}{\lambda - 1} \right|$$

Now, RHL = LHL = L

$$\left| \frac{1}{\lambda} \right| = \left| \frac{1}{\lambda - 1} \right| = L$$

So,

$$\left| \frac{1}{\lambda} \right| = \left| \frac{1}{\lambda - 1} \right|$$

TAH 5
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ODISHA

S.B.S

$$\frac{1}{\lambda^2} = \frac{1}{(\lambda - 1)^2}$$

$$\Rightarrow \lambda^2 + 1 - 2\lambda = \lambda^2$$

$$\Rightarrow \boxed{\lambda = \frac{1}{2}}$$

So,

$$L = \left| \frac{1}{1/2} \right| = |2| = 2 \quad \text{(Ans)}$$

QUESTION [JEE Mains 2020 (3 Sep)]

TAH 6

$$\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}},\ (a\neq 0) \text{ is equal to:}$$

A $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

B $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

C $\left(\frac{2}{3}\right)^{\frac{4}{3}}$

D $\left(\frac{2}{9}\right)^{\frac{4}{3}}$

Ans. B

TAH6 $\lim_{x \to a} \frac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$, $a \neq 0$ is equal to?

Ans $\lim_{x \to a} \frac{(a+2x)^{1/3} - (3x)^{1/3}}{(3a+x)^{1/3} - (4x)^{1/3}}$

$\lim_{x \to a} \frac{a+2x-3x}{3a+x-4x} \times \frac{\{(3a+x)^{2/3} + (4x)^{2/3} + (3a+x)^{1/3}(4x)^{1/3}}{(a+2x)^{2/3} + (3x)^{2/3} + (a+2x)^{1/3}(3x)^{1/3}}$

$\lim_{x \to a} \frac{(a-x) \times \{(3a+x)^{2/3} + (4x)^{2/3} + (3a+x)^{1/3}(4x)^{1/3}\}}{3(a-x) \times \{(a+2x)^{2/3} + (3x)^{2/3} + (a+2x)^{1/3}(3x)^{1/3}\}}$

$\frac{1}{3} \frac{(4a)^{2/3} + (4a)^{2/3} + (4a)^{2/3}}{(3a)^{2/3} + (3a)^{2/3} + (3a)^{2/3}}$

$\frac{1}{3} \times \frac{3(4a)^{2/3}}{3(3a)^{2/3}}$

$\frac{1}{3} \times \frac{4^{2/3}}{3^{2/3}}$

$\frac{2}{3} \left(\frac{2}{9}\right)^{1/3}$

TAH 6

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FROM HARIDWAR

QUESTION [JEE Mains 2022 (27 June)]

TAH 7

Let a be an integer such that $\lim_{x\to 7} \frac{18-[1-x]}{[x-3a]}$ exists,

where [t] is greatest integer $\leq$ t. Then a is equal to:

(A) –6

(B) –2

(C) 2

(D) 6

Ans. A

Let a be an integer such that $\lim_{x\to 7}$ $\lim_{x\to 7}$ $\frac{18-[1-x]}{[x-3a]}$ Exists,
Where [t] is greatest integer $\leq t$. Then a is Equal to:
(2022)

$$\Rightarrow \lim_{x\to 7} \frac{18-(1+[-x])}{[x]-3a}$$

$$\lim_{x\to 7} \cdot \frac{17-[-x]}{[x]-3a}$$

RHL,

$$\lim_{x\to 7^+} \frac{17+8}{7-3a} = \frac{25}{7-3a}$$

LHL,

$$\lim_{x\to 7^-} \frac{17+7}{6-3a} = \frac{24}{6-3a}$$

Now,

LHL = RHL

$$\Rightarrow \frac{25}{7-3a} = \frac{24}{6-3a}$$

$$\Rightarrow 25(6-3a) = 24(7-3a)$$
$$\Rightarrow 150-75a = 168-72a$$
$$\Rightarrow -18 = 3a$$
$$\Rightarrow a = -6 \quad (\text{Ans})$$

TAH 7
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

TAH 9

The value of $\lim_{x \to \pi/6} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$ is

A 1/12

B 1/24

C 1/36

D 1/48

TAH ⑨ Sol^n:- $\lim_{x\to \pi/6} \frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}$ $\quad x=\frac{\pi}{6}+t$

$= \lim_{t\to 0} \frac{2-\sqrt{3}\cos(\pi/6+t)-\sin(\pi/6+t)}{\{6(\pi/6+t)-\pi\}^2} \Rightarrow \lim_{t\to 0} \frac{2-\sqrt{3}\cos t-\sin t}{(6t)^2}$

$= \lim_{t\to 0} \frac{2-\sqrt{3}(\cos\pi/6\cos t-\sin\pi/6\sin t)-(\sin\pi/6\cos t+\cos\pi/6\sin t)}{(6t)^2}$

$\lim_{t\to 0} \frac{2-\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos t-\frac{1}{2}\sin t\right)-\left(\frac{1}{2}\cos t+\frac{\sqrt{3}}{2}\sin t\right)}{36t^2}$

$\lim_{t\to 0} \frac{2-\frac{3}{2}\cos t+\frac{\sqrt{3}}{2}\sin t-\frac{1}{2}\cos t-\frac{\sqrt{3}}{2}\sin t}{36t^2} \Rightarrow \lim_{t\to 0} \frac{2-2\cos t}{36t^2} \Rightarrow \lim_{t\to 0} \frac{2(1-\cos t)}{36t^2}$ (1/2)

$= \frac{2}{36}\cdot\frac{1}{2} = \boxed{\frac{1}{36}}$ Ans:

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From bihar

The value of $\lim_{x \to \frac{\pi}{6}} \frac{2-\sqrt{3}\cos x - \sin x}{(6x-\pi)^2}$ is

$\Rightarrow$ Let $x = \frac{\pi}{6} + t$

Now,

$$\lim_{t \to 0} \frac{2-\sqrt{3}\cos\left(\frac{\pi}{6}+t\right) - \sin\left(\frac{\pi}{6}+t\right)}{\left(6\left(\frac{\pi}{6}+t\right)-\pi\right)^2}$$

$$\lim_{t \to 0} \frac{2-\sqrt{3}\cos\left(\frac{\pi}{6}+t\right) - \sin\left(\frac{\pi}{6}+t\right)}{(\pi+6t-\pi)^2}$$

$$\lim_{t \to 0} \frac{2-\frac{\sqrt{3}}{2}\cos t + \frac{1}{2}\sin t - \frac{1}{2}\cos t - \frac{\sqrt{3}}{2}\sin t}{36t^2}$$

$$\lim_{t \to 0} \frac{2(1-\cos t)}{36t^2}$$

$$= 2 \times \frac{1}{2} \times \frac{1}{36} = \frac{1}{36} \quad \text{(Ans)}$$

TAH 9
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

$$\lim_{x\to 0}\frac{\tan^3 x+\tan^3 2x+\tan^2 3x}{\tan^3 4x+\tan^2 5x+\tan^3 6x}$$

TAH 10

TAH (10) $\lim_{x\to 0} \dfrac{\tan^3 x + \tan^3 2x + \tan^2 3x}{\tan^3 4x + \tan^2 5x + \tan^3 6x}$

Soln:-

$$= \lim_{x\to 0} \frac{\dfrac{\tan^2 x}{x^2}\cdot\dfrac{\tan x}{x} + \dfrac{\tan^2 2x}{4x^2}\cdot 4\tan 2x + \dfrac{\tan^2 3x}{9x^2}\cdot 9}{\dfrac{\tan^2 4x}{16x^2}\tan 4x\cdot 16 + \dfrac{\tan^2 5x}{25x^2}\cdot 25 + \dfrac{\tan^2 6x}{36x^2}\cdot \tan x\, 36}$$

$$= \frac{1\cdot 0 + 1\cdot 0 + 9}{1\cdot 0 + 1\cdot 25 + 1\cdot 0}$$

$$= \frac{9}{25}$$ Ans

Shivani
From bihar

TAH 10
ABHIJEET KUMAR PATRA
ODISHA

$$\lim_{x\to 0} \frac{\tan^3 x + \tan^3 2x + \tan^2 3x}{\tan^3 4x + \tan^2 5x + \tan^3 6x}$$

$$\Rightarrow \lim_{x\to 0} \frac{9\left(\frac{\tan^3 x}{9x^2} + \frac{\tan^3 2x}{9x^2} + \frac{\tan^3 3x}{9x^2}\right)}{25\left(\frac{\tan^3 4x}{25x^2} + \frac{\tan^2 5x}{25x^2} + \frac{\tan^3 6x}{25x^2}\right)}$$

$$\lim_{x\to 0} \frac{\left(\frac{9\tan^3 x}{9x^2 \cdot x} \cdot x + \frac{9\tan^3 2x}{9 \cdot 8x^2} \cdot x \cdot 8 + 9\frac{\tan^2 3x}{9x^2}\right)}{\left(25\frac{\tan^3 4x}{25\,x^2 \cdot x \cdot 64} \cdot 64x + 25\frac{\tan^2 5x}{25x^2} + \frac{25\tan^3 6x}{25\,x^3 \cdot 216} x \cdot 216\right)}$$

$$= \frac{9}{25}$$

QUESTION [JEE Mains 2021 (March)]

TAH 11

The value of the limit $\lim\limits_{\theta \to 0} \frac{\tan(\pi \cos^2\theta)}{\sin(2\pi \sin^2\theta)}$ is equal to:

A $-1/2$

B $-1/4$

C 0

D $1/4$

Shivani
From bihar

TAH (11)

Soln:- $$\lim_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$$

$$=\lim_{\theta\to 0}\frac{-\tan(\pi-\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}\Rightarrow\lim_{\theta\to 0}\frac{-\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}$$

$$=\lim_{\theta\to 0}-\frac{\tan(\pi\sin^2\theta)}{\pi\sin^2\theta}\cdot\frac{\pi\sin^2\theta}{\sin(2\pi\sin^2\theta)}\times 2\pi\sin^2\theta$$

$\tan(\pi\sin^2\theta)/\pi\sin^2\theta \to 1$, $\sin(2\pi\sin^2\theta)/2\pi\sin^2\theta \to 1$

$$\Rightarrow\frac{-1\cdot 1}{1\cdot 2}=\frac{-1}{2}$$ Ans

The value of the limit $\lim_{\theta \to 0} \frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is Equal to: (2021)

$\Rightarrow \lim_{\theta \to 0} \frac{\tan(\pi(1-\sin^2\theta))}{\sin(2\pi\sin^2\theta)}$

$\lim_{\theta \to 0} \frac{-\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}$

$\lim_{\theta \to 0} -\left[\frac{\tan(\pi\sin^2\theta)}{\pi\sin^2\theta}\right]\left[\frac{2\pi\sin^2\theta}{\sin(2\pi\sin^2\theta)}\right] \times \frac{1}{2} = -\frac{1}{2}$

TAH 11
ABHIJEET KUMAR PATRA
ODISHA

Bhoolnaa Mat Yeh Batayaa thaa

In any quadratic expression f(x) if f(a).f(b)<0 then f(x) = 0 has exactly one root between a & b

Mann ki Baat Ashish Sir ke Saath

How to revise???

Revision : Quad Revise till 3rd Aug

Maths Revision is different

① Read short notes
- Imp. points
- Imp formula
- Chotay Chotay points jo sir bol detay hai

+

ASRQ+KCLS

Short notes = handwritten notes given in class + choti choti baatien.


THANK
YOU

PRAYAS

JEE 2025

Lecture-06 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Functions with Built in Limit
2. Geometrical Problems

Recap *of previous lecture*

(1) $\lim_{x\to 0} \frac{x^2}{1-\cos x} = 2$

(2) $\lim_{x\to 0} \frac{x}{\ln(1+3x)} = \lim_{x\to 0} \frac{3x}{\ln(1+3x)} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$

(3) $\lim_{x\to 0} \left[\frac{\tan x}{x}\right] + \left[\frac{2^2 \tan 2x}{2x}\right] + \left[\frac{3^2 \tan 3x}{3x}\right] = 1^2 + 2^2 + 3^2 = 14$

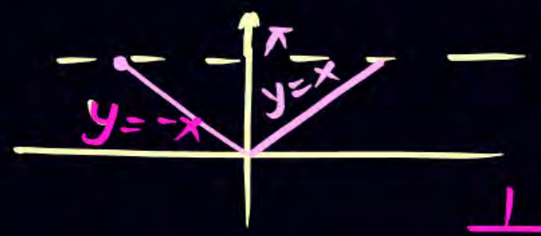
(4) $\lim_{x\to 0} \left[\frac{\tan^{-1} x}{x}\right] + \left[\frac{2^2 \tan^{-1} 2x}{2x}\right] + \left[\frac{3^2 \tan^{-1} 3x}{3x}\right] = 1^2 - 1 + 2^2 - 1 + 3^2 - 1 = 11$

(5) $\lim_{\theta\to 0^-} \frac{\sin\theta}{\cos^{-1}(\cos\theta)} = \lim_{\theta\to 0^-} \frac{\sin\theta}{-\theta} = -1$

(6) $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

(7) $\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x = e$

$y = \cos^{-1}(\cos x)$



$\lim_{x\to a} (1+\square)^{\frac{1}{\square}} = e$.

as $x \to a$, $\square \to 0$

Recap of previous lecture

(8) $\lim_{x\to 0}(1+2x)^{\frac{1}{x}} = \lim_{x\to 0}\left((1+2x)^{\frac{1}{2x}}\right)^{2} = e^2$

(9) $\lim_{x\to 0}(1+5\sin x)^{\operatorname{cosec} x} = \lim_{x\to 0}\left((1+5\sin x)^{\frac{1}{5\sin x}}\right)^{5} = e^5$

(10) $\lim_{x\to 0}\frac{\sin^5 x+\sin^3 x+x^5}{\tan^5 x+\tan^3 x-x^4} = \frac{1}{1}$

(11) $\lim_{x\to 0}\frac{\sin^3 x+\tan^2 x-x^3+x}{\tan^3 x+\tan x+x} = 1/2$

(12) $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x} = \lim_{x\to 0}\frac{|\sin x|\cdot\sqrt{2}}{x} = \text{D.N.E}$

(13) $\lim_{n\to\infty}\frac{5^n+2^n+3^{2n}}{6\cdot 3^{2n}-2^n+7^n} = \lim_{n\to\infty}\frac{5^n+2^n+9^n}{6\cdot 9^n-2^n+7^n} = \lim_{n\to\infty}\frac{(5/9)^n+(2/9)^n+1}{6-(2/9)^n+(7/9)^n} = 1/6.$

($5/9 \to 0$, $2/9 \to 0$, $2/9 \to 0$, $7/9 \to 0$)

Four Golden Points

(1) $\lim_{x\to 0}\frac{e^x-1}{x}=1$

(2) $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$

(3) $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$

(4) $\lim_{x\to 0}(1+x)^{1/x}=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$



Generalized 1^{∞} form

If $\lim_{x \to a} f(x) = 1$ & $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} (f(x))^{g(x)} = e^{\lim_{x \to a} g(x) \cdot (f(x) - 1)}$

Limit using Series Expansion

Expansion of function like binomial expansion, exponential & logarithmic expansion, expansion of sin x, cos x, tan x should be remembered by heart which are given below :

(a) $a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + \ldots\ldots\ a > 0$ ★★★★★

(b) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$ ★★★★★★

(c) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots \text{ for } -1 < x \leq 1$ ★★★★★

(d) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$ ★★★★★

Limit using Series Expansion

(e) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots$

(f) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\ldots$

(g) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$

(h) $\sin^{-1} x = x + \frac{1^2}{3!}x^3 + \frac{1^2 \cdot 3^2}{5!}x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!}x^7 + \ldots\ldots$

(i) $\sec^{-1} x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \ldots\ldots$

(j) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots,$

(k) $(1+x)^{1/x} = e^{\frac{1}{x}\ln(1+x)} = e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 - \frac{21}{48}x^3 + \ldots.\right]$

Galti Batao

Tah:06

(a) $\lim_{x\to 0} \dfrac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$

Don't Apply limit partially

Sol:

$\lim_{x\to 0} \dfrac{-1+\cos(xe^x)+1-\cos xe^{-x}}{x^2}$

$\lim_{x\to 0} \dfrac{1}{x}\left[-\left(\dfrac{1-\cos(xe^x)}{x^2(e^x)^2}\cdot e^{x^2}\right) + \left(\dfrac{1-\cos xe^{-x}}{x^2(e^{-x})^2}\cdot(e^{-x})^2\right)\right]$

$\lim_{x\to 0} \dfrac{1}{x}\left[-(\ )\cdot e^{x^2} + (\ )\cdot e^{-x^2}\right]$

$\lim_{x\to 0} -(\ )\left[\dfrac{e^{2x}-e^{-2x}}{x}\right]$

$\lim_{x\to 0} -(\ )\dfrac{(e^{4x}-1)}{e^{2x}\cdot x} \Rightarrow \lim_{x\to 0} -(\ )\dfrac{(e^{4x}-1)}{4x\cdot e^{2x}}\cdot 4$

$= \dfrac{-1}{2}\times\dfrac{4}{e^0} = -2$

Ans

Our Three Friends in Limits

★ $\lim_{x\to 0}\frac{x-\sin x}{x^3} = \lim_{x\to 0}\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\infty\right)}{x^3} = \frac{1}{3!}=\frac{1}{6}$

★ $\lim_{x\to 0}\frac{\tan x-x}{x^3} = \lim_{x\to 0}\frac{\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+\cdots\infty\right)-x}{x^3} = \frac{1}{3}$

as $x\to 0$, $\tan x - x$, $x-\sin x$ ki aatma ki shaanti ke liyay x^3 ki jarroorat hoti hai

★ $\lim_{x\to 0}\frac{e^x-1-x}{x^2} = \lim_{x\to 0}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\infty\right)-1-x}{x^3} = \frac{1}{2}$.

as $x\to 0$ $x+\sin x$, $x+\tan x$ ki aatma ki shanti ke liyay x^1 ki jarcorat hai

ASNC

Question:Expansion kab lagana Chahiye??

Answer: Jahan Numerator aur denominator may unequal power ki zaroorat Lage toh Expansion Lagado

Homework Discussion

QUESTION [JEE Mains 2023 (8 April)] TAH 3

If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x \to \frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{1/2} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$, then k is equal to

(A) $\beta 2$

(B) β

(C) α

(D) 2α

$\alpha > \beta > 0$

$\frac{1}{\alpha} < \frac{1}{\beta}$

$ax^2+bx+1=0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix} \right. \xrightarrow{x \to \frac{1}{x}} x^2+bx+a=0 \left\langle \begin{matrix}\frac{1}{\alpha} \\ \frac{1}{\beta}\end{matrix} \right.$

$\alpha\beta = \frac{1}{a}$ (+ve, +ve)

$x^2+bx+a = 1\cdot\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right) = \frac{(\alpha x-1)(\beta x-1)}{\alpha\beta}$

$x^2+bx+a = a(\alpha x-1)(\beta x-1)$

$$\lim_{x \to 1/\alpha} \left(\frac{1-\cos\left(a(\alpha x-1)(\beta x-1)\right)}{2a^2(1-\alpha x)^2(\beta x-1)^2} \cdot (\beta x-1)^2 \cdot a^2\right)^{\frac{1}{2}}$$

$$= \left(\frac{1}{2}\cdot\frac{1}{2}\left(\frac{\beta}{\alpha}-1\right)^2 \cdot a^2\right)^{1/2} = \frac{1}{2}\left(a^2\beta^2\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2\right)^{1/2} = \frac{1}{2}|a\beta|\left|\frac{1}{\alpha}-\frac{1}{\beta}\right| = -\frac{1}{2}a\beta\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)$$

$$= \frac{1}{2a}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$

$a\alpha\beta = 1 \Rightarrow a\beta = \frac{1}{\alpha}$

QUESTION

TAH 6

(a) $\lim_{x\to 0} \dfrac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$

$= \lim_{x\to 0} \dfrac{2\sin\frac{x(e^x+e^{-x})}{2}\sin\frac{x(e^{-x}-e^x)}{2}}{x^3}$

(b) $\lim_{x\to\infty} \dfrac{e^{1/x^2} - 1}{2\arctan x^2 - \pi}$

$= \lim_{x\to 0} 2 \cdot \dfrac{\sin\frac{x(e^x+e^{-x})}{2}}{\frac{x(e^x+e^{-x})}{2}} \cdot \dfrac{\sin\frac{x(e^{-x}-e^x)}{2}}{x\cdot\frac{(e^{-x}-e^x)}{2}}$

(c) $\lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

$\cdot \dfrac{x^2}{x^3} \dfrac{e^x+e^{-x}}{2} \cdot \dfrac{e^{-x}-e^x}{2}$

$= \lim_{x\to 0} 2(\ )(\ ) \dfrac{1}{x} \dfrac{(e^{-x}-1-(e^x-1))}{2} \dfrac{e^x+e^{-x}}{2}$

$= \lim_{x\to 0} 2(\ )(\ ) \left(\dfrac{(e^{-x}-1)}{-x} - \dfrac{e^x-1}{x}\right)\dfrac{1}{2} \dfrac{e^x+e^{-x}}{2} = 2\cdot 1\cdot 1\,(-1-1)\cdot\dfrac{1}{2}\cdot\dfrac{2}{2} = -2$

QUESTION

KTK 9

Let $\lim_{x\to 1}\frac{[x]}{x} = l$ and $\lim_{x\to 1}\frac{x}{[x]} = m$ where [] denotes the greatest integer function, then

(A) l exist but m does not exist

(B) m exist but l does not exist

(C) l and m both exist

(D) neither l nor m exist

$$l = \lim_{x\to 1}\frac{[x]}{x}$$

RHL: $\lim_{h\to 0}\frac{[1+h]}{1+h} = \lim_{h\to 0}\frac{1}{1+h} = 1$

LHL: $\lim_{h\to 0}\frac{[1-h]}{1-h} = \lim_{h\to 0}\frac{0}{1-h} = 0$

$f(x) = \frac{x}{[x]}$ clearly f is not defined in $[0,1)$

$$\Rightarrow \lim_{x\to 1^+}\frac{x}{[x]} = \lim_{x\to 1^+}\frac{x}{[x]} = \lim_{h\to 0}\frac{1+h}{[1+h]} = \lim_{h\to 0} 1+h = 1 = m$$

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION

(Q) $\lim_{x\to\pi/2}(1+3\cot^2 x)^{\tan^2 x}$ $\quad\hookrightarrow 1^\infty$

M① $e^{\lim_{x\to\pi/2}\tan^2 x\,(1+3\cot^2 x-1)} = e^{\lim_{x\to\pi/2}\tan^2 x\cdot 3\cot^2 x} = e^3$

(Q) $\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x$ $\quad\hookrightarrow 1^\infty$

M② $\lim_{x\to\pi/2}\left((1+3\cot^2 x)^{\frac{1}{3\cot^2 x}}\right)^{3\cot^2 x\cdot\tan^2 x} = e^3$

$= e^{\lim_{x\to\infty} x(\sin 1/x+\cos 1/x-1)}$

$= \lim_{e^{x\to\infty}} \frac{\cos 1/x - 1 + \sin 1/x}{1/x} = \lim_{x\to\infty} -\frac{(1-\cos 1/x)}{(1/x)^2}\cdot\frac{1}{x} + \frac{\sin 1/x}{1/x} = e^{0+1} = e^1$

$\Gamma\frac{1}{2}\ \Gamma 0 \qquad \Gamma 1$

QUESTION

Tan 1?

If $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = e^c$, then the value of c is

1^∞

A 5

B 4

C 3

D 2

QUESTION

Tanz

$$\lim_{x\to 0}\left(\frac{5}{2+\sqrt{9+x}}\right)^{\operatorname{cosec} x}$$

A $e^{-1/6}$

B $e^{-1/5}$

C $e^{-1/25}$

D $e^{-1/30}$

QUESTION

$$\lim_{n\to\infty}\left(5^{\frac{1}{n}}+3^{\frac{1}{n}}-1\right)^{n}$$

$5^0+3^0-1 \to 1^\infty$

$$= e^{\lim_{n\to\infty} n\left(5^{\frac{1}{n}}+3^{\frac{1}{n}}-1-1\right)}$$

$$= e^{\lim_{n\to\infty} \frac{\left(5^{1/n}-1+3^{1/n}-1\right)}{1/n}}$$

$$= e^{\lim_{n\to\infty}\left(\frac{5^{1/n}-1}{1/n}+\frac{3^{1/n}-1}{1/n}\right)}$$

$$= e^{\ln 5+\ln 3} = e^{\ln 15} = 15 \text{ Ans}$$

QUESTION [JEE Advanced 2024 (Paper 2)]

Tah 3

Let $k \in \mathbb{R}$. If $\lim\limits_{x \to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

1^∞

(A) 1

(B) 2

(C) 3

(D) 4

Ans. B

QUESTION [JEE Mains 2024 (8 April)]

Tan 4

If $\alpha = \lim\limits_{x \to 0^+} \left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and $\beta = \lim\limits_{x \to 0} (1 + \sin x)^{\frac{1}{2} \cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then $12 \log_e (a + b)$ is equal to

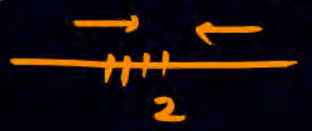
Ans. 6

QUESTION [JEE Advanced 2014] ASRQ+KCLS

1-ve no:) $\frac{31}{16}$

power → 2

(+ve no:)



The largest value of the non-negative integer a for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$ is

If $a=2$

$$\lim_{x\to 1}\left(\frac{-2+\frac{\sin(x-1)}{x-1}}{1+\frac{\sin(x-1)}{x-1}}\right)^{1+\sqrt{x}}$$

−ve

$$\lim_{x\to 1}\left(\frac{a(1-x)+\sin(x-1)}{(x-1)+\sin(x-1)}\right)^{\frac{(1-\sqrt{x})(1+\sqrt{x})}{1-\sqrt{x}}}$$

$$\lim_{x\to 1}\left(\frac{-a+\frac{\sin x-1}{x-1}}{1+\frac{\sin(x-1)}{x-1}}\right)^{(1+\sqrt{x})}=1/4$$

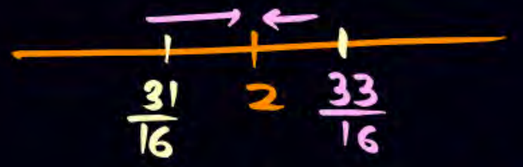
$$\Rightarrow\left(\frac{1-a}{2}\right)^2=1/4$$

$$\frac{1-a}{2}=1/2,-1/2$$

$$1-a=1,-1\Rightarrow a=0,2$$

Ans: a=0

$(-ve)^{\frac{1}{2n}}$ is not defined in real no:s. $(n \in N)$



(-ve no:) power → 2

$2-\frac{1}{128}=\frac{255}{128}$, $2+\frac{1}{128}=\frac{257}{128}$

$2-\frac{1}{2^{10}}=\frac{2047}{1024}$, $2+\frac{1}{2^{10}}=\frac{2049}{1024}$

QUESTION

$$\lim_{x\to\infty}\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+\cdots+n^{\frac{1}{x}}}{n}\right)^{nx};\ n\in N$$

$(\frac{n}{n})^{\infty}$ 1^{∞} form.

$$= \lim_{x\to\infty} e^{nx\left(\frac{1^{\frac{1}{x}}+2^{\frac{1}{x}}+3^{\frac{1}{x}}+\cdots+n^{\frac{1}{x}}}{n}-1\right)}$$

$$= e^{\lim_{x\to\infty} nx\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}+\cdots+n^{1/x}-n}{n}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{1^{1/x}-1+2^{1/x}-1+3^{1/x}-1+\cdots+n^{1/x}-1}{1/x}\right)} =$$

$$e^{\ln n!} = n!$$

$$= e^{\ln(1\cdot 2\cdot 3\cdots n)}$$

$$= e^{\ln 1+\ln 2+\cdots+\ln n}$$

QUESTION

Tan 5

$$\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n \quad a>0, b>0, n\in N$$

A $a^{1/b}$

B $b^{1/a}$

C a^b

D b^a

QUESTION [JEE Mains 2020]

Tan 6

$$\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}} \text{ is equal to:}$$

A e

B 2

C e^2

D 1

QUESTION

$$\lim_{x\to 0}\frac{1-\cos(\tan x - x)}{x^6} \quad (0)$$

$$= \lim_{x\to 0}\frac{1-\cos(\tan x - x)}{(\tan x - x)^2}\left(\frac{\tan x - x}{x^3}\right)^2 = \frac{1}{2}\cdot\left(\frac{1}{3}\right)^2 = \frac{1}{18}$$

$$\lim_{x\to 0}\frac{\tan^2 x - x^2}{x^2\tan^2 x}$$

$$= \lim_{x\to 0}\frac{(\tan x - x)}{x^3}\cdot\frac{(\tan x + x)}{x}\cdot\frac{x^4}{x^3+\tan x}$$

$$= 1/3 \times 2 \times 1 = \underline{2/3}$$

QUESTION

$$\lim_{x\to 0} \frac{1}{(\sin^{-1}x)^2} - \frac{1}{x^2}$$

put $x = \sin\theta$

as $x \to 0$, $\theta \to 0$

$$= \lim_{\theta\to 0} \left(\frac{1}{\theta^2} - \frac{1}{\sin^2\theta}\right)$$

$$= \lim_{\theta\to 0} \frac{\sin^2\theta - \theta^2}{\theta^2 \sin^2\theta} = -\lim_{\theta\to 0} \frac{(\theta - \sin\theta)(\theta + \sin\theta)}{\theta^3 \cdot \frac{\sin^2\theta}{\theta^2} \cdot \theta}$$

$$= \frac{-\left(\frac{1}{6}\cdot 2\right)}{1} = -1/3.$$

$$\lim_{x\to 0}\left(\left(\frac{x}{\sin^{-1}x}\right)^2 \cdot \frac{1}{x^2} - \frac{1}{x^2}\right)$$

($\frac{x}{\sin^{-1}x} \to 1$)

$$\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x^2}\right) = 0$$

Agar Apnay bhi yahi kiya Toh app ——— hai !!

Ans. -1/3

QUESTION

If $\lim_{x\to 0} \frac{A\cos x + Bx\sin x - 5}{x^4}$ exists & finite. Find A & B and also the limit.

$$\lim_{x\to 0} \frac{A\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\infty\right)+Bx\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\infty\right)-5}{x^4}$$

$$\lim_{x\to 0} \frac{\overset{0}{\overset{\|}{(A-5)}}+x^2\overset{0}{\overset{\|}{\left(-\frac{A}{2}+B\right)}}+x^4\left(\frac{A}{24}-\frac{B}{6}\right)+x^6\left(-\frac{A}{6!}+\frac{B}{5!}\right)+\cdots\infty}{x^4}$$

for limit to exist $A-5=0 \Rightarrow A=5$

$-A/2+B=0 \Rightarrow B=5/2$

$L=\frac{A}{24}-\frac{B}{6}$

$=5/24-\frac{5}{12}=-5/24$

QUESTION

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$$

$$= \lim_{x\to 0}\frac{e^{\ln(1+x)^{\frac{1}{x}}}-e}{x} = \lim_{x\to 0}\frac{e^{\frac{1}{x}\ln(1+x)}-e}{x}$$

$\frac{1}{x}\ln(1+x)-1 = M$

$$= e\lim_{x\to 0}\frac{e^{\frac{1}{x}\ln(1+x)-1}-1}{x}$$

as $x\to 0 \Rightarrow M\to 0$

b'coz $\frac{\ln(1+x)}{x}\to 1$

$$= e\lim_{x\to 0}\frac{e^M-1}{M}\cdot\frac{\left(\frac{1}{x}\cdot\ln(1+x)-1\right)}{x}$$

$$= e\lim_{x\to 0}\frac{e^M-1}{M}\cdot\frac{\ln(1+x)-x}{x^2}$$

$$= e\lim_{x\to 0}\left(\frac{e^M-1}{M}\right)\left(\frac{(x-x^2/2+x^3/3-\cdots)-x}{x^2}\right) = -e/2.$$

QUESTION

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$$

$$= \lim_{x\to 0}\frac{e\left(1-\frac{x}{2}+\frac{11}{24}x^2-\cdots\right)-e}{x}$$

$$= e\lim_{x\to 0}\frac{\left(1-\frac{x}{2}+\frac{11}{24}x^2-\cdots\right)-1}{x}$$

$$= -e/2.$$

QUESTION [JEE Mains 2024 (9 April)]

Tan 1

$$\lim_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$$ is equal to

(A) $-\frac{2}{e}$

(B) $e - e^2$

(C) 0

(D) e

Ans. D

QUESTION [JEE Advanced 2020]

TAH 8

Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit $\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^n}$ is equal to nonzero real number, is

Ans : 1

QUESTION [JEE Mains 2024 (31 Jan)]

$$\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$$

$$L=\lim_{x\to 0}\left(\frac{e^{2|\sin x|}-2|\sin x|-1}{\sin^2 x}\cdot\frac{\sin^2 x}{x^2}\right)$$

$x^2=|x|^2$

(A) is equal to 1

(B) does not exist

(C) is equal to -1

(D) is equal to 2

$$\text{Let } l=\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{|\sin x|^2}$$

Let $|\sin x|=t$

as $x\to 0 \Rightarrow t\to 0^+$

$$l=\lim_{t\to 0^+}\frac{e^{2t}-2t-1}{t^2}=\lim_{t\to 0^+}\frac{\left(1+2t+\frac{(2t)^2}{2!}+\cdots\infty\right)-1-2t}{t^2}$$

$$=2.$$

$$L=\lim_{x\to 0}\left(\frac{e^{2|\sin x|}-2|\sin x|-1}{\sin^2 x}\right)\cdot\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2=2\cdot 1=2$$

Ans. D

QUESTION

Tan9

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^3}{\sin^6 2x}$$

Ans. 1/128

QUESTION [JEE Mains 2024 (31 Jan)]

Tan 10

If $\lim\limits_{x\to 0}\dfrac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$, then $16(a^2+b^2+c^2)$ is equal to

Ans. 81

QUESTION [JEE Mains 2021 (20 July)]

TAH !!

If $\lim\limits_{x\to 0} \dfrac{\alpha x e^{x} - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10, \alpha, \beta, \gamma \in R$, then the value of $\alpha + \beta + \gamma$ is

QUESTION

$$\lim_{x\to\infty}\left\{x - x^2 \ln\left(1+\frac{1}{x}\right)\right\} = \lim_{x\to\infty} x^2\left(\frac{1}{x} - \ln\left(1+\frac{1}{x}\right)\right)$$

$$= \lim_{x\to\infty} x^2\left(\frac{1}{x} - \left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \frac{1}{4x^4} + \cdots - \infty\right)\right)$$

$$= \frac{1}{2} \text{ Ans}$$

Ans. 1/2

QUESTION [JEE Mains 2021]

TAH 12

If $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of $a-2b$ is

QUESTION [JEE Mains 2023 (13 April)]

TAH 12

If $\lim\limits_{x\to 0} \dfrac{e^{ax} - \cos(bx) - \dfrac{cx\, e^{-cx}}{2}}{1 - \cos(2x)} = 17$, then $5a^2 + b^2$ is equal to

A 64

B 68

C 72

D 76

Functions with Built in Limit

To find limit of a fn which already has a limit in it

Limit at $x=a$ is all about the fn in nbd of $x=a$.

QUESTION

$$f(x)=\lim_{n\to\infty}\frac{\tan \pi x^2+(x+1)^n \sin x}{x^2+(x+1)^n} \text{ then find } \lim_{x\to 0} f(x).$$

★ $0<a<1$ $\quad a^x\to 0$ as $x\to\infty$

★ $a>1$ $\quad a^x\to\infty$ as $x\to-\infty$

clearly for $\lim_{x\to 0} f(x)$ f should be defined in some nbd of $x=0$



$$f(x)=\begin{cases}\lim\limits_{n\to\infty}\dfrac{\tan\pi x^2+\sin x}{\frac{x^2}{(x+1)^n}+1} & x\in(0,\Delta) \quad x+1>1\\ \lim\limits_{n\to\infty}\dfrac{\tan\pi x^2+(x+1)^n\sin x}{x^2+(x+1)^n} & x\in(-\Delta,0) \quad 0<x+1<1\end{cases}$$

$(x+1)^n \to 0$ as $n\to\infty$

$$f(x)=\begin{cases}\sin x & x\in(0,\Delta)\\ \dfrac{\tan\pi x^2}{x^2} & x\in(-\Delta,0)\end{cases}$$

$$f(x)=\begin{cases}\sin x & x\in(0,\Delta)\\ \dfrac{\tan\pi x^2}{x^2} & x\in(-\Delta,0)\end{cases}$$

$\lim\limits_{x\to 0} f(x)$

- $\lim\limits_{x\to 0^+} \sin x = 0$
- $\lim\limits_{x\to 0^-} \dfrac{\tan\pi x^2}{\pi x^2}\cdot\pi = \pi$

$\Rightarrow \lim\limits_{x\to 0} f(x)$ D.N.E

QUESTION

$$f(x)=\lim_{n\to\infty}\frac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}\ \text{find}\ \lim_{x\to1}f(x).$$

To find $\lim_{x\to1} f(x)$ we need to define f in nbd of $x=1$



$$f(x)=\begin{cases}\lim\limits_{n\to\infty}\dfrac{\frac{\cos\pi x}{x^{2n+1}}-\frac{\sin(x-1)}{x}}{\frac{1}{x^{2n+1}}+1-\frac{1}{x}} & x\in(1,1+\Delta)\\ \lim\limits_{n\to\infty}\dfrac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}} & x\in(1-\Delta,1)\end{cases}$$

($\frac{1}{x^{2n+1}}\to0$, $\frac{\cos\pi x}{x^{2n+1}}\to0$, $x^{2n}\to0$ for $x\in(1-\Delta,1)$)

$$=\begin{cases}\dfrac{-\sin(x-1)}{x(1-1/x)} & x\in(1,1+\Delta)\\ \cos\pi x & x\in(1-\Delta,1)\end{cases}$$

$\lim_{x\to1} f(x)$:

- $\lim_{x\to1^+}\frac{-\sin(x-1)}{x(1-1/x)}=\lim_{x\to1^+}-\frac{\sin(x-1)}{x-1}=-1$
- $\lim_{x\to1^-}\cos\pi x=\cos\pi=-1$

$\lim_{x \to -1} f(x) = -1$ — Ans

QUESTION

Tah 13

$(n \in N)$

Let $f(x) = \lim\limits_{n\to\infty} \dfrac{2x^{2n}\sin\frac{1}{x}+x}{1+x^{2n}}$ then which of the following alternative(s) is/are correct?

(A) $\lim\limits_{x\to\infty} x f(x) = 2$

(B) $\lim\limits_{x\to 1} f(x)$ does not exist.

(C) $\lim\limits_{x\to 0} f(x)$ does not exist.

(D) $\lim\limits_{x\to-\infty} f(x)$ is equal to zero.

If x is very large +ve no:

(A) $f(x) = \lim\limits_{n\to\infty} \dfrac{2\sin 1/x + \frac{x}{x^{2n}}}{\frac{1}{x^{2n}}+1} = 2\sin\dfrac{1}{x}$

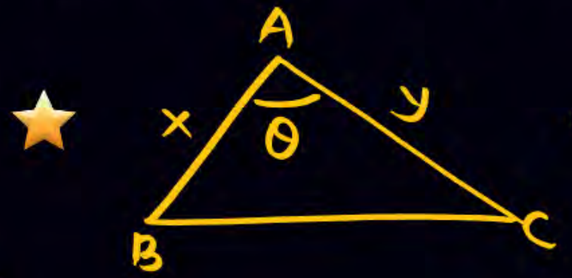
$\lim\limits_{x\to\infty} x f(x) = \lim\limits_{x\to\infty} 2x\sin 1/x$

$= \lim\limits_{x\to\infty} 2\dfrac{\sin 1/x}{1/x} = 2.$

Geometrical Problems

QUESTION

An arc PQ of a circle subtends a central angle θ as shown. Let A(θ) be the area between the chord PQ and the arc PQ. Let B(θ) be the area between the tangent lines PR and QR and the are PQ. Find $\lim_{\theta \to 0} \frac{A(\theta)}{B(\theta)}$



$ar(\Delta ABC) = \frac{1}{2} \cdot x \cdot y \sin\theta$

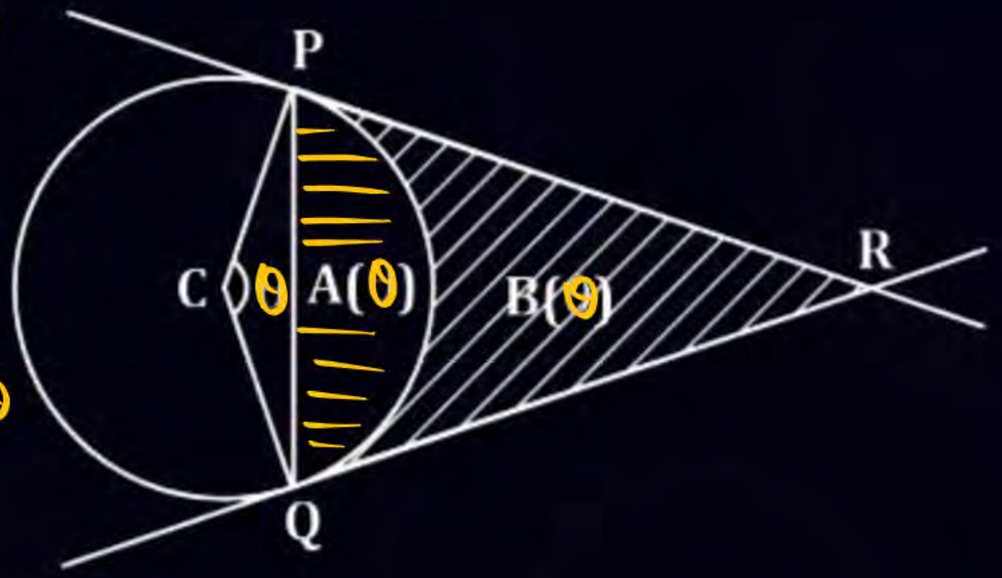


PA = PB

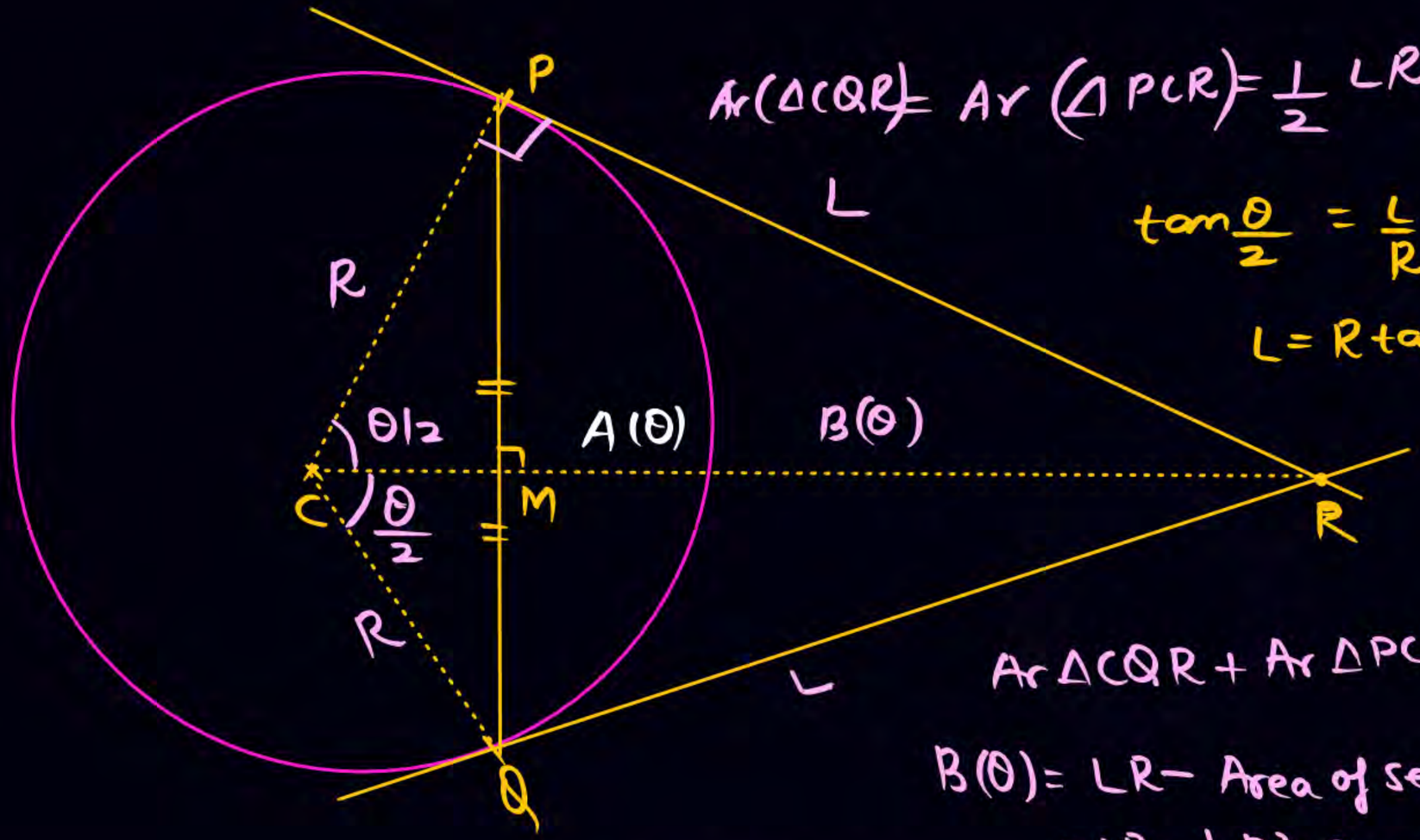
AM = MB



$\text{area of sector} = \frac{1}{2} r^2 \theta$



Ans. 2



$Ar(\Delta CQR) = Ar(\Delta PCR) = \frac{1}{2} LR$

$\tan\frac{\theta}{2} = \frac{L}{R}$

$L = R\tan\theta/2$

$Ar\,\Delta CQR + Ar\,\Delta PCR = LR$

$B(\theta) = LR - \text{Area of sector}$

$= LR - \frac{1}{2}R^2\theta = R^2\tan\frac{\theta}{2} - \frac{1}{2}R^2\theta$

$A(\theta) = \text{Area of sector} - Ar(\Delta PCQ)$

$= \frac{1}{2}R^2\theta - \frac{1}{2}R^2\sin\theta = \frac{1}{2}R^2(\theta - \sin\theta)$

$$\lim_{\theta \to 0} \frac{B(\theta)}{A(\theta)} = \lim_{\theta \to 0} \frac{R^2 \tan\frac{\theta}{2} - \frac{1}{2} R^2 \theta}{\frac{1}{2} R^2 (\theta - \sin\theta)}$$

$$= \lim_{\theta \to 0} \frac{2\left(\tan\frac{\theta}{2} - \frac{\theta}{2}\right)}{\theta - \sin\theta}$$

$$= \lim_{\theta \to 0} \frac{2\left(\frac{\tan(\theta/2) - \theta/2}{(\theta/2)^3 \cdot 2^3}\right)}{\frac{\theta - \sin\theta}{A3}}$$

$$= 2 \cdot \frac{\frac{1}{3} \cdot \frac{1}{8}}{\frac{1}{6}} = \frac{1}{2}$$

$$\lim_{\theta \to 0} \frac{B(\theta)}{A(\theta)} = \frac{1}{2} \Rightarrow \lim_{\theta \to 0} \frac{A(\theta)}{B(\theta)} = 2$$

QUESTION

Suppose that circle of equal diameter are packed tightly in n rows inside an equilateral triangle. (The figure illustrates the case $n = 4$.) If A is the area of the triangle and A_n is the total area occupied by the circles in n rows then $\lim_{n\to\infty} \frac{A_n}{A}$ equals

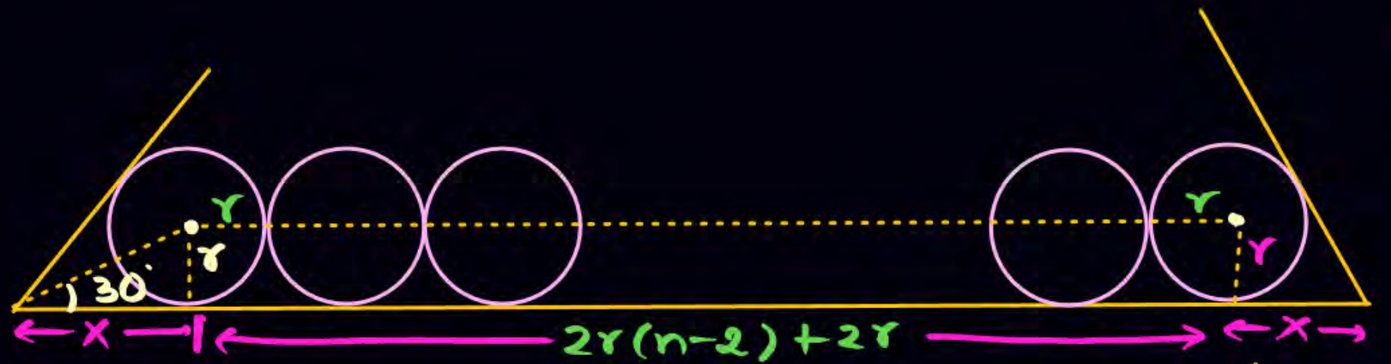


A $\pi/\sqrt{6}$

B $\pi\sqrt{3}/6$

C $\pi/2\sqrt{3}$

D $\pi/6$



Let side of equilateral $\Delta = a$

$X = r\cot 30^\circ = r\sqrt{3}$

$$a = X + 2r(n-2) + 2r + X$$

$$a = 2X + 2nr - 4r + 2r$$

$$a = 2r\sqrt{3} + 2(n-1)r = 2r(\sqrt{3} + n - 1)$$

$$A_n = \pi r^2(1+2+3+\cdots+n)$$

$$A_n = \pi r^2 \frac{n(n+1)}{2}$$

$$A = \frac{\sqrt{3}}{4}a^2$$

$$\lim_{n\to\infty} \frac{A_n}{A} = \lim_{n\to\infty} \frac{\pi r^2 \frac{n(n+1)}{2}}{\frac{\sqrt{3}}{4}a^2} = \lim_{n\to\infty} \frac{\pi r^2 \frac{n^2+n}{2}}{\frac{\sqrt{3}}{4}4r^2(\sqrt{3}+n-1)^2} = \frac{\pi \cdot 1/2}{\sqrt{3}\cdot 1} = \frac{\pi}{2\sqrt{3}}$$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 07

The number of value of k for which $(x^2 - (k - 2)x - 2k)(x^2 + kx + 2k - 4)$ is a perfect square is

A 1

B 2

C 3

D 0

Ans. A

(Revision Practice Problems)

QUESTION

(RPP 1)

If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

A $(-\infty, \infty)$

B $R - \left\{\frac{1}{2}\right\}$

C $R - \left\{\frac{4}{9}, \frac{1}{2}\right\}$

D $R - \left\{\frac{3}{2}\right\}$

Ans. C

QUESTION

(RPP 2)

If the quadratic polynomial $f(x) = (a-3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in

A $[0, 2]$

B $[3, 5]$

C $[4, 6)$

D $[5, 7]$

Ans. C

QUESTION

(RPP 3)

If the function $f: R^+ \to A$ defined by $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$ is surjective then find A.

Ans. $[6, \infty]$

Homework From Module

LCD

PRARAMBH : Complete. (for limit)

(Solution to TAH)

QUESTION

TAH 1

Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2 \sin 2x}{x}\right] + ... + \left[\frac{10 \sin 10x}{x}\right]$

(where [y] is the largest integer $\leq y$). The value of $\lim_{x \to 0} f(x)$ equals

A 55

B 164

C 165

D 375

Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \dots + \left[\frac{10\sin 10x}{x}\right]$

(where $[Y]$ is the largest integer $\le Y$). The value of

$\lim_{x\to 0} f(x)$ Equals:

$$\Rightarrow \lim_{x\to 0} \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{2x}\right] + \dots + \left[\frac{10^2 \sin 10x}{10x}\right]$$

($\frac{\sin x}{x} \to 1^-$, $\frac{2\sin 2x}{2x} \to 1^-$, $\frac{10^2\sin 10x}{10x} \to 1^-$)

$$= (1^2-1) + (2^2-1) + \dots + (10^2-1)$$
$$= (1^2 + 2^2 + 3^2 + \dots + 10^2) - (10)$$
$$= \frac{10(10+1)(20+1)}{6} - 10$$
$$= \frac{\overset{5}{\cancel{10}} \times 11 \times \overset{7}{\cancel{21}}}{\cancel{6}\,\cancel{2}} - 10$$
$$= 385 - 10 = 375$$

TAH 1
ABHIJEET KUMAR PATRA
ODISHA

1st August

Tah-(01)

Let $f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \text{----} + \left[\frac{10\sin 10x}{x}\right]$

(Where $[y]$ is the largest integer $\leq y$) The value of $\lim_{x\to 0} f(x)$ equals.

Soln: $\lim_{x\to 0} f(x)$

$$\lim_{x\to 0} \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \left[\frac{3\sin 3x}{x}\right] + \text{----} + \left[\frac{10\sin 10x}{x}\right]$$

$$\lim_{x\to 0} \left[\frac{\sin x}{x}\right] + \left[\frac{2^2\sin 2x}{2x}\right] + \left[\frac{3^2\sin 3x}{3x}\right] + \text{----} + \left[\frac{10^2\sin 10x}{10x}\right]$$

$$= 0 + (2^2-1) + (3^2-1) + \text{----} + (10^2-1)$$

$$= (2^2+3^2+4^2+\text{---}+10^2) + 9(-1)$$

$$= (1^2+2^2+3^2+\text{---}+10^2) - 9 - 1 \qquad [\text{Add \& Sub } 1]$$

$$= \frac{5}{6}\frac{10(10+1)(20+1)}{3} - 10$$

$$= \frac{5\times 11\times 7}{3} - 10$$

$$= 385 - 10$$

$$= 375 \text{ Answer}$$

Kalpana From Bihar...

QUESTION

TAH 2

If $\lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+\ldots+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)=100$, then the value of n, is

Note: [k] denotes the greatest integer less than or equal to k

A 2

B 3

C 4

D 5

If $\lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+\ldots+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)$

$=100$, then value of n, is

Note: $[k]$ deotoes GIF.

$\Rightarrow \lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]^{\to 1^+}+\left[\frac{2^3\sin^{-1}2x}{2x}\right]^{\to 1^+}+\ldots+\left[\frac{n^3\sin^{-1}nx}{nx}\right]^{\to 1^+}\right)=100$

$1^3+2^3+3^3+\cdots+n^3=100$

$\Rightarrow \frac{n^2(n+1)^2}{4}=100$

$\Rightarrow \frac{n(n+1)}{2}=10 \Rightarrow n(n+1)=20=4(4+1)$ $\Rightarrow \boxed{n=4}$ Ans

TAH 2
ABHIJEET KUMAR PATRA
ODISHA

Tah - (02)

If $\lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+---+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)$

$= 100$, then the value of n is, Note $[k]$ = GIF.

Solⁿ:- $\lim_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+---+\left[\frac{n^2\sin^{-1}nx}{x}\right]=100$

$\lim_{x\to 0}\left[\frac{1^3\sin^{-1}x}{1x}\right]+\left[\frac{2^3\sin^{-1}2x}{2x}\right]+\left[\frac{3^3\sin^{-1}3x}{3x}\right]+---+\left[\frac{n^3\sin^{-1}nx}{nx}\right]=100$

$(1^3+2^3+3^3+---+n^3)=100$

$\left(\frac{n(n+1)}{2}\right)^2=100$

$n^2(n+1)^2=400$

Taking square root both sides, we get

$n(n+1)=20$

$n(n+1)=4\times 5$

$n(n+1)=4(5+1)$

Hence $\boxed{n=4}$ Answer

Kalpana From Bihar

QUESTION [JEE Mains 2023 (8 April)]

TAH 3

If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim\limits_{x \to \frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{1/2} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$, then k is equal to

(A) $\beta 2$

(B) β

(C) α

(D) 2α

If $\alpha > \beta > 0$ are the roots of the Equation $ax^2+bx+1=0$, and

$$\lim_{x\to\frac{1}{\alpha}} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{1/2} = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right),$$ then k is Equal to (2023)

(A) 2β
(B) β
(C) α
(D) 2α

$$\lim_{x\to\frac{1}{\alpha}} \left(\frac{1-\cos((x-1/\alpha)(x-1/\beta)}{(x-1/\alpha)^2(x-1/\beta)^2} \cdot \frac{(x-1/\alpha)^2(x-1/\beta)^2}{2(1-\alpha x)^2}\right)^{1/2}$$

$$\lim_{x\to\frac{1}{\alpha}} \left(\left(\quad\right) \cdot \frac{(x-1/\alpha)^2(x-1/\beta)^2}{2\alpha^2(1/\alpha-x)^2}\right)^{1/2}$$

TAH 3
ABHIJEET KUMAR PATRA
ODISHA

$$= \left(\frac{1}{2}\frac{\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2}{2\alpha^2}\right)^{1/2} = \left(\frac{1}{4\alpha^2}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2\right)^{1/2}$$

$$= \frac{1}{2\alpha}\left|\frac{1}{\alpha}-\frac{1}{\beta}\right| \quad (\text{-ve}) = \frac{1}{2\alpha}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$

$\alpha > \beta > 0 \Rightarrow \frac{1}{\alpha} < \frac{1}{\beta}$

Tab:03

If $\alpha > \beta > 0$ are the roots of the Eqn $ax^2+bx+1=0$ and $\lim_{x \to 1/\alpha} \left(\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right)^{1/2} = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$, then $k=?$

Sol^n :- $ax^2+bx+1=0 \left[\begin{matrix}\alpha \\ \beta\end{matrix}\right> \boxed{\alpha\beta = \frac{1}{a}}$

$\downarrow x \to \frac{1}{x}$

$\frac{a}{x^2}+\frac{b}{x}+1=0 \Rightarrow \frac{x^2+bx+a}{x^2}=0$; $x^2+bx+a=0$ ($1/\alpha$, $1/\beta$)

$x^2+bx+a = \therefore \left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right) = \frac{(\alpha x-1)(\beta x-1)}{\alpha\beta}$

$\boxed{x^2+bx+a = a(\alpha x-1)(\beta x-1)}$

$\lim_{x \to 1/\alpha} \left[\frac{1-\cos(x^2+bx+a)}{2(1-\alpha x)^2}\right]^{1/2}$

$= \lim_{x \to 1/\alpha} \left[\frac{1-\cos(a(\alpha x-1)(\beta x-1))^{1/2}}{a^2(\alpha x-1)^2(\beta x-1)^2} \times \frac{a^2\cdot(\beta x-1)^2}{2}\right]^{1/2}$

$\lim_{x \to 1/\alpha} \left[(\,)\ a^2 \cdot \frac{(1-\beta x)^2}{2}\right]^{1/2}$ $\quad \because (1-\beta x)^2 = (\beta x-1)^2$

$\left[\frac{1}{2} \times \frac{1}{2} \times a^2\left(1-\frac{\beta}{\alpha}\right)^2\right]^{1/2}$

$\frac{1}{2} \times a \times \left(1-\frac{\beta}{\alpha}\right)$ $\quad \because \frac{1}{\alpha} = \alpha\beta$

$\frac{1}{2} \times \frac{1}{\alpha\beta}\left(\frac{\alpha-\beta}{\alpha}\right) = \frac{1}{2\alpha}\left(\frac{\alpha-\beta}{\alpha\beta}\right)$

$\frac{1}{2\alpha}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right) = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$

$\therefore k = 2\alpha$ (D) Ans //

Ritesh Singh, from Devbhoomi Uttarakhand

QUESTION [JEE Mains 2020]

TAH 4

If $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is

If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is (2020)

$$\Rightarrow \lim_{x\to 0}\frac{1}{x^8}\left[\left(1-\cos\left(\frac{x^2}{2}\right)\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)\right]$$

$$\lim_{x\to 0}\frac{1}{x^8}\left(1-\cos x^2/2\right)\left(1-\cos x^2/4\right)$$

$$\lim_{x\to 0}\left(\frac{1-\cos(x^2/2)}{\frac{x^4}{4}\cdot 4}\right)\left(\frac{1-\cos(x^2/4)}{\frac{x^4}{16}\cdot 16}\right)$$

$$=\frac{1}{8}\cdot\frac{1}{32}=\frac{1}{2^8}=2^{-8}=2^{-k}$$

So, $\boxed{k=8}$

TAH 4
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

TAH 5

(a) $\lim_{x \to \pi/2} \frac{\tan x}{\tan 3x}$

(b) $\lim_{x \to \pi/2} \frac{\tan x}{\tan 5x}$

(c) $\lim_{x \to -\infty} \frac{x^4 \sin(1/x) + x^2}{1 + |x|^2}$

Tah - (5)

(a) $\lim_{x \to \frac{\pi}{2}} \frac{\tan x}{\tan 3x}$

let $x = \frac{\pi}{2} + t$ as $x \to \frac{\pi}{2}$, $t \to 0$

$\lim_{t \to 0} \frac{\tan\left(\frac{\pi}{2} + t\right)}{\tan 3\left(\frac{\pi}{2} + t\right)}$

$\lim_{t \to 0} \frac{-\cot t}{-\cot 3t}$

$\lim_{t \to 0} \frac{\tan 3t}{\tan t}$

$\lim_{t \to 0} 3 \cdot \frac{\frac{\tan 3t}{3t}}{\frac{\tan t}{t}}$ ($\frac{\tan 3t}{3t} \to 1$, $\frac{\tan t}{t} \to 1$)

$= 3 \cdot$

Answer = 3

(b) $\lim_{x \to \pi/2} \frac{\tan x}{\tan 5x}$

let $x = \frac{\pi}{2} + t$ as $x \to \frac{\pi}{2}$, $t \to 0$

$\lim_{t \to 0} \frac{\tan\left(\frac{\pi}{2} + t\right)}{\tan 5\left(\frac{\pi}{2} + t\right)}$

$\lim_{t \to 0} \frac{-\cot t}{-\cot 5t}$

$\lim_{t \to 0} \frac{\tan 5t}{\tan t}$

$\lim_{t \to 0} \frac{5 \cdot \frac{\tan 5t}{5t}}{\frac{\tan t}{t}} = 5$ Answer

Kalpana from Bihar

TAH 5

ABHIJEET KUMAR PATRA
ODISHA

(a) $\lim_{x \to \pi/2} \frac{\tan x}{\tan 3x}$

$x = \pi/2 + t$

As $x \to \pi/2$, $t \to 0$

$\lim_{t \to 0} \frac{\tan(\pi/2 + t)}{\tan 3(\frac{\pi}{2} + t)}$

$\lim_{t \to 0} \frac{-\cot t}{-\cot 3t} = \lim_{t \to 0} \frac{3 \cdot \frac{\tan 3t}{3t}}{\frac{\tan t}{t}} = 3$

(b) $\lim_{x \to \frac{\pi}{2}} \frac{\tan x}{\tan 5x}$

$x = \pi/2 + t$

As, $x \to \frac{\pi}{2}$, $t \to 0$

$\lim_{t \to 0} \frac{-\cot t}{-\cot 5t} = \lim_{t \to 0} \frac{5 \cdot \frac{\tan 5t}{5t}}{\frac{\tan t}{t}} = 5$

QUESTION

TAH 6

(a) $\lim_{x\to 0} \frac{\cos(xe^x) - \cos(xe^{-x})}{x^3}$

(b) $\lim_{x\to \infty} \frac{e^{1/x^2} - 1}{2 \arctan x^2 - \pi}$

(c) $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$

Kalpana From Bihar..

Date
Page

Tah-06

(c) $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1 + 1 - \cos x}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2}$

$1 + \frac{1}{2} = \frac{3}{2}$ Answer

(b) $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$ part 2

Sol^n: $\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2}$

$1 + \frac{1}{2} \Rightarrow \boxed{\frac{3}{2}}$ Ans

Bhavy Khandelwal
Gangapur Rajasthan

QUESTION [JEE Mains 2023 (8 April)]

TAH 7

$$\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right) \text{ is equal to}$$

A 15

B 18

C 9

D 24

Tah- (07)
(Mains 2023)

$$\lim_{x\to 0}\left(\left(\frac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right) \text{ is equal to}$$

$$\lim_{x\to 0}\frac{(1-\cos 3x)(1+\cos 3x)}{9x^2}\times 9x^2\times\frac{\sin^3 4x}{(4x)^3}\times\frac{64x^3\times(2x)^5}{2^5x^5(\log_e(1+2x))^5}$$

$$\lim_{x\to 0}\left(\frac{1-\cos 3x}{(3x)^2}\right)\left(\frac{\sin 4x}{4x}\right)^3\left(\frac{2x}{\log_e(1+2x)}\right)^5(1+\cos 3x)\frac{9x^2\times 64x^3}{9\times 2^5x^5}$$

$$\frac{1}{2}\times 1^3\times\left(\frac{1}{1}\right)^5\times(1+\cos 0)\times\frac{9\times 64}{32}^2$$

$$\frac{1}{2}\times 1\times 1\times(2)\times 18$$

18 Answer.

Kalpana From Bihar...

$$\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1)^5}\right)\right)$$ is Equal to (2023)

(A) 15

(B) 18

(C) 9

(D) 24

$$\lim_{x\to 0} 9.\frac{(1-\cos 3x)}{9x^2}(1+\cos 3x)\frac{\sin^3 4x}{64x^3}64\,\frac{1}{\cos^3 4x}\,\frac{32x^5}{32(\log(1+2x))^5}$$

$$\lim_{x\to 0} 9\left(\frac{1-\cos 3x}{9x^2}\right).(1+\cos 3x)\left(\frac{\sin 4x}{4x}\right)^3.64.\frac{1}{\cos^3 4x}\,\frac{1}{32}\left(\frac{2x}{\log_e(1+2x)}\right)^5$$

$$= 9.\frac{1}{2}.2.1.64.1.\frac{1}{32}.1 = 18$$

TAH 7
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

(ASRQ) + KCLS

TAH 8

The value of $\lim_{x \to 2} \frac{\sec^x \theta - \tan^x \theta - 1}{x-2}$ is equal to

(A) $\sec^2 \theta \cdot \ln \sec \theta + \tan^2 \theta \cdot \ln \tan \theta$

(B) $\sec^2 \theta \cdot \ln \tan \theta + \tan^2 \theta \cdot \ln \sec \theta$

(C) $\sec^2 \theta \cdot \ln \tan \theta - \tan^2 \theta \cdot \ln \sec \theta$

(D) $\sec^2 \theta \cdot \ln \sec \theta - \tan^2 \theta \cdot \ln \tan \theta$

The value of $\lim_{x \to 2} \frac{\sec^x\theta - \tan^x\theta - 1}{x-2}$ is Equal to

$\Rightarrow x = 2+t$

TAH (8)
ABHIJEET KUMAR PATRA
ODISHA

$$\lim_{t \to 0} \frac{\sec^{2+t}\theta - \tan^{2+t}\theta - 1}{t}$$

$$\lim_{t \to 0} \frac{\sec^{2+t}\theta - \tan^{2+t}\theta - (\sec^2\theta - \tan^2\theta)}{t}$$

$$\lim_{t \to 0} \frac{(\sec^{2+t}\theta - \sec^2\theta) - \tan^{2+t}\theta + \tan^2\theta}{t}$$

$$\lim_{t \to 0} \sec^2\theta \left(\frac{\sec^t\theta - 1}{t}\right) - \frac{\tan^2\theta(\tan^t\theta - 1)}{t}$$

$$= \sec^2\theta \log\sec\theta - \tan^2\theta \log\tan\theta$$

tah8

$$\lim_{n \to \frac{\pi}{2}} \frac{\sec^n\theta - \tan^n\theta - 1}{n-2}$$

Niranjan tiwari
Ballia wale ❤️❤️

put $n \to 2+t$

$$\lim_{t \to 0} \frac{\sec^{2+t}\theta - \tan^{2+t}\theta - (\sec^2\theta - \tan^2\theta)}{t}$$

$$\lim_{t \to 0} \frac{\sec^{2+t}\theta - \sec^2\theta - \tan^{2+t}\theta + \tan^2\theta}{t}$$

$$\lim_{t \to 0} \left(\sec^2\theta\left(\frac{(\sec\theta)^t - 1}{t}\right) - \tan^2\theta\left(\frac{(\tan\theta)^t - 1}{t}\right)\right.$$

$$\sec^2\theta \ln\sec\theta - \tan^2\theta \ln\tan\theta$$

Ans

Tah-8 (ASRQ+KCLS)

Ruchita Dwivedi
(From UP)

Q. $\lim_{x \to 2} \frac{\sec^n\theta - \tan^n\theta - 1}{x-2}$ equal to:

Soln. Let $x = 2+t$

as $x \to 2$

$t \to 0$

$$\lim_{x \to 2} \frac{\sec^n\theta - \sec^2\theta - \tan^n\theta + \tan^2\theta}{x-2}$$

$$\lim_{t \to 0} \frac{\sec^2\theta[\sec^t - 1] + \tan^2\theta[1 - \tan^t\theta]}{t}$$

$$\lim_{t \to 0} \sec^2\theta \frac{[\sec^t - 1]}{t} - \frac{\tan^2\theta[\tan^t - 1]}{t}$$

$$\boxed{\sec^2\theta \ln(\sec\theta) - \tan^2\theta \ln|\tan\theta|}$$ Answer.

Ques: $\lim_{x \to 2} \frac{\sec^x\theta - \tan^x\theta - 1}{x - 2}$ (KCLS + ASRO)

Put $x = 2 + t$

Sol^n: $\lim_{t \to 0} \frac{\sec^{(2+t)}\theta - \tan^{(2+t)}\theta - 1}{2 + t - 2}$ $\{1 + \tan^2\theta = \sec^2\theta\}$

$\lim_{t \to 0} \frac{\sec^{(2+t)}\theta - \tan^{(2+t)}\theta - (\sec^2\theta - \tan^2\theta)}{t}$

$\lim_{t \to 0} \frac{\sec^{(2+t)}\theta - \sec^2\theta - \tan^{(2+t)}\theta + \tan^2\theta}{t}$

$\lim_{t \to 0} \frac{\sec^2\theta(\sec^t\theta - 1) - \tan^2\theta(\tan^t\theta - 1)}{t}$

$\lim_{t \to 0} \sec^2\theta \left(\frac{\sec^t\theta - 1}{t}\right) - \tan^2\theta \left(\frac{\tan^t\theta - 1}{t}\right)$ $\left\{\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a\right\}$

$\boxed{\sec^2\theta \cdot \ln\sec\theta - \tan^2\theta \cdot \ln\tan\theta}$ (D)

Bhavy Khandelwal,Gangapur Rajasthan

(Solution to KTK)

QUESTION

(KTK 1)

#Q. The signum (or sign) function, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

. Compute the following.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$

(ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$

(iv) $\lim_{x \to 0} |\operatorname{sgn} x|$

Ans. (i) 1; (ii) –1, (iii) does not exist; (iv) 1

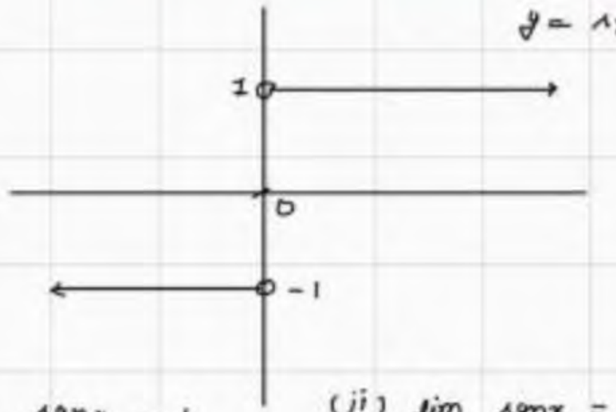
KTK 1

Q The signum (or sign) func., denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x > 0 \end{cases}$$

. compute the following

Sol:



(i) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ = D.N.E (iv) $\lim_{x \to 0} |\operatorname{sgn} x| = 1$

mod make -ve part +ve

$\lim_{x \to 0^-} |\operatorname{sgn} x| = 1$

$\lim_{x \to 0^+} |\operatorname{sgn} x| = 1$

KTK 1RAHUL DHAKAD ❤️❤️ FROM AGRA UP

QUESTION

(KTK 2)

#Q. For the function g whose graph is given, state the value of each quantity, if it exists.

(a) $\lim_{t \to 0^-} g(t)$

(b) $\lim_{t \to 0^+} g(t)$

(c) $\lim_{t \to 0} g(t)$

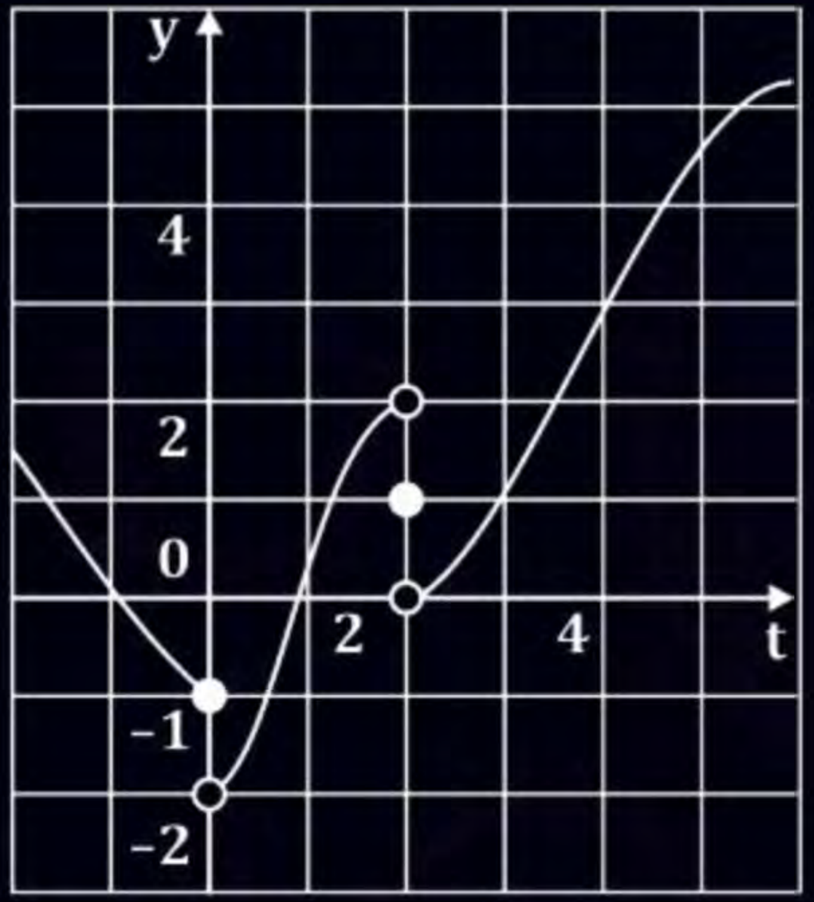
(d) $\lim_{t \to 2^-} g(t)$

(e) $\lim_{t \to 2^+} g(t)$

(f) $\lim_{t \to 2} g(t)$

(g) $g(2)$

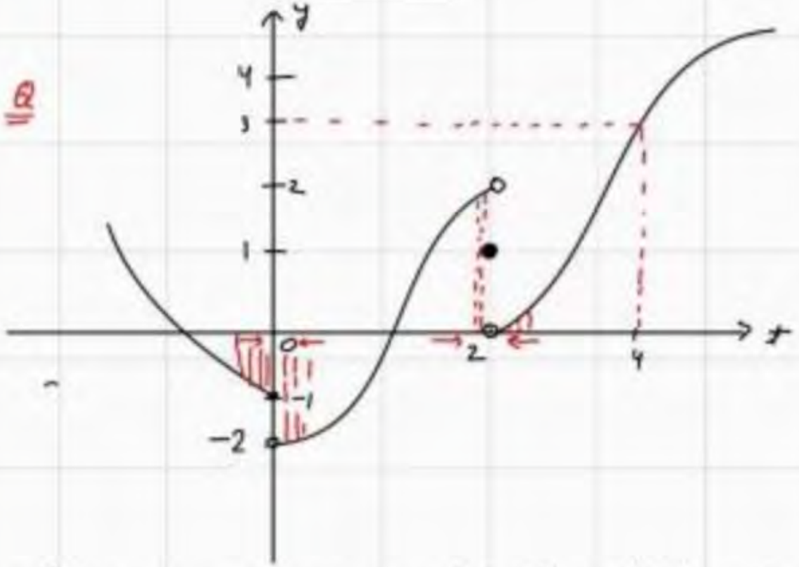
(h) $\lim_{t \to 4} g(t)$



Ans. (a) –1; (b) –2, (c) does not exist; (d) 2,; (e) 0; (f) does not exits; (g) 1; (h) 3

KTK-2

Q



(a) $\lim_{t \to 0^-} g(t) = -1$

(b) $\lim_{t \to 0^+} g(t) = -2$

(c) $\lim_{t \to 0} g(t) = D.N.E$

(d) $\lim_{t \to 2^-} g(t) = 2$

(e) $\lim_{t \to 2^+} g(t) = 0$

(f) $\lim_{t \to 2} g(t) = D.N.E$

(g) $g(2) = 1$

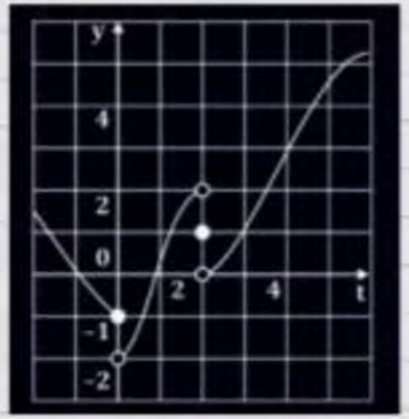
(h) $\lim_{t \to 4} g(t) = 3$

KTK 2 RAHUL DHAKAD

❤️❤️

FROM AGRA UP

For the following graph is given, State the value of each quantity, if it exists.



(a) $\lim_{t \to 0^-} g(t) = -1$

(b) $\lim_{t \to 0^+} g(t) = -2$

(c) $\lim_{t \to 0} g(t)$

⇒ As, RHL ≠ LHL

So, $\lim_{t \to 0} g(t)$ does not Exist

(d) $\lim_{t \to 2^-} g(t) = 2$

KTK 2
ABHIJEET KUMAR PATRA
ODISHA

(e) $\lim_{t \to 2^+} g(t) = 0$

(f) $\lim_{t \to 2} g(t)$

⇒ As, LHL ≠ RHL

So, $\lim_{t \to 2} g(t)$ does not Exist

(g) $g(2) = 1$

(h) $\lim_{t \to 4} g(t) = 3$

QUESTION [JEE Mains 2019 (10 April)]

KTK 3

If $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1}=\lim\limits_{x\to k}\dfrac{x^3-k^3}{x^2-k^2}$, then k is:

A $\frac{3}{2}$

B $\frac{8}{3}$

C $\frac{4}{3}$

D $\frac{3}{8}$

Ans. B

KTK-3

Q. If $\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-k^3}{x^2-k^2}$

Solu.

$\lim_{x\to 1} \frac{x^4-1^4}{x-1}$ = $\lim_{x\to k} \frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}$

$= 4.1^3$ = $\frac{3.k^2}{2.k^1}$

$= 4$ = $\frac{3}{2}k$

They are equal

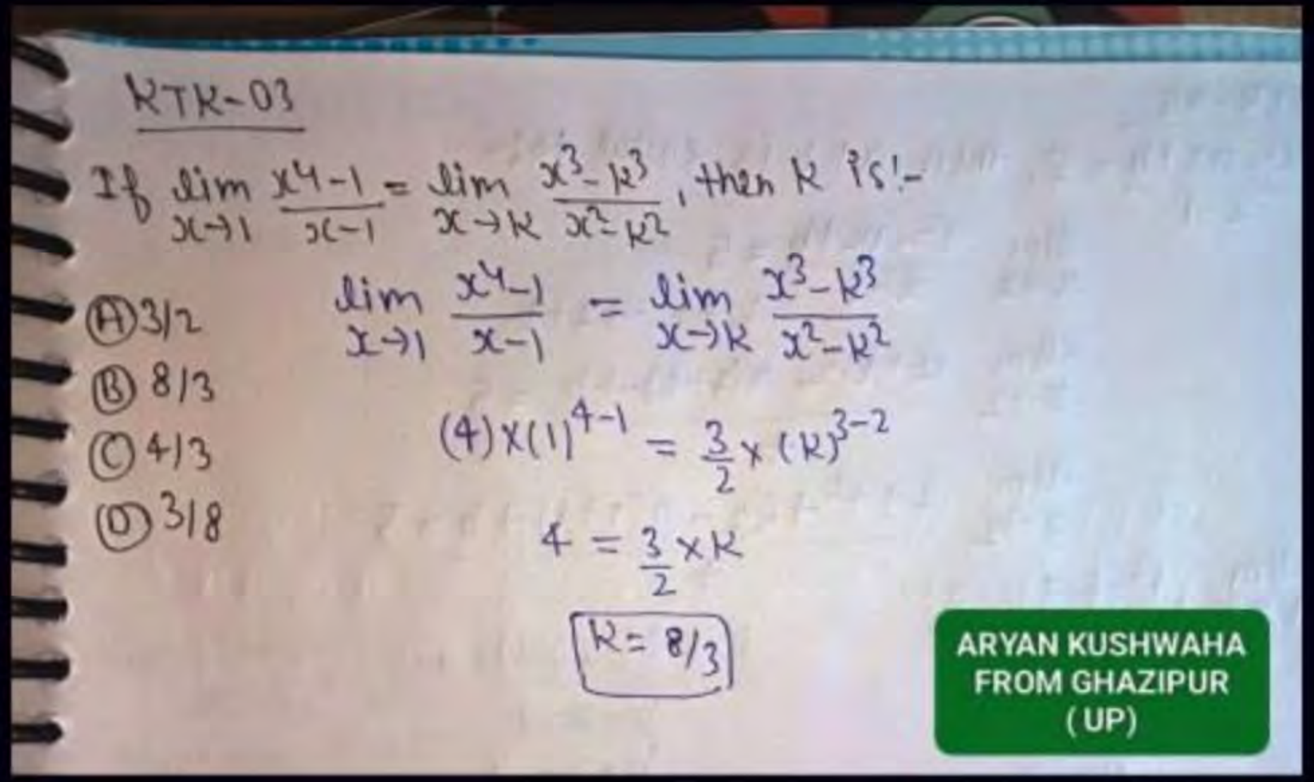
$\frac{3}{2}k = 4$

$k = \frac{8}{3}$ Ans.

KTK 3 RAHUL DHAKAD

❤️❤️

FROM AGRA UP



KTK - 3

$$\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$$

$$\frac{x^4 - (1)^4}{x - 1} = \frac{\frac{x^3 - k^3}{x - k}}{\frac{x^2 - k^2}{x - k}}$$

$$4 \cdot (1)^3 \Rightarrow \frac{3 \cdot k^2}{2 \cdot k}$$

4 — $\frac{3}{2}k$

$$4 = \frac{3}{2}k$$

$$\boxed{k = \frac{8}{3}} \quad \text{Ans}$$

Prakhar
From Delhi 🔥

QUESTION [JEE Mains 2017 (8 April)]

KTK 4

$\lim\limits_{x\to 3}\dfrac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to:

(A) $\sqrt{3}$

(B) $\dfrac{1}{\sqrt{2}}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{1}{2\sqrt{2}}$

Ans. B

KTK-4

Q

$$\lim_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x-4} - \sqrt{2}}$$

$$\lim_{x \to 3} \frac{(3x-9)(\sqrt{2x-4} + \sqrt{2})}{(2x-4-2)(\sqrt{3x}+3)}$$

$$= \lim_{x \to 3} \frac{3(x-3)}{2(x-3)} \quad \begin{matrix} \to 2\sqrt{2} \\ \to 6 \end{matrix}$$

$$= \frac{\cancel{6}}{\cancel{2}} \times \frac{2\sqrt{2}}{\cancel{6}_2} = \frac{1}{\sqrt{2}}$$ Ans

KTK 4 RAHUL DHAKAD

❤️❤️

FROM AGRA UP

$\lim_{x \to 3} \frac{\sqrt{3x-3}}{\sqrt{2x-4}-\sqrt{2}}$ is Equal to : (2017)

KTK 4
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$\Rightarrow \lim_{x \to 3} \frac{\sqrt{3x-3}}{\sqrt{2x-4}-\sqrt{2}} \cdot \frac{(\sqrt{3x}+3)}{(\sqrt{3x}+3)} \cdot \frac{(\sqrt{2x-4}+\sqrt{2})}{(\sqrt{2x-4}+\sqrt{2})}$

$\lim_{x \to 3} \frac{3x-9}{2x-4-2} \cdot \frac{(\sqrt{2x-4}+\sqrt{2})}{(\sqrt{3x}+3)} = \lim_{x \to 3} \frac{3(x-3)}{2(x-3)} \cdot \frac{(\sqrt{2x-4}+\sqrt{2})}{(\sqrt{3x}+3)}$

$\lim_{x \to 3} \frac{3}{2} \frac{(\sqrt{2x-4}+\sqrt{2})}{(\sqrt{3x}+3)} = \frac{3}{2} \frac{(\sqrt{6-4}+\sqrt{2})}{(\sqrt{9}+3)} = \frac{3}{2}\left(\frac{2\sqrt{2}}{6}\right) = \frac{1}{\sqrt{2}}$

ktk4

(4) $\lim_{x \to 3} \frac{\sqrt{3x} - 3}{\sqrt{2x-4} - \sqrt{2}}$

$\lim_{x \to 3} \frac{\sqrt{3x} - 3 \times \sqrt{3x} + 3 \times \sqrt{2x-4} + \sqrt{2}}{(\sqrt{2x-4} - \sqrt{2}) \times \sqrt{2x-4} + \sqrt{2} \times \sqrt{3x} + 3)}$

$\lim_{x \to 3} \frac{3x - 9 \times \sqrt{2x-4} + \sqrt{2}}{2x - 4 - 2 \times \sqrt{3x} + 3}$

$\lim_{x \to 3} \frac{3(x-3) \times \sqrt{2x-4} + \sqrt{2}}{2(x-3) \times \sqrt{3x} + 3}$

$\frac{3}{2} \times \frac{\sqrt{2}}{6} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ Ans

Niranjan tiwari
Ballia wale ❤️❤️



QUESTION [JEE Mains 2021 (27 August)]

KTK 5

If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is:

A $\left(1,\frac{1}{2}\right)$

B $\left(1,-\frac{1}{2}\right)$

C $\left(-1,\frac{1}{2}\right)$

D $\left(-1,-\frac{1}{2}\right)$

Ans. B

KTK-5

Q If $\lim_{x\to\infty}(\sqrt{x^2-x+1}-ax)=b$, then the ordered pair (a,b) is :-

Solution → $\lim_{x\to\infty}\left(\frac{x^2-x+1-a^2x^2}{\sqrt{x^2-x+1}+ax}\right)=b$

$\lim_{x\to\infty}\left(\frac{x^2(1-a^2)+1-x}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}\right)=b$

for limit exsist coefficient of x^2 is zero

$1-a^2=0$

$a=\pm 1 \Rightarrow a=1,-1$ ✗

$\lim_{x\to\infty}\left(\frac{x\left(\frac{1}{x}-1\right)}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}\right)=b$

$\frac{-1}{1+a}=b$, $a\neq -1$

$a=1$

$b=-\frac{1}{2}$

$(a,b)=\left(1,-\frac{1}{2}\right)$ Ans

KTK 5 RAHUL DHAKAD ❤❤ FROM AGRA UP

QUESTION [JEE Mains 2019 (10 April)]

KTK 6

If $\lim\limits_{x\to 1}\dfrac{x^2-ax+b}{x-1}=5$, then $a+b$ is equal to:

(A) 1

(B) -4

(C) -7

(D) 5

Ans. C

KTK- 6

Q If $\lim_{x\to 1} \frac{x^2-ax+b}{x-1} = 5$, then $a+b$ is equal to :→

Value → If $\lim_{x\to 1} \frac{x^2-ax+b}{x-1} = 5$

RHL

for limit exist

x^2-ax+b

if $x=1$

$1-a+b=0$

$\boxed{b = a-1}$

$\lim_{x\to 1} \frac{x^2-ax+a-1}{x-1} = 5$

$\lim_{x\to 1} \frac{(x-1)(x+1)-a(x-1)}{(x-1)} = 5$

$\lim_{x\to 1} \frac{(x-1)(x+1-a)}{(x-1)} = 5$

$2-a=5$

$\boxed{a=-3}$

$\boxed{b=-4}$

$\boxed{a+b=-7}$ Ans

KTK 6 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION [JEE Mains 2019 (9 Jan)]

KTK 7

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

(A) exists and equals $\frac{1}{2\sqrt{2}}$

(B) exists and equals $\frac{1}{4\sqrt{2}}$

(C) exists and equals $\frac{1}{2\sqrt{2}(1+\sqrt{2})}$

(D) does not exists

Ans. B

$$\lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \quad (2019)$$

$$\Rightarrow \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4} \cdot \frac{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$\lim_{y\to 0} \frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$\lim_{y\to 0} \frac{\sqrt{1+y^4}-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)} \cdot \frac{\left(\sqrt{1+y^4}+1\right)}{\left(\sqrt{1+y^4}+1\right)}$$

$$\lim_{y\to 0} \frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \frac{1}{2\cdot 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$$

Therefore, limit Exist & Equal to $\frac{1}{4\sqrt{2}}$

KTK 7
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

KTK 8

If $f(x) = \begin{cases} x+2; & x<1 \\ 4x-1; & 1 \le x \le 3 \\ x^2+5, & \text{where } x>3 \end{cases}$; then

(a) $\lim\limits_{x \to 1^+} f(x) = 3$

(b) $\lim\limits_{x \to 1^+} f(x) =$ does not exist

(c) $\lim\limits_{x \to 3^+} f(x) = 11$

(d) $\lim\limits_{x \to 3} f(x) =$ does not exist

If $f(x) = \begin{cases} x+2 & x < 1 \\ 4x-1 & 1 \leq x \leq 3 \\ x^2+5 & \text{where } x > 3 \end{cases}$; then

KTK 8

ABHIJEET KUMAR PATRA

ODISHA

(a) $\lim_{x \to 1^+} f(x) = 3$

(b) $\lim_{x \to 1^+} f(x) =$ Does not Exist

(c) $\lim_{x \to 3^+} f(x) = 11$

(d) $\lim_{x \to 3} =$ does not Exist

(a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 4x-1 = 3$

(b) ⊗

(c) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2+5 = 14$

(d) For this.

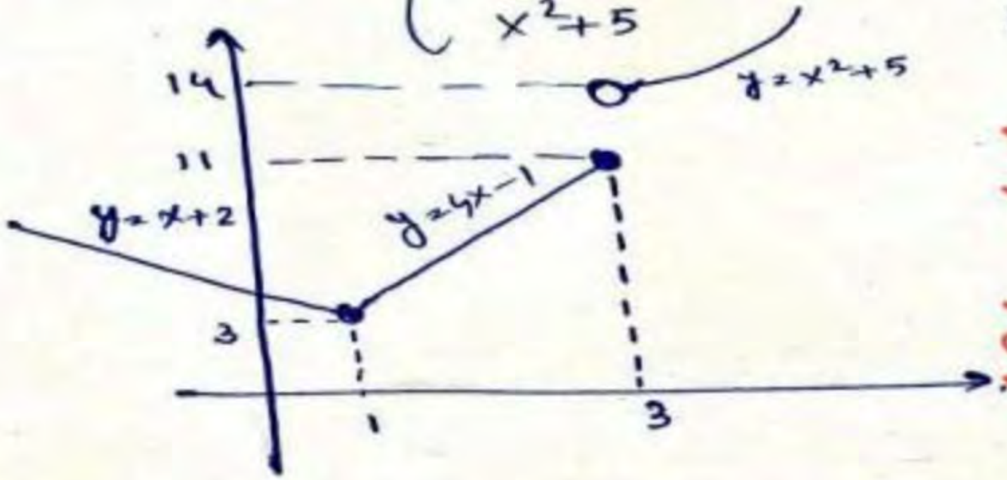
$\lim_{x \to 3^-} 4x-1 = 11$

Since $\lim_{x \to 3^-} 4x-1 \neq \lim_{x \to 3^+} x^2+5$

LHL ≠ RHL ⇒ D.N.E

[kTK-8]

$$f(x) = \begin{cases} x+2 & x<1 \\ 4x-1 & 1 \le x \le 3 \\ x^2+5 & x>3. \end{cases}$$



(i) $\lim_{x \to 1^+} f(x) = 3$ ✓

(ii) $\lim_{x \to 1^-} f(x) =$ does N.E ✗

(iii) $\lim_{x \to 3^+} f(x) = 11$ ✗

(iv) $\lim_{x \to 3} f(x) =$ D.N.E ✓

Souvik Mondal
West Bengal
Murshidabad

QUESTION

KTK 9

Let $\lim_{x\to 1}\frac{[x]}{x} = l$ and $\lim_{x\to 1}\frac{x}{[x]} = m$ where [] denotes the greatest integer function, then

A l exist but m does not exist

B m exist but l does not exist

C l and m both exist

D neither l nor m exist

QUESTION [JEE Mains 2018 (16 April)]

KTK 10

$$\lim_{x \to 0} \frac{(27+x)^{\frac{1}{3}} - 3}{9 - (27+x)^{\frac{2}{3}}} \text{ equals}$$

A $\frac{1}{3}$

B $-\frac{1}{3}$

C $-\frac{1}{6}$

D $\frac{1}{6}$

Ans. C

KTK-10 :→

Q $\lim_{x \to 0} \dfrac{(27+x)^{\frac{1}{3}} - (27)^{\frac{1}{3}}}{9 - (27+x)^{2/3}}$

Solve → $\lim_{x \to 0} \dfrac{(27+x)^{1/3} - 3}{(3+(27+x)^{1/3})(3-(27+x)^{1/3})}$

$= \lim_{x \to 0} \dfrac{-1}{(3+(27+x)^{1/3})}$

$= -\frac{1}{6}$ Ans

KTK 10 RAHUL DHAKAD❤️❤️ FROM AGRA UP

$\lim_{x \to 0} \dfrac{(27+x)^{1/3} - 3}{9 - (27+x)^{2/3}}$ Equals (2018)

$\Rightarrow \lim_{x \to 0} \dfrac{(27+x)^{1/3} - 3}{(3+(27+x)^{1/3})(3-(27+x)^{1/3})}$

$\lim_{x \to 0} \dfrac{-1}{(3+(27+x)^{1/3})} = \dfrac{-1}{6}$

KTK 10
ABHIJEET KUMAR PATRA
ODISHA

(Solution to RPP)

QUESTION

RPP 1

If $3p\sin x-(p+\sin x)(p^2-p\sin x+\sin^2 x)=1$ has a solution for x, then the number of integral values of p are

Ans. 4.0

$3p\sin x - (p+\sin x)(p^2 - p\sin x + \sin^2 x) = 1$

$3p\sin x - (p^3 + \sin^3 x) = 1$

$p^3 + \sin^3 x + 1 - 3p\sin x = 0$

$(p)^3 + (\sin x)^3 + 1^3 - 3 \cdot p \cdot \sin x \cdot 1 = 0$

$\Rightarrow \quad p + \sin x + 1 = 0 \quad$ or $\quad p = \sin x = 1$

$p = -(1+\sin x)$

$[-2, 0]$

$\Rightarrow p = -2, -1, 0$

$p = 1$

$p = 1, 0, -1, -2$

4 values

concept :

$\because a^3 + b^3 + c^3 - 3abc = 0$

if $a = b = c$

or

$a + b + c = 0$

QUESTION

RPP2

Possible integral value(s) of m for the equation $\sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$ can be valid for some $x \in [0, 2\pi]$ is

A -1

B 0

C 1

D 2

Ans. A, B, C, D

Possible integral value(s) of m for the eqn $\sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$ can be valid for some $x \in [0, 2\pi]$ is.

$\Rightarrow \sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$

$2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) = \frac{4m-6}{4-m}$

$2\left(\sin(x - \pi/3)\right) = \frac{4m-6}{4-m}$

RPP 2
ABHIJEET KUMAR PATRA
ODISHA

$\sin(x - \frac{\pi}{3}) = \frac{2m-3}{4-m}$

Since,

$-1 \leq \sin(x - \frac{\pi}{3}) \leq 1$

$-1 \leq \frac{2m-3}{4-m} \leq 1$

$\frac{2m-3}{4-m} \geq 0$

$\frac{m+1}{4-m} \geqslant 0$

$m \in [-1, 4]$

$\frac{2m-3-4+m}{4-m} \leqslant 0$

$\Rightarrow \frac{3m-7}{4-m} \leqslant 0$

$m \in (-\infty, \frac{7}{3}] \cup [4, \infty)$

$m \in [-1, \frac{7}{3}]$ $\therefore m = 0, -1, 1, 2$ (Answer)

RPP-2 $\sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$ $x \in [0, 2\pi]$

we know

$a\sin x - b\cos x \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$.

$\therefore \sin x - \sqrt{3}\cos x \in [-2, 2]$.

$\therefore -2 \le \frac{4m-6}{4-m} \le 2$

$\frac{4m-6}{4-m} \ge -2$ & $\frac{4m-6}{4-m} \le 2$

$\frac{4m-6}{4-m} + 2 \ge 0$

$\frac{4m-6+8-2m}{4-m} \ge 0$

$\frac{2m+2}{m-4} \le 0$

$\frac{m+1}{m-4} \le 0$

$m \in [-1, 4]$

$\frac{4m-6}{4-m} - 2 \le 0$

$\frac{4m-6-8+2m}{4-m} \le 0$

$\frac{6m-14}{4-m} \le 0$

$\frac{3m-7}{m-4} \ge 0$

$m \in (-\infty, \frac{7}{3}] \cup (4, \infty)$

$m \in [-1, \frac{7}{3}]$

$\therefore m = -1, 0, 1, 2$

all are correct

SouvikMondal
West Bengal
Murshidaabad

QUESTION

RPP3

If $\sin x + \cos x + \tan x + \cot x + \sec x + \text{cosec} = 7$ and $\sin 2x = a - b\sqrt{7}$, then $a - b + 14$ is divisible by

A 2

B 3

C 4

D 7

Ans. A, C, D

If $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$
and $\sin 2x = a - b\sqrt{7}$, then $a - b + 14$ is divisible by

(A) 2
(B) 3
(C) 4
(D) 7

RPP 3
ABHIJEET KUMAR PATRA
ODISHA

$$\Rightarrow \sin x + \cos x + \frac{1}{\sin x \cos x} + \frac{\sin x + \cos x}{\sin x \cos x} = 7$$

S.B.S

$$(\sin x + \cos x)\left[1 + \frac{2}{\sin 2x}\right] = 7 - \frac{2}{\sin 2x}$$

$$(\sin x + \cos x)^2\left[1 + \frac{2}{\sin 2x}\right]^2 = 49 + \frac{4}{\sin^2 2x} - \frac{28}{\sin 2x}$$

$$(1 + \sin 2x)\left[1 + \frac{2}{\sin 2x}\right]^2 = 49 + \frac{4}{\sin^2 2x} - \frac{28}{\sin 2x}$$

Put $\sin 2x = t$,

$$(1+t)\left[1 + \frac{2}{t}\right]^2 = 49 + \frac{4}{t^2} - \frac{28}{t}$$

$\Rightarrow t^2 - 44t + 36 = 0$
$\Rightarrow t = 22 \pm 8\sqrt{7}$
So, $\sin 2x = 22 - 8\sqrt{7}$)
$\therefore a = 22, b = 8$
So, $a - b + 14 = 28$, which is divisible by 2, 4, 7

(Solution to Home Challenge)

Home Challenge - 06

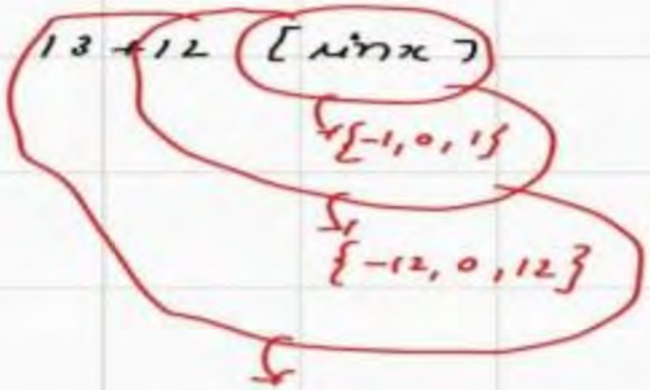
The number of solutions of equation $8[x^2 - x] + 4[x] = 13 + 12[\sin x]$ is (here [.] represents greatest integer less than or equal to 'x')

Ans. 0.00

Home CHALLANGE → 6

Q ÷ The no. of solutions of equation 8 [x²−x] + 4 [x] = 13 + 12 [sinx] in

Solve 8 [x²−x] + 4 [x] = 13 + 12 [sinx]



Now

8 [x²−x] + 4 [x] ∉ {1, 13, 25}

8 [x²−x] → even integer

4 [x] → even integer

even + even = even

⇒ No solution Ans

RAHUL DHAKAD ❤❤
FROM AGRA UP
HOME CHALLENGE 6

Bhoolnaa Mat Yeh Batayaa thaa

If A is an Idempotent matrix, then $A^n = A$ for all $n \varepsilon N$


THANK
YOU

PRAYAS

JEE 2025

Lecture-07 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Sandwich Theorem and Some Important Points
2. Introduction to Continuity

Recap *of previous lecture*

1. $\lim\limits_{x\to 0}\dfrac{\sin x^\circ}{x} = \lim\limits_{x\to 0}\dfrac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}\cdot\dfrac{\pi}{180} = \dfrac{\pi}{180}$

2. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e$, $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x} = e$

3. $\lim\limits_{x\to\infty}(a^x+b^x+c^x)^{\frac{1}{x}} = \lim\limits_{x\to\infty}(a^x)^{1/x}\left(1+\left(\frac{b}{a}\right)^x+\left(\frac{c}{a}\right)^x\right)^{1/x}$ (where $a > b > c > 0$)
$= a$

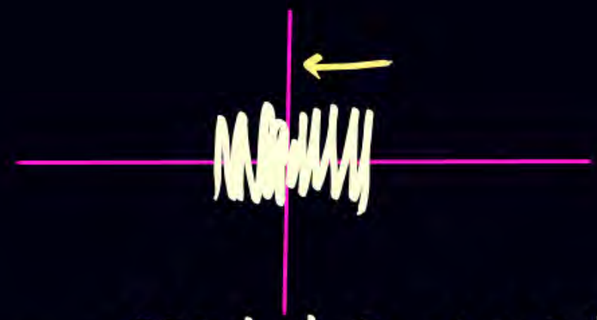
4. Let $g(x) = (\cos x)^{\tan x}$, find $\lim\limits_{x\to\frac{\pi}{2}^-} g(x)$. $= \lim\limits_{x\to\pi/2}(\cos x)^{\tan x} = 0$. (0^∞)

5. Let $h(x) = (\sin^2 x)^{\cos^2 x}$, find $\lim\limits_{x\to 0} h(x)$. $= \lim\limits_{x\to 0}(\sin^2 x)^{\cos^2 x} = 0$. (0^1)

Recap *of previous lecture*

6. $\lim_{x\to 0} \frac{\cos\frac{1}{x}}{\cos\frac{1}{x}} = 1$ [T/F] (False)

★ $\lim_{x\to 0} \frac{\sin x}{\sin x} = 1$



$\cos\frac{1}{x} \neq 0 \Rightarrow \frac{\cos\frac{1}{x}}{\cos\frac{1}{x}} = 1$

$\cos 1/x = 0 \Rightarrow \frac{\cos 1/x}{\cos 1/x}$ (N.D)

7. $\lim_{x\to 1} \log_x \sqrt{x}$ does not exist. [T/F] (False)

★ $\lim_{x\to 0} \frac{\sin 1/x}{\sin 1/x}$ DNE

$\lim_{x\to 1} \frac{\ln\sqrt{x}}{\ln x} = \lim_{x\to 1} \frac{1}{2} \frac{\ln x}{\ln x} = \frac{1}{2}$

8. $\lim_{x\to\infty} \frac{3^x}{\pi^x} = \lim_{x\to\infty} \left(\frac{3}{\pi}\right)^x = 0$

b/w 0 & 1

9. If $\lim_{x\to 1} \frac{f(x)-4}{x^3-1} = 2$, find $\lim_{x\to 1} f(x)$. = 4

$\lim_{x\to 1} \frac{f(x)-4}{x^3-1} = 2$ for limit to exist

$\lim_{x\to 0} (f(x)-4) = 0 \Rightarrow \lim_{x\to 0} f(x) = 4$

Recap *of previous lecture*

10. $\lim_{x\to 1} \frac{\frac{\pi}{4}-\tan^{-1}x}{e^{\sin(\ln x)}-x^n}$ exists and has the value equal to $\frac{1}{8}$, then find n.

Tan 1@

$$\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}} = \sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\cos^2\theta/2} = \cos\theta/2$$

$$\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}} = \sqrt{\frac{1}{2}+\frac{1}{2}\cos\theta/2} = \cos\theta/4$$

11. The value of $\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\cdot\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\ldots\ldots\infty$ is

$$l = \lim_{n\to\infty}\left(\sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\text{ --- upto n terms}\right)$$

Put $\frac{1}{\sqrt{2}} = \cos\theta$ $\quad \theta = \pi/4$

$$\lim_{n\to\infty}(\cos\theta\cdot\cos\theta/2\cos\theta/2^2\text{ --- upto n terms})$$

$$l = \lim_{n\to\infty} \left(\cos\theta \cdot \cos\frac{\theta}{2} \cdot \cos\frac{\theta}{2^2} \cdots \cos\frac{\theta}{2^{n-1}}\right)$$

$$= \lim_{n\to\infty} \left(\cos\theta \cdot \cos\frac{\theta}{2} \cdot \cos\frac{\theta}{2^2} \cdots \cos\frac{\theta}{2^{n-1}}\right)$$ ← Sabse chhota

$$= \lim_{n\to\infty} \frac{\sin 2\theta}{2^n \cdot \sin\left(\theta/2^{n-1}\right)}$$

$$= \lim_{n\to\infty} \frac{\sin 2\theta}{2 \cdot 2^{n-1} \sin\left(\theta/2^{n-1}\right)}$$

$$= \lim_{n\to\infty} \left(\frac{\sin 2\theta}{2\theta}\right) \cdot \left(\frac{\theta/2^{n-1}}{\sin\left(\theta/2^{n-1}\right)}\right) \pm 1$$

Agar Apne bhi yahi Sochaa toh aap —— hai

$$= \frac{\sin 2\theta}{2\theta} = \frac{\sin \pi/2}{\pi/2} = 2/\pi \quad \text{Ans.}$$ (1)

Homework Discussion

QUESTION

TAH 13

$(n \in N)$

Let $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin\frac{1}{x}+x}{1+x^{2n}}$ then which of the following alternative(s) is/are correct?

(A) $\lim_{x\to\infty} x f(x) = 2$ (Already done)

(B) $\lim_{x\to 1} f(x)$ does not exist.

(C) $\lim_{x\to 0} f(x)$ does not exist.

(D) $\lim_{x\to-\infty} f(x)$ is equal to zero.

(B) if $x \in (1, 1+\Delta)$ $f(x) = \lim_{n\to\infty} \frac{2\sin 1/x + \frac{x}{x^{2n}}}{\frac{1}{x^{2n}}+1}$

$= 2\sin\frac{1}{x}$

if $x \in (1-\Delta, 1)$ $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin 1/x + x}{1+x^{2n}} = x.$

$$f(x) = \begin{cases} 2\sin 1/x & x \in (1, 1+\Delta) \\ x & x \in (1-\Delta, 1) \end{cases}$$

$\lim_{x\to 1} f(x)$ { RHL $= 2\sin 1$, LHL $= 1$ } $\Downarrow$ D.N.E

If x is a very large −ve no:

$f(x) = \lim_{n\to\infty} \frac{2\sin 1/x + x/x^{2n}}{1/x^{2n}+1} = 2\sin 1/x \Rightarrow f(x) = 2\sin 1/x \Rightarrow \lim_{x\to-\infty} 2\sin 1/x = 2\sin 0 = 0$

(c) $\lim_{x \to 0} f(x)$

$$f(x) = \lim_{n \to \infty} \frac{2x^{2n} \sin\frac{1}{x} + x}{1 + x^{2n}} \qquad \begin{matrix} x^{2n} \to 0 \\ x^{2n} \to 0 \end{matrix}$$

$x \in (0, \Delta)$ OR $x \in (-\Delta, 0)$

$$f(x) = x \qquad x \in (-\Delta, \Delta) - \{0\}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x = 0 .$$

QUESTION [JEE Advanced 2020] TAH 8

Let e denote the base of the natural logarithm. The value of the real number n for which the right hand limit $\lim\limits_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^n}$ is equal to nonzero real number, is

$$\lim_{x\to 0^+} \frac{e^{\ln(1-x)^{\frac{1}{x}}} - e^{-1}}{x^n}$$

$$\lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)}{x}+1} - 1}{x^n e}$$

$M = \frac{\ln(1-x)}{x} + 1$

$$\lim_{x\to 0^+} \frac{\ln(1-x)}{x} = \lim_{x\to 0^+} \frac{-\ln(1-x)}{-x} = -1$$

$\Rightarrow M = \frac{\ln(1-x)}{x} + 1$

as $x \to 0^+$, $M \to 0$.

$$\lim_{x\to 0^+} \frac{e^M - 1}{M} \cdot \frac{\frac{\ln(1-x)}{x}+1}{x^n e} = \lim_{x\to 0^+} (\quad) \frac{\ln(1-x)+x}{x^{n+1}.e}$$

Ans : 1

$$\ell = \lim_{x \to 0^+} (\quad) \frac{-\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \infty\right) + x}{x^{n+1} \cdot e}$$

$$= \lim_{x \to 0^+} (\quad) \frac{-x^2\left(\frac{1}{2} + \frac{x^3}{3} + \cdots \infty\right)}{e \cdot x^{n+1}}$$

$n+1 = 2$

$n = 1$

QUESTION [JEE Mains 2024 (31 Jan)] TAH 10

If $\lim\limits_{x\to 0}\dfrac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$, then $16(a^2+b^2+c^2)$ is equal to

$$\lim_{x\to 0}\frac{ax^2\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\infty\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots\infty\right)+cx\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\frac{x^5}{5!}+\cdots\infty\right)}{x^2\sin x}$$

Ans. 81

QUESTION [JEE Mains 2021]

TAH 12 A

If $\lim\limits_{x\to 0}\dfrac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a-2b$ is

$$\lim_{x\to 0}\frac{ax-\left(\left(1+4x+\frac{(4x)^2}{2!}+\frac{(4x)^3}{3!}+\cdots\infty\right)-1\right)}{4ax^2\ \frac{(e^{4x}-1)}{4x}}$$

$$\lim_{x\to 0}\frac{(a-4)x-\left(\frac{16}{2!}x^2+\frac{64x^3}{3!}+\cdots\infty\right)}{4ax^2\ (\ )}\overset{0}{=}$$

$a-4=0$

$a=4$

$2b=-1$

$a-2b=5.$

$$=-\frac{8}{4a}=\frac{-2}{a}=\frac{-1}{2}=b.$$

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

Galti Batao

RPP2 $f(x) = (a-3)x^2 - 2ax + 3a - 7 \in [-1, \infty)$ — Range

(a ≠ 3) $\because R \in [-1, \infty)$ Now



Range of $f(x) \in \left[-\frac{D}{4a}, \infty\right)$

So $-\frac{D}{4a} \geq -1$ $\quad -\frac{D}{4a} = -1$ & $a - 3 > 0$

$D \geq 4(a-3) \rightarrow 4a^2 - 4(a-3)(3a-7) \geq 4(a-3)$

$\Rightarrow -2a^2 + 16a - 21 \geq a - 3$

$2a^2 - 15a + 18 \leq 0$

$(a-6)(2a-3) \leq 0$

$\therefore a \in \left[\frac{3}{2}, 6\right]$

From (i) & (ii)

So $a \in (3, 6]$

(i) ∩ (ii) So $a \in [4, 6]$ Ans

QUESTION

KCLS

1. $\lim_{n\to\infty} \cos^2\left(\pi\left(\sqrt[3]{n^3+n^2+2n}\right)\right)$ where n is an integer, equals

2. $\lim_{n\to\infty} n \sin\left(2\pi\sqrt{1+n^2}\right)\ (n \in N)$ equals

Body gaurd ← Netaji → Body guard

Body guard ← Netaji → Body guard

Body guard - Netaji - Body gaurd

Body guard Netaji Body gaurd

In nbd of $x=a$

$f(x) \le g(x) \le h(x)$

($f(x)$: lower bound; $h(x)$: upper bound)

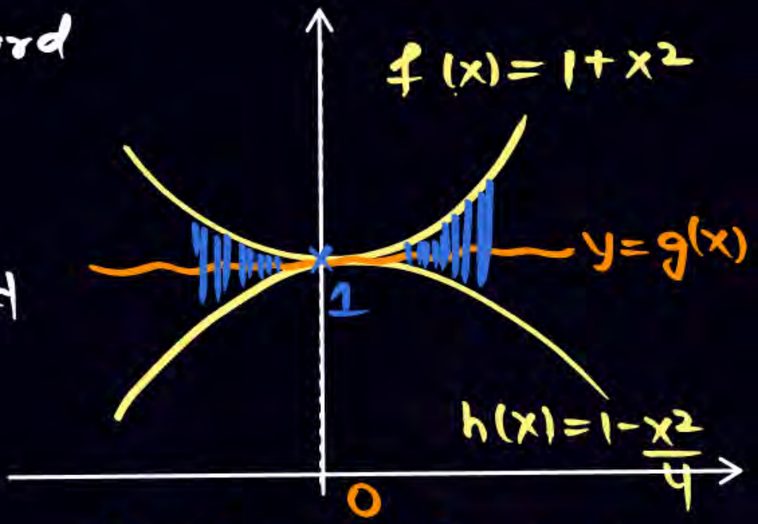
& $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = \ell$

then

$\lim_{x\to a} g(x) = \ell$.

★ a can be $+\infty$ or $-\infty$

★ lly ℓ can be ∞ or $-\infty$



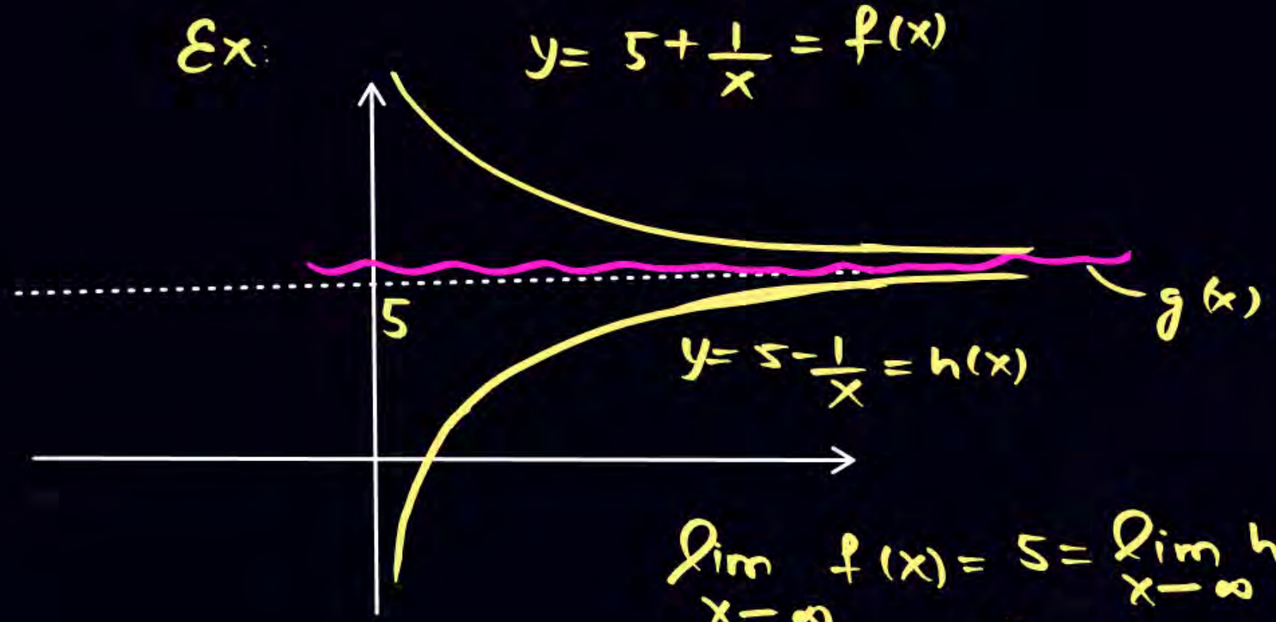
$\lim_{x\to 0} f(x) = 1 = \lim_{x\to 0} h(x)$

⇓

By S. Thm

$\lim_{x\to 0} g(x) = 1$

Ex:



$$\lim_{x \to -\infty} f(x) = 5 = \lim_{x \to -\infty} h(x)$$

$\Downarrow$

By Sandwich Thm

$\Downarrow$

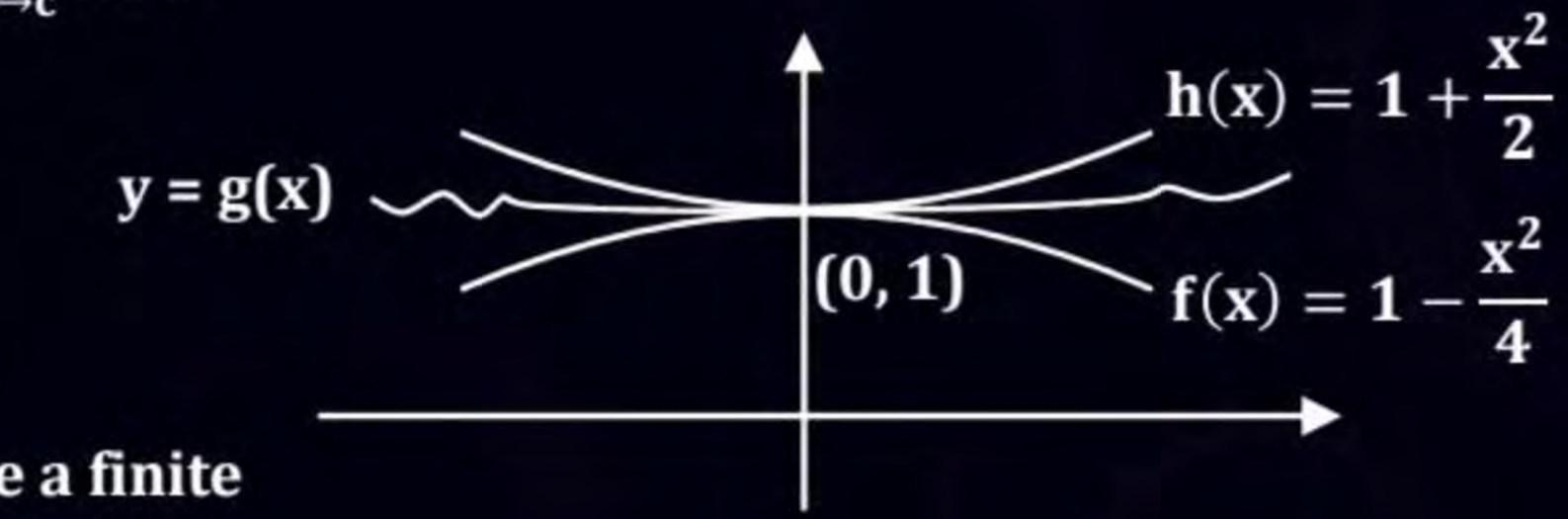
$$\lim_{x \to -\infty} g(x) = 5.$$

Sandwich Theorem/Squeeze Play Theorem

Statement :

If f, g and h are 3 functions such that $f(x) \leq g(x) \leq h(x)$ for all x in some deleted neighbourhood of $x = a$ and $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} g(x) = L$

From the figure note that $\lim_{x \to c} g(x) = 1$



Note: The quantity c may be a finite number, $+\infty$ or $-\infty$. Similarly L may be finite number, $+\infty$ or $-\infty$.

QUESTION

Evaluate: $\lim_{n\to\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+2}+\frac{n}{n^2+3}+\cdots+\frac{n}{n^2+n}\right)$ $\sim 0\times\infty$

$$\frac{n}{n^2+n}+\frac{n}{n^2+n}+\cdots+\frac{n}{n^2+n} \le \frac{n}{n^2+1}+\frac{n}{n^2+2}+\frac{n}{n^2+3}+\cdots+\frac{n}{n^2+n} \le \frac{n}{n^2+1}+\frac{n}{n^2+1}+\cdots+\frac{n}{n^2+1}$$

$$\| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \|$$

$$n\cdot\frac{n}{n^2+n}=\frac{n^2}{n^2+n} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{n}{n^2+1}\cdot n=\frac{n^2}{n^2+1}$$

$$\Downarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Downarrow$$

$$\lim_{n\to\infty}\frac{n^2}{n^2+n}=1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \lim_{n\to\infty}\frac{n^2}{n^2+1}=1$$

By S.Thm

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+2}+\cdots+\frac{n}{n^2+n}\right)=1$$

Ans. 1

QUESTION

Tan⁻¹ⓑ

Evaluate: $\lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+4}+\frac{9}{n^3+9}+\cdots+\frac{n^2}{n^3+n^2}\right)$

Ans. 1/3

QUESTION [JEE Mains 2023 (6 April)]

$$\lim_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\} \text{ is equal to}$$

(A) $\sqrt{2}$

(B) 1

(C) $1/\sqrt{2}$

(D) 0

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)$$

smallest

$$\left(2^{\frac{1}{2}}-2^{1/3}\right)\left(2^{\frac{1}{2}}-2^{1/5}\right)\ldots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)$$



$2^{1/3} > 2^{1/5} > 2^{1/7} \ldots > 2^{\frac{1}{2n+1}}$ Biggest

Sandwich Theorem is generally used when we have a product or Sum series

But apnay Dimaag kaa use Karnaa zarooori hai

Ans. D

$$\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\cdots\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right) \le \left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\cdots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right) \le \left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)-\left(2^{\frac{1}{3}}-2^{\frac{1}{2n+1}}\right)$$

$$\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n \le \left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\cdots\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right) \le \left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n$$

$$\Downarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Downarrow$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n = 0 \qquad\qquad \lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n = 0$$

By S.Thm

$$\lim_{n\to\infty}\left(2^{1/2}-2^{1/3}\right)\left(2^{1/2}-2^{1/5}\right)\cdots\left(2^{1/2}-2^{1/2n+1}\right)=0 .$$

QUESTION

Let $f : (1, \infty) \to (0, \infty)$ be a continuous decreasing function with $\lim_{x\to\infty} \frac{f(4x)}{f(8x)} = 1$. Then $\lim_{x\to\infty} \frac{f(6x)}{f(8x)}$ is equal to

(A) 4/8

(B) 4/6

(C) 6/8

(D) 1

Domain

Codomain.

$\lim_{x\to\infty} \frac{f(4x)}{f(8x)} = 1$, $\lim_{x\to\infty} \frac{f(6x)}{f(8x)} =$

$4x < 6x < 8x \qquad x \in (1, \infty)$

$f(8x) < f(6x) < f(4x)$ (+ve)

$1 < \frac{f(6x)}{f(8x)} < \frac{f(4x)}{f(8x)}$

By Sandwich Thm.

Now $\lim_{x\to\infty} \frac{f(4x)}{f(8x)} = 1$ (given) & $\lim_{x\to\infty} 1 = 1 \Rightarrow \lim_{x\to\infty} \frac{f(6x)}{f(8x)} = 1$

QUESTION

we know $x-1 < [x] \le x, \quad x \in R$

$$\lim_{n\to\infty} \frac{[1^2x]+[2^2x]+[3^2x]+\ldots+[n^2x]}{n^3} \text{ where [ ] = step up function and } x \in R$$

$$(1^2x-1)+(2^2x-1)+\cdots+(n^2x-1) < [1^2x]+[2^2x]+[3^2x]+\cdots+[n^2x] \le 1^2x+2^2x+\cdots+n^2x.$$

$$(1^2+2^2+\cdots+n^2)x-n < [1^2x]+[2^2x]+[3^2x]+\cdots+[n^2x] \le (1^2+2^2+\cdots+n^2)x$$

$$\frac{\frac{n(n+1)(2n+1)}{6}x-n}{n^3} < \frac{[1^2x]+[2^2x]+\cdots+[n^2x]}{n^3} \le \frac{\frac{n(n+1)(2n+1)}{6}x}{n^3}$$

$$\lim_{n\to\infty}\left(\frac{\frac{n(n+1)(2n+1)}{6}x-n}{n^3}\right) = 2x/6 = x/3$$

$$\lim_{n\to\infty}\left(\frac{n(n+1)(2n+1)}{6n^3}x\right) = x/3$$

By Sandwich

$$\lim_{n\to\infty}\frac{[1^2x]+[2^2x]+\cdots+[n^2x]}{n^3} = x/3$$

Golden Points

Panch Parmeshwar

1. If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both exists then limit of $f(x) \pm g(x)$; $f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$ as $x \to c$ must exists.

   (where $\lim_{x\to c} g(x) \neq 0$), (Provided f is not a point function)

$f(x)=\sqrt{x}$, $g(x)=\sqrt{-x}$ $\quad \lim_{x\to 0}\sqrt{x}=0$, $\lim_{x\to 0}\sqrt{-x}=0$

$h(x)= f(x)+g(x)=\sqrt{x}+\sqrt{-x}$ $\quad$ But $\lim_{x\to 0}(\sqrt{x}+\sqrt{-x})$ = Not def

$x \geq 0$ $\quad -x \geq 0$ $\quad x \leq 0$

Domain $h=\{0\}$ $\quad x=0$

Golden Points

2. If $\lim_{x\to a} f(x)$ exist and $\lim_{x\to a} g(x)$ does not exist then $\lim_{x\to a}(f(x)\pm g(x))$ can not exist. however nothing definite can be said for product or quotient.

$\lim_{x\to a} f(x)$ = exists, $\lim_{x\to a} g(x)$ = D·N·E $\Rightarrow \lim_{x\to a}(f(x)\pm g(x))$ D·N·E

proof: let $\lim_{x\to a}(f(x)-g(x))$ exist & equals L

Now $\lim_{x\to a} f(x)$ exists (given) & equals M

$\lim_{x\to a}(f(x)-g(x)) - \lim_{x\to a} f(x) = L-M$

$\lim_{x\to a}(f(x)-g(x)-f(x)) = L-M$

$\lim_{x\to a} g(x) = M-L$ ✗ contradiction

$f(x) = [x]$

$g(x) = \sin x$

★ $\lim_{x \to 0} (f(x) + g(x))$ — D·N·E + 0 = D·N·E

★ $\lim_{x \to 0} ([x] - \sin x)$ — D·N·E + 0 = D·N·E

★ $\lim_{x \to 0} f(x) \cdot g(x) = \lim_{x \to 0} [x] \cdot \sin x$ — D·N·E, Exist

RHL $\lim_{h \to 0} [h] \sin h$

$= 0$

$\lim_{h \to 0} 0 \cdot \sin h = 0$

LHL $\lim_{h \to 0} [-h] \sin h$

$= \lim_{h \to 0} -\sin h = 0$

★ $\lim_{x \to 1} f(x) \cdot g(x) = \lim_{x \to 1} [x] \cdot \sin x$ = D·N·E — D·N·E, Exist

RHL: $1 \cdot \sin 1$

LHL: 0

NICHOD

$\lim_{x \to a} f(x)$ Exist & $\lim_{x \to a} g(x)$ D·N·E

then

★ $\lim_{x \to a} (f(x) \pm g(x))$ D·N·E

★ $\lim_{x \to a} f(x) \cdot g(x)$

★ $\lim_{x \to a} \frac{f(x)}{g(x)}$

Kuch nahi kahaa jaa saktaa LHL RHL Evaluate Karnaa padegaa.

Golden Points

3. If both $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ does not exist then nothing definite can be said about the sum and product.

$\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ both D.N.E

then $\lim_{x\to a} f(x) \pm g(x)$

$\lim_{x\to a} f(x)\cdot g(x)$

$\lim_{x\to a} \frac{f(x)}{g(x)}$

Kuch nahi kahaa jaa saktaa LHL & RHL nikalnaa padegaa.

Ex: $f(x)=[x]$

$g(x)=\{x\}$

$\lim_{x\to 0} f(x)$, $\lim_{x\to 0} g(x)$ both D.N.E

But $\lim_{x\to 0} f(x)+g(x) = \lim_{x\to 0} [x]+\{x\}$

$= \lim_{x\to 0} x = 0$.

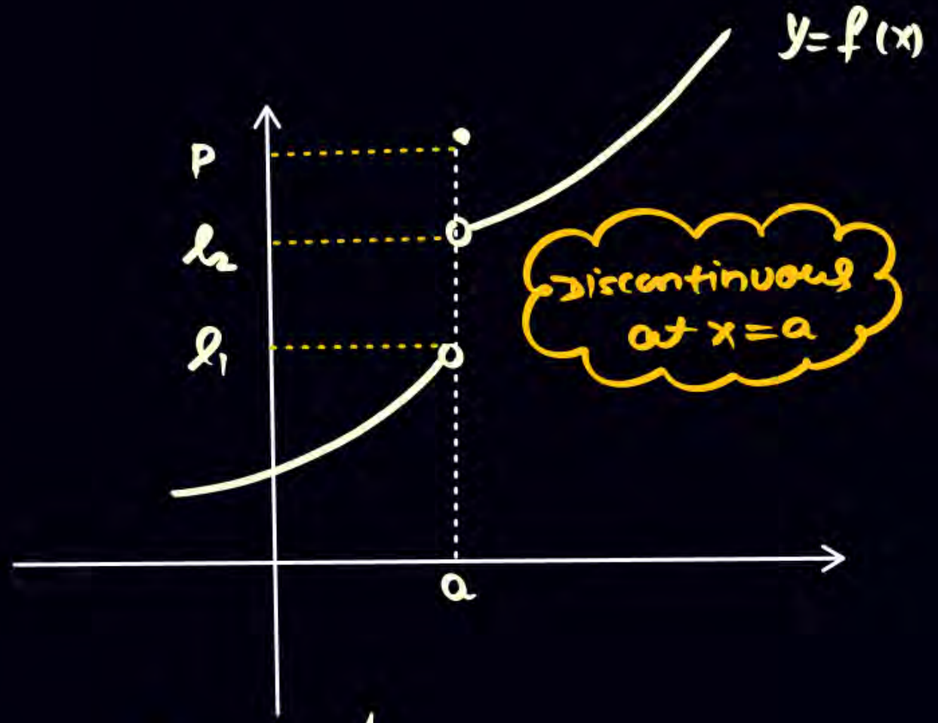
Continuity



Continuity

$\lim_{n\to\infty} x^n$ = DNE $\quad -1 < x < 0$, $\lim_{n\to\infty} x^n = 0 \quad (-1 < x < 0, n \in N)$

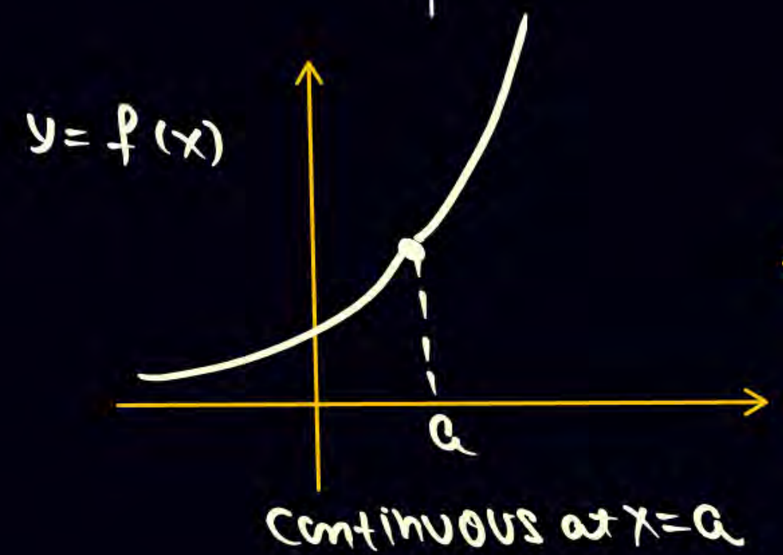
$\lim_{n\to\infty} x^n = 0 \quad 0 < x < 1$



Discontinuous at $x=a$

LHL $\lim_{x\to a} f(x) = \ell_1$

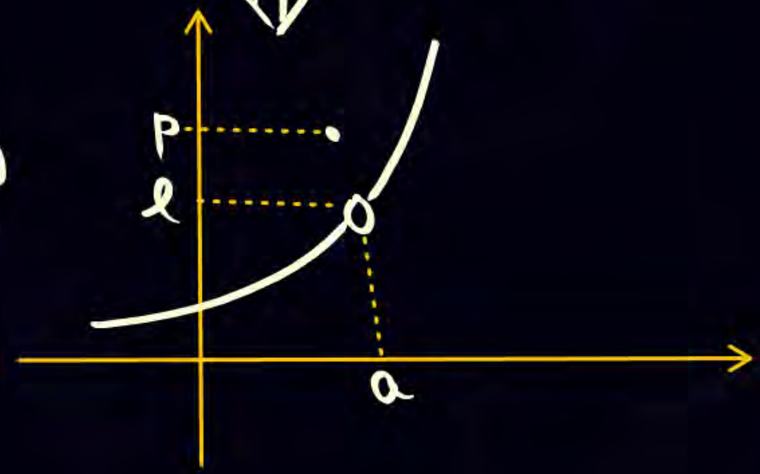
RHL $\lim_{x\to a} f(x) = \ell_2$



Continuous at $x=a$

$RHL = LHL = \ell = f(a)$

$RHL = LHL = \ell$



f is continuous at $x=a$ if LHL at $x=a$ = RHL at $x=a$ = $f(a)$

KAAM KI BAAT

★ If a function is continuous at $x=a \Rightarrow f$ is defined at $x=a$ but not the converse.

★ We talk of continuity of a function only in Domain of fn.

★ All Trigonometric fns are continuous in their Domain.

★ All polynomial fns are continuous on R,

★ Exponential fn, logarithmic fn are continuous in their Domain.

Ex: $f(x) = \text{sgn}\, x$

$0 \in D_f$

$\lim_{x \to 0^+} \text{sgn}\, x = 1$

$\lim_{x \to 0^-} \text{sgn}\, x = -1$

$\Big\}$ discontinuous at $x = 0$

KAAM KI BAAT

★ We can talk of Discontinuity at $x = a$ for a fn 'f' if $y = f(x)$ has a graph in atleast one nbd of $x = a$ i.e it is defined in either left or right nbd of $x = a$.

★ If f, g are continuous at $x = a$ then $f \pm g$, $f \cdot g$, $\frac{f}{g}$ are also continuous at $x = a$ provided for $\frac{f}{g}$, $g(a) \neq 0$.

Continuity in an Interval



A function f is said to be continuous in (a, b) if f is continuous at each & every point $\in (a, b)$.

A function f is said to be continuous in a closed interval [a, b] if :

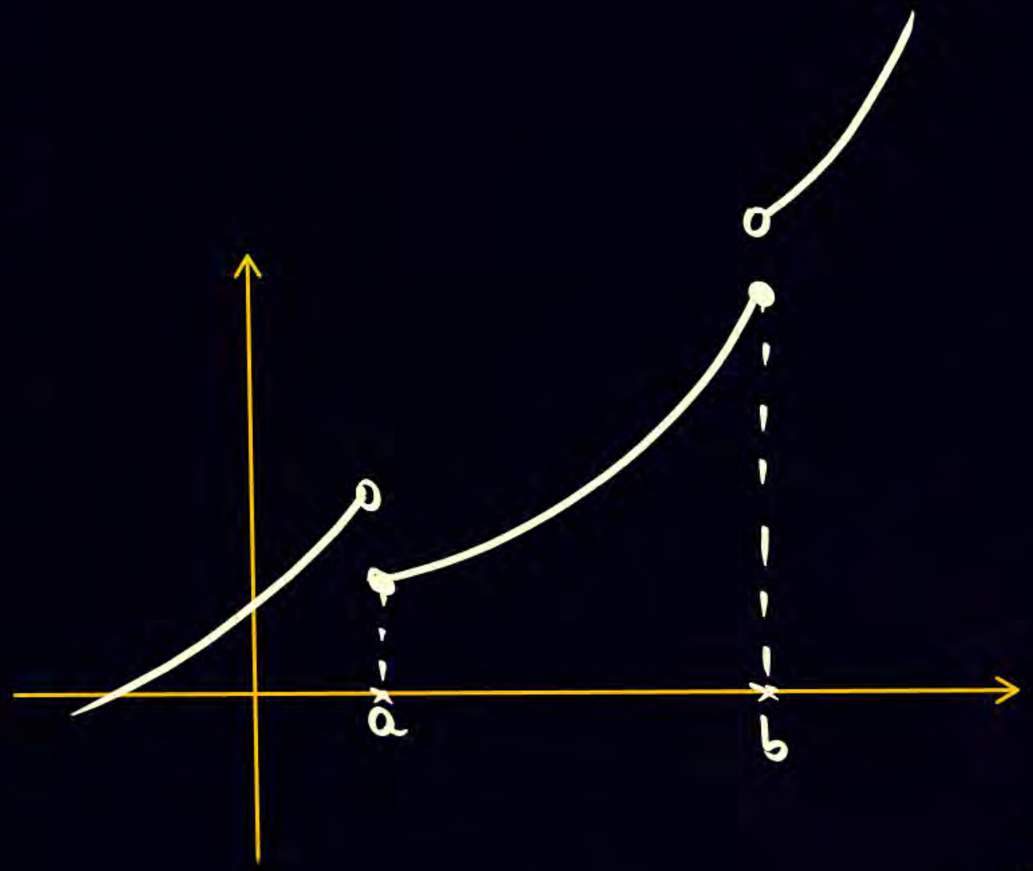
(i) f is continuous in the open interval (a, b) &

(ii) f is right continuous at 'a' i.e. $\lim_{x \to a^+} f(x) = f(a) =$ a finite quantity .

(iii) f is left continuous at 'b' i.e. $\lim_{x \to b^-} f(x) = f(b) =$ a finite quantity .

f is continuous in $[a,b]$ if

- f is continuous in (a,b)
- $\lim_{x \to a^+} f(x) = f(a)$
- $\lim_{x \to b^-} f(x) = f(b)$



QUESTION [JEE Mains 2024 (9 April)]

Let $f:(0,\pi)\to R$ be a function given by $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$

where $a, b \in Z$. If f is continuous at $x=\frac{\pi}{2}$, then a^2+b^2 is equal to

AT $x=\frac{\pi}{2}$ RHL

$$\lim_{x\to \pi/2^+}(1+|\cot x|)^{\frac{b}{a}|\tan x|}$$

$$\lim_{h\to 0}(1+|\cot(\pi/2+h)|)^{\frac{b}{a}|\tan(\pi/2+h)|}$$

$$=\lim_{h\to 0}(1+|-\tanh|)^{\frac{b}{a}|-\coth|} = \lim_{h\to 0}\left((1+\tanh)^{\frac{1}{\tanh}}\right)^{\frac{b}{a}} = e^{b/a}$$

Ans. 81

LHL

$$\lim_{x \to \pi/2^-} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \lim_{h \to 0} \left(\frac{8}{7}\right)^{\frac{\tan 8\left(\frac{\pi}{2}-h\right)}{\tan 7\left(\pi/2-h\right)}}$$

$$= \lim_{h \to 0} \left(\frac{8}{7}\right)^{\frac{-\tan 8h}{\cot 7h}}$$

$$= \lim_{h \to 0} \left(\frac{8}{7}\right)^{-\tan 8h \cdot \tan 7h}$$

$$= (8/7)^0 = 1 .$$



RHL = LHL = $f(\pi/2)$

$e^{b/a} = 1 = a - 8$

$a = 9$

$e^{b/9} = 1$

$b = 0$

Ans $a^2 + b^2 = 81$

QUESTION [JEE Mains 2024 (8 April)]

Tan 2

For $a, b > 0$, let $f(x) = \begin{cases} \frac{\tan((a+1)x)+b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$ be a continuous function at $x = 0$. Then $\frac{b}{a}$ is equal to:

(A) 4

(B) 5

(C) 8

(D) 6

Ans. D

QUESTION [JEE Mains 2024 (27 Jan)]

Tan 3

Consider the function, $f(x)=\begin{cases}\frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x<3 \\ 2^{\frac{\sin(x-3)}{x-|x|}}, & x>3 \\ b, & x=3,\end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x=3$, then the number of elements in S is:

(A) Infinitely many

(B) 4

(C) 2

(D) 1

Ans. D

QUESTION [JEE Mains 2024 (4 April)]

Tan 4

If the function $f(x)=\begin{cases}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, & x\neq 0\\ a\log_e 2\log_e 3, & x=0\end{cases}$ is continuous at $x=0$, then the value of a^2 is equal to

(A) 968

(B) 1250

(C) 1152

(D) 746

Ans. C

QUESTION [JEE Mains 2024 (4 April)]

Tan 5

Let $f: R \to R$ be a function given by $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$

where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

A 48

B 6

C 3

D 12

Ans. D

If a fn is continous at $x=a$

then $\left(\begin{matrix}\text{LHL}\\ \text{at } x=a\end{matrix}\right) = \left(\begin{matrix}\text{RHL}\\ \text{at } x=a\end{matrix}\right) = f(a)$

$$\lim_{x \to a} f(x) = f(a)$$

QUESTION [JEE Mains 2024 (5 April)]

If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$, is continuous at $x = 0$, then $f(0)$ is equal to:

(A) 4

(B) -2

(C) -4

(D) 2

$\lim_{x \to 0} f(x) = f(0)$

$\left(3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \cdots\right) + \alpha\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right) - \beta\left(1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} + \cdots\right)$

$\Rightarrow \lim_{x \to 0} \frac{\quad}{x^3}$

$f(0) = \lim_{x \to 0} \frac{-\beta + x(3+\alpha) + \beta\frac{9x^2}{2!} + x^3\left(-\frac{27}{3!} - \frac{\alpha}{3!}\right) + \cdots}{x^3}$ (terms: $-\beta = 0$, $3+\alpha = 0$, $\beta\frac{9x^2}{2!} = 0$)

Clearly $\beta = 0$

$3 + \alpha = 0 \Rightarrow \alpha = -3$

$= -\frac{27}{6} - \frac{\alpha}{6}$

Ans. C

$$f(0) = -\frac{27}{6} - \frac{(-3)}{6}$$

$$f(0) = -\frac{24}{6} = -4$$

QUESTION [JEE Mains 2023 (25 Jan)]

TAH 6

If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & , 0 < x < \frac{\pi}{2} \\ \mu & , \quad x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & , \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$,

then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(A) 11

(B) 10

(C) 8

(D) $2e^4 + 8$

QUESTION [JEE Mains 2021 (March)]

TAH 7

If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is

QUESTION [JEE Mains 2021 (July)]

TAH 8

Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to R$ be defined as, $f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}}, & -\frac{\pi}{4}<x<0\\ b, & x=0\\ e^{\cot 4x/\cot 2x}, & 0<x<\frac{\pi}{4}\end{cases}$

If f is continuous at $x=0$, then the value of $6a+b^2$ is equal to:

(A) $1-e$

(B) $e-1$

(C) $1+e$

(D) e

Five Golden Points to remember

T-1 : Sum, difference, product and quotient of two continuous functions is always a continuous function. However $h(x) = \frac{f(x)}{g(x)}$ is continuous at $x = a$ only if $g(a) \neq 0$.

T-2 : If $f(x)$ is continuous and $g(x)$ is discontinuous then $f(x) \pm g(x)$ must be discontinuous function.

$f(x)$ is continuous at $x=a$
$g(x)$ is discontinuous at $x=a$ } (given)

claim: $h(x) = f(x) + g(x)$
$\phi(x) = f(x) - g(x)$ } discontinuous at $x=a$

proof: let if possible $h(x)$ be continuous at $x=a$
Also $f(x)$ is continuous at $x=a$ (given)
$\Rightarrow h(x) - f(x) = g(x)$ is also continuous at $x=a$ ✗ contradiction

Five Golden Points to remember

T-3 : If $f(x)$ is continuous & $g(x)$ is discontinuous at $x = a$ then nothing definite can be said about the product function $\phi(x) = f(x) \cdot g(x)$

Ex: $g(x) = [x]$ – discontinuous at $x = 0$

$f(x) = \sin x$ – continuous at $x = 0$

$h(x) = f(x) \cdot g(x) \Rightarrow \lim_{x \to 0} f(x) \cdot g(x) = \lim_{x \to 0} [x] \sin x = 0 \Rightarrow \lim_{x \to 0} h(x) = h(0) \Rightarrow h(x) = f(x) \cdot g(x)$ is continuous at $x = 0$

$h(0) = f(0) g(0)$

$h(0) = 0.$

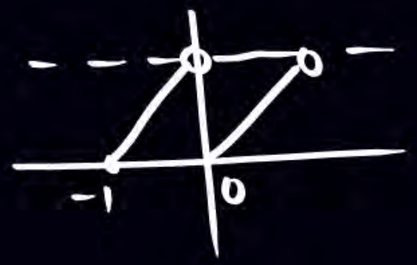
If f is continuous at $x=a$ & $f(a)=0$

while g is discontinuous at $x=a$ with a finite LHL

& RHL & $g(a)$ is also defined then

their product fn

$h(x)=f(x)\cdot g(x)$ is continuous at $x=a$



Ex: $f(x)=\tan x$ — continuous at $x=0$, $f(0)=0$

$g(x)=\{x\}$ — discontinuous at $x=0$

LHL = 1

RHL = 0

also defined at $x=0$

$\Rightarrow h(x)=f(x)\cdot g(x)$ is continuous at $x=0$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 08

Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0, 2\pi)$, holds.

If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to:

A $\frac{\pi}{16}$

B $\frac{\pi}{48}$

C $\frac{\pi}{8}$

D $\frac{\pi}{12}$

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

$$\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)}$$

Ans. $\frac{1}{\sqrt{2}}$

QUESTION

(KTK 2)

$$\lim_{x \to \frac{\pi}{2}} (2x \tan x - \pi \sec x)$$

Ans. –2

QUESTION

(KTK 3)

$$\lim_{x \to -1} \frac{\sqrt{\pi} - \sqrt{\arccos x}}{\sqrt{x+1}}$$

Ans. $\frac{1}{\sqrt{2\pi}}$

QUESTION

(KTK 4)

If $\lim_{x\to\infty}\left(\ln\left(x^2+5x\right)-2\ln(cx+1)\right)=-2$ then

A $c=e^{-1}$

B $c=e^{-2}$

C $c=e$

D $c=e^{\frac{-1}{2}}$

Ans. D

QUESTION

(KTK 5)

The value of $\lim_{x \to 3} \frac{\sqrt{x+6}-\sin(x-3)-3}{(x-3)\cos(x-3)}$ is equal to

A $\frac{1}{5}$

B 5

C $\frac{-3}{4}$

D $\frac{-5}{6}$

Ans. C

(Revision Practice Problems)

QUESTION

(RPP 1)

If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is

A $x^2 + 10x - 11 = 0$

B $11x^2 + 10x + 1 = 0$

C $x^2 + 10x + 11 = 0$

D $11x^2 - 10x + 1 = 0$

Ans. B

QUESTION

(RPP 2)

If α, β be the roots of $x^2 + x + 2 = 0$ and γ, δ be the roots of $x^2 + 3x + 4 = 0$, then $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$ is equal to

A -18

B 18

C 24

D 44

Ans. D

QUESTION

(RPP 3)

The value of 'a' for which the quadratic expression $ax^2 + |2a - 3|x - 6$ is positive for exactly three integral values of x is

(A) $\left(-\frac{3}{5}, -\frac{1}{2}\right]$

(B) $\left[-\frac{3}{5}, -\frac{1}{2}\right]$

(C) $\left[-\frac{3}{5}, -\frac{1}{6}\right]$

(D) None of these

Ans. A

Homework From Module

LCD

PRARAMBH : Q1 TO Q43
PRABAL : Q1 TO Q39
PARIKSHIT : Q1 TO Q8

(Solution to TAH)

QUESTION

TAH 1

If $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = e^c$, then the value of c is

A 5

B 4

C 3

D 2

If $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = e^c$, then the value of c is

(A) 5
(B) 4
(C) 3
(D) 2

$\to 1^\infty$

$$= e^{\lim_{x\to\infty}(x+4)\left(\frac{x+6}{x+1}-1\right)}$$

$$= e^{\lim_{x\to\infty}(x+4)\left(\frac{x+6-x-1}{x+1}\right)}$$

$$= e^{\lim_{x\to\infty}(x+4)\left(\frac{5}{x+1}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{5x+20}{x+1}\right)}$$

$$= e^5$$

TAH 1
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

TAH 2

$$\lim_{x \to 0} \left(\frac{5}{2 + \sqrt{9 + x}} \right)^{\text{cosec } x}$$

A $e^{-1/6}$

B $e^{-1/5}$

C $e^{-1/25}$

D $e^{-1/30}$

TAH 2
ABHIJEET KUMAR PATRA
ODISHA

$$\lim_{x\to 0}\left(\frac{5}{2+\sqrt{9+x}}\right)^{\operatorname{cosec} x}$$

$\to 1^{\infty}$

(A) $e^{-1/6}$
(B) $e^{-1/5}$
(C) $e^{-1/25}$
(D) $e^{-1/30}$

$$= e^{\lim_{x\to 0} \operatorname{cosec} x \cdot \left(\frac{5}{2+\sqrt{9+x}} - 1\right)}$$

$$= e^{\lim_{x\to 0} \frac{1}{\sin x}\left(\frac{3-\sqrt{9+x}}{(2+\sqrt{9+x})}\right)}$$

$\to 5$

$$= e^{\lim_{x\to 0} \frac{1}{\sin x (2+\sqrt{9+x})} \cdot \frac{9-(9+x)}{3+\sqrt{9+x}}}$$

$$= e^{\lim_{x\to 0} \frac{-x}{\sin x} \cdot \frac{1}{2+\sqrt{9+x}} \cdot \frac{1}{3+\sqrt{9+x}}}$$

$$= e^{-1 \cdot 1/5 \cdot 1/6} = e^{-1/30}$$

TAH-2

$$\lim_{x\to 0}\left[\frac{5}{2+\sqrt{9+x}}\right]^{\operatorname{cosec} x} = 1^{\infty}$$

$$\lim_{x\to 0} \operatorname{cosec} x\left[\frac{5}{2+\sqrt{9+x}} - 1\right] = e^{\lim_{x\to 0} \operatorname{cosec} x\left[\frac{5-2-\sqrt{9+x}}{2+\sqrt{9+x}}\right]}$$

$$e^{\lim_{x\to 0} \operatorname{cosec} x\left[\frac{3-\sqrt{9+x}}{2+\sqrt{9+x}}\right]} = e^{\lim_{x\to 0} \operatorname{cosec} x\left[\frac{9-9-x}{(2+\sqrt{9+x})(3+\sqrt{9+x})}\right]}$$

$$e^{\lim_{x\to 0} \frac{-x \operatorname{cosec} x}{(2+\sqrt{9+x})(3+\sqrt{9+x})}}$$

$$e^{-\lim_{x\to 0} (x \operatorname{cosec} x)\left[\frac{1}{(2+\sqrt{9+x})(3+\sqrt{9+x})}\right]}$$

$(x \operatorname{cosec} x) \to 1$

$$e^{-1/30}$$ Ans.

Name Sanjeet kumar
From – Varanasi (U.P)

QUESTION [JEE Advanced 2024 (Paper 2)] TAH 3

Let $k \in \mathbb{R}$. If $\lim\limits_{x \to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

(A) 1

(B) 2

(C) 3

(D) 4

Ans. B

$\lim_{x\to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$

$\lim_{x\to 0^+} \frac{2}{x}[\sin(\sin kx) + \cos x + x - 1] = e^6$

$\lim_{x\to 0^+} \frac{2}{x}[\sin(\sin kx) + x - 1 + \cos x] = e^6$

$\lim_{x\to 0^+} 2\left[\frac{\sin(\sin kx)}{x} + 1 - \left(\frac{1-\cos x}{x}\right)\right] = e^6$

$\lim_{x\to 0^+} 2\left[\frac{\sin(\sin kx)}{\sin kx} \cdot \frac{\sin kx}{kx} k + 1 - \left[\frac{1-\cos x}{x^2} \times x\right]\right] = e^6$

$\lim_{x\to 0^+} 2\left[k + 1 - \frac{x}{2}\right] = e^6$

$\lim_{x\to 0^+} [2k + 2 - x] = e^6$

$e^{2(k+1)} = e^6$

$2(k+1) = 6$

$k+1 = 3$

$\boxed{k = 2}$

TAH-3

Name Sanjeet kumar
From – Varanasi (U.P)

Let $k \in R$, If

$\lim_{x\to 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is

$\to 1^\infty$ (2024) Adv.

$e^{\lim_{x\to 0^+} \frac{2}{x}(\sin(\sin kx) + \cos x + x - 1)} = e^6$

$e^{\lim_{x\to 0} 2\left(\frac{\sin(\sin kx)}{x} + \frac{\cos x}{x} + 1 - \frac{1}{x}\right)} = e^6$

$e^{\lim_{x\to 0} \frac{2}{x}\left(\frac{\sin(\sin kx)}{\sin kx} \cdot \frac{\sin kx}{kx} \cdot kx + \left(\frac{1-\cos x}{x^2}\right)x^2 + x\right)} = e^6$

$e^{\lim_{x\to 0} 2\left(\frac{\sin(\sin kx)}{\sin kx} \cdot \frac{\sin kx}{kx} \cdot k - \left(\frac{1-\cos x}{x^2}\right)x + 1\right)} = e^6$

$e^{2(k-0+1)} = e^6$

$e^{2(k+1)} = e^6$

$2(k+1) = 6$

$k+1 = 3$

$k = 2$ Answer.

TAH 3
ABHIJEET KUMAR PATRA
ODISHA

QUESTION [JEE Mains 2024 (8 April)]

TAH 4

If $\alpha = \lim\limits_{x \to 0^+} \left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and $\beta = \lim\limits_{x \to 0} (1 + \sin x)^{\frac{1}{2} \cot x}$ are the roots of the quadratic equation $a^2 + bx - \sqrt{e} = 0$, then $12 \log_e (a + b)$ is equal to

Ans. 6

TAH-4

$$\alpha = \lim_{x\to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right), \quad \beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\cot x}$$

$$\alpha = \lim_{x\to 0^+} \frac{e^{\sqrt{x}}\left(e^{\sqrt{\tan x}-\sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}} = e^0(1) = 1$$

$$\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\cot x}$$

$$\beta = \lim_{x\to 0}(1+\sin x)^{\frac{\cos x}{2\sin x}}$$

$$= \lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}\times\frac{\cos x}{2}} = e^{\frac{1}{2}} = \sqrt{e}$$

$$\Rightarrow ax^2 + bx - \sqrt{e} = 0 \quad (\alpha, \beta)$$

$$\Rightarrow x^2 - (1+\sqrt{e})x + \sqrt{e}$$

$$\Rightarrow x^2 - x + \sqrt{e}x + \sqrt{e} = 0$$

$$\Rightarrow -x^2 + x(1+\sqrt{e}) - \sqrt{e} = 0$$

$$a = -1, \quad b = 1+\sqrt{e}$$

$$\Rightarrow 12\log_{\sqrt{e}}(-1+1+\sqrt{e}) = 12\ln(e^{\frac{1}{2}}) = \underline{6}$$

JM-29

Tah-4 $\alpha = \lim_{x\to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ & $\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\cot x}$ are roots of the quadratic eqn $ax^2+bx+\sqrt{e}=0$ (α, β) then $12\log_e(a+b)$ is equal to : $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{-\sqrt{e}}{a}$

$$\alpha = \lim_{x\to 0^+} \frac{e^{\sqrt{x}}\left(e^{\sqrt{\tan x}-\sqrt{x}} - 1\right)}{(\sqrt{\tan x} - \sqrt{x})}$$

$$\alpha = e^0 \cdot 1 = 1$$

$$\beta = e^{\lim_{x\to 0}\frac{1}{2}\cot x(1+\sin x-1)}$$

$$\beta = e^{\lim_{x\to 0}\frac{1}{2}\cot x}$$

$$\beta = e^{\frac{1}{2}}$$

$$\alpha\beta = \frac{-\sqrt{e}}{a}, \quad \alpha+\beta = \frac{-b}{a}$$

$$e^{\frac{1}{2}} = \frac{-e^{\frac{1}{2}}}{a} \qquad 1+e^{\frac{1}{2}} = \frac{-b}{a}$$

$$a = -1 \qquad a + ae^{\frac{1}{2}} = -b$$

$$a+b = -ae^{\frac{1}{2}}$$

$$a+b = e^{\frac{1}{2}}$$

$$12\log_e(a+b) = 12\log_e e^{\frac{1}{2}} = \frac{12}{2} = 6 \quad \text{Ans}$$

tah4
kshitij from up

QUESTION

TAH 5

$$\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n \quad a>0, b>0, n\in N$$

A $a^{1/b}$

B $b^{1/a}$

C a^b

D b^a

Tah:05

$$\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n \quad , a>0, b>0, n\in N$$

Sol^n

$$= e^{\lim_{n\to\infty} n\cdot\left(\frac{a-1+\sqrt[n]{b}}{a}-1\right)}$$

$$e^{\lim_{n\to\infty}\frac{n}{a}\cdot(a-1+\sqrt[n]{b}-a)}$$

$$e^{\lim_{n\to\infty}\frac{n}{a}(b^{1/n}-1)}$$

$$e^{\lim_{n\to\infty}\frac{1}{a}\left(\frac{b^{1/n}-1}{1/n}\right)} = e^{\frac{1}{a}\ln b}$$

$$= e^{\ln b^{1/a}} = b^{1/a}$$

(B) Ans

Ritesh Singh, from Devbhoomi Uttarakhand 🙏

Q5 $\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n$ $a>0, b>0, n\in N$.

$\to 0$

$\left(1+\left(\frac{b^{1/n}-1}{a}\right)\right)^n \to 1^\infty$

$= e^{\lim_{n\to\infty} n\cdot\left(\frac{a-1+b^{1/n}-1}{a}\right)}$

$= e^{\lim_{n\to\infty} n\cdot\left(\frac{a+b^{1/n}-1-a}{a}\right)}$

$= e^{\lim_{n\to\infty} \frac{(b^{1/n}-1)}{1/n}\cdot\frac{1}{a}} = e^{\ln b^{1/a}} = b^{1/a}$.

Priyanka Keshri W.B

QUESTION [JEE Mains 2020]

TAH 6

$\lim_{x \to 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{\frac{1}{x}}$ is equal to:

A e

B 2

C e^2

D 1

M-26

Tah-6

$$\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}}.$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)}$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x}{1-\tan x}-1\right)}$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{2\tan x}{1-\tan x}\right)} = e^{\lim_{x\to 0}\frac{2}{1-\tan x}\left(\frac{\tan x}{x}\right)^{\to 1}} = e^{2}$$ Ans

tah6
kshitij from up

Tah:06

Ritesh Singh, from Devbhoomi Uttarakhand 🙏

$$\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$$

1^∞ form

$$e^{\lim_{x\to 0}\frac{1}{x}\times\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$$

$$e^{\lim_{x\to 0}\frac{1}{x}\times\left(\frac{1+\tan x}{1-\tan x}-1\right)} = e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{1+\tan x-1+\tan x}{1-\tan x}\right)}$$

$$e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{2\tan x}{1-\tan x}\right)}$$

$$e^{\lim_{x\to 0}\frac{2}{1-\tan x}\cdot\left(\frac{\tan x}{x}\right)}$$

$$e^{\frac{2}{1-0}\times 1} = e^2$$ (C)

Ans //

TAH-6 → Mains-2020

→ $\lim_{x\to 0} \left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$

$= e^{\lim_{x\to 0} \frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)}$

$= e^{\lim_{x\to 0} \frac{1}{x}\left(\frac{\cos x+\sin x}{\cos x-\sin x}-1\right)}$

$= e^{\lim_{x\to 0} \frac{1}{x}\left(\frac{\cos x+\sin x-\cos x+\sin x}{\cos x-\sin x}\right)}$

$= e^{\lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)\cdot\frac{1}{\cos x-\sin x}}$

$\boxed{= e^2}$ Ans option (c) is correct

Wasima Khatri
From Gujarat

QUESTION [JEE Mains 2024 (9 April)]

TAH 7

$$\lim_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$$ is equal to

(A) $-\frac{2}{e}$

(B) $e - e^2$

(C) 0

(D) e

Ans. D

Ritesh Singh ,from Devbhoomi Uttarakhand 🙏

Tah:07

$$\lim_{x\to 0} \frac{e-(1+2x)^{1/2x}}{x}$$

Sol^n

$$\lim_{x\to 0} \frac{e-e^{\ln(1+2x)^{1/2x}}}{x}$$

$$\lim_{x\to 0} \frac{e-e^{\frac{1}{2x}\ln(1+2x)}}{x} - \lim_{x\to 0} e\left(\frac{1-e^{\frac{1}{2x}\ln(1+2x)-1}}{x}\right)$$

$\frac{1}{2x}\ln(1+2x)-1 = M$

as $x\to 0 \Rightarrow M\to 0$
bcoz $\frac{\ln(1+2x)}{2x} \to 1$

$$= e\lim_{x\to 0} \frac{1-e^M}{M}\left(\frac{\frac{1}{2x}\cdot\ln(1+2x)-1}{x}\right)$$

$$= -e\lim_{x\to 0}\left(\frac{e^M-1}{M}\right)\frac{\ln(1+2x)-2x}{2x^2}$$

$$= -e\lim_{x\to 0}\left(\frac{e^M-1}{M}\right)\left(\frac{2x-4x^2/2+8x^3/3+\cdots\infty-2x}{2x^2}\right)$$

$$= \boxed{(+e)(-1) = e} \quad (D) \quad Ans//$$

M-24

Tah-7 $\lim_{x\to 0} \frac{e-(1+2x)^{\frac{1}{2x}}}{x}$

$= \lim_{x\to 0} \frac{e-e^{\ln(1+2x)^{\frac{1}{2x}}}}{x}$

$= \lim_{x\to 0} \frac{e-e^{\frac{1}{2x}\ln(1+2x)}}{x}$

let $\frac{1}{2x}\ln(1+2x) = M$.

as $x\to 0$, $M\to 0$

b'cos $x\cdot\frac{\ln(1+x)}{x} \to 1$

$= e \lim_{x\to 0} \frac{1-e^{\frac{1}{2x}\ln(1+2x)-1}}{x}$

tah7
kshitij from up

$= e \lim_{M\to 0} \frac{1-e^{M}}{x}$

$= -e \lim_{M\to 0} \frac{e^{M}-1}{M}\cdot\frac{\frac{1}{2x}\ln(1+2x)-1}{x}$

$= -e \lim_{M\to 0} \left(\frac{e^{M}-1}{M}\right)\cdot\frac{\ln(1+2x)-2x}{2x^2}$

$= -e \lim_{M\to 0} \left(\frac{e^{M}-1}{M}\right)\left(\frac{\left(2x-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}+\cdots\right)-2x}{2x^2}\right)$

$\to 1$

$= -e\,(1)\cdot(-1)$

$= e$ Ans

$$\lim_{x\to 0} \frac{e-(1+2x)^{1/2x}}{x}$$ is Equal to (2024)

$$\Rightarrow \lim_{x\to 0} \frac{e-e\left[1-\frac{2x}{2}+\frac{11}{24}(2x)^2-\frac{21}{48}(2x)^3+\cdots\right)}{x}$$

$$\lim_{x\to 0} \frac{e\left[x-x+\frac{2x}{2}-\frac{11}{24}(2x)^2+\frac{21}{48}(2x)^3+\cdots\right]}{x}$$

$$\lim_{x\to 0} \frac{ex\left[1-\frac{11}{24}(2^2x)+\frac{21}{48}(2)^3x^2+\cdots\right]}{x}$$

$= e \cdot 1 = e$

TAH 7
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

QUESTION [JEE Advanced 2020]

TAH 8

Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit $\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^n}$ is equal to nonzero real number, is

Ans : 1

Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit $\lim_{x\to 0^+} \frac{(1-x)^{1/x} - e^{-1}}{x^a}$ is Equal to a non zero real number, is (JEE adv. 2020).

TAH 8
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

$\Rightarrow l = \lim_{x\to 0^+} \frac{(1-x)^{1/x} - e^{-1}}{x^a}$

$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}\ln(1-x)} - \frac{1}{e}}{x^a} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}\ln(1-x)} - 1/e}{x^a}$

$= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}\ln(1-x)+1} - 1}{e x^a} = \lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)}{x}+1} - 1}{e x^a}$

We analyze $\lim_{x\to 0^+} \frac{\ln(1-x)}{x} + 1 = \lim_{x\to 0^+} \left(\frac{-\ln(1+(-x))}{-x}\right) + 1$

$= -1 + 1 = 0$

Hence, $l = \lim_{x\to 0^+} \left(\frac{e^{\frac{\ln(1-x)}{x}+1} - 1}{\frac{\ln(1-x)}{x}+1}\right) \cdot \frac{\frac{\ln(1-x)}{x}+1}{e x^a}$

$= \lim_{x\to 0^+} (\quad) \frac{\ln(1-x)+x}{e x^{a+1}}$

$= \lim_{x\to 0^+} (\quad) \frac{\left[(-x - \frac{x^2}{2} - \frac{x^3}{3} + \dots \infty) + x\right]}{e x^{a+1}}$

$= \lim_{x\to 0^+} (\quad) \frac{x^2\left[-\frac{1}{2} - \frac{x}{3} + \dots \infty\right]}{e x^{a+1}}$

If $a+1 < 2$ $\quad l = 0$
$a+1 > 2$ $\quad l = -\infty$
$a+1 = 2$ $\quad l = 1 \cdot \frac{-1}{2e} = \frac{-1}{2e}$

⇓

For finite non-zero limit $a+1 = 2$
i.e. $a = 1$

TAH-8

Let c denote the base of the natural logarithm. The value of real nu. a. for which the right hand limit

$\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$ is equal to non-zero real nu., is.

(m-1)

Solve → $\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$

methode

$\lim_{x\to 0^+} \frac{e^{\ln(1-x)^{\frac{1}{x}}} - e^{-1}}{x^a}$

$\lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)}{x}} - \frac{1}{e}}{x^a}$

$\lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)}{x}+1} - 1}{e \cdot x^a}$

$\lim_{x\to 0^+} \frac{e^{\frac{\ln(1-x)+x}{x}} - 1}{\frac{\ln(1-x)+x}{x} \cdot e x^a} \cdot \frac{\ln(1-x)+x}{x}$

$\lim_{x\to 0^+} \frac{1}{e} (\quad) \cdot \frac{-(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \infty) + x}{x^{a+1}}$

TAH 8 PART 1

for → RHL is a non zero real nu.

$q = 1$ Ans

if $q=1$ $\lim_{x\to 0^+} \frac{1}{e} (\quad) \left(-\frac{x^2(1 + \frac{x}{2} + \dots \infty)}{x^2}\right)$

$= -\frac{1}{2e}$

methode 2

TAH 8. RAHUL DHAKAD❤️❤️ FROM AGRA UP

(m-2)

$$\lim_{x\to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$$

$$\lim_{x\to 0^+} \frac{\left[(1+(-x))^{\frac{1}{(-x)}}\right]^{(-1)} - e^{-1}}{x^a}$$

$$\lim_{x\to 0^+} \frac{\left(e\left[1-\frac{(-x)}{2}+\frac{11}{24}(-x)^2+\cdots\infty\right]\right)^{-1} - e^{-1}}{x^a}$$

$$\lim_{x\to 0^+} \frac{e^{-1}\left(\left[1+\frac{x}{2}+\frac{11}{24}x^2+\cdots\infty\right]^{-1}-1\right)}{x^a}$$

$$\lim_{x\to 0^+} \frac{e^{-1}\left(-\left[\frac{x}{2}+\frac{11}{24}x^2+\cdots\infty\right]\right)}{x^a\left[1+\frac{x}{2}+\frac{11}{24}x^2+\cdots\infty\right]}$$

for finite RHL in a non zero real no.

$$a=1$$ Ans

if $a=1$

$$\lim_{x\to 0^+} \frac{e^{-1}\left(-\left[\frac{1}{2}+\frac{11}{24}x+\cdots\infty\right]\right)}{\left(1+\frac{x}{2}+\frac{11}{24}x^2+\cdots\infty\right)}$$

$$= -\frac{1}{2e}$$

QUESTION

TAH 9

$$\lim_{x \to 0} \frac{e^{x^3} - 1 - x^3}{\sin^6 2x}$$

Ans. 1/128

Tah-9 $\lim_{x\to 0} \frac{e^{x^3}-1-x^3}{\sin^6 2x}$

tah9
kshitij from up

$= \lim_{x\to 0} \frac{e^{x^3}-1-x^3}{(2x)^6} \times \left(\frac{2x}{\sin 2x}\right)^6$

$= \lim_{x\to 0} \frac{\left(1+x^3+\frac{x^6}{2!}+\frac{x^9}{3!}+\cdots\infty\right)-1-x^3}{(2x)^6} \cdot \left(\frac{2x}{\sin 2x}\right)^6$ ($\left(\frac{2x}{\sin 2x}\right)^6 \to 1$)

$= \frac{1}{2!}\cdot\frac{1}{2^6}\times 1 = \frac{1}{2\times 64} = \frac{1}{128}$ Ans

TAH-9

Q $\lim_{x \to 0} \dfrac{e^{x^3} - 1 - x^3}{\sin^6 2x}$ $\left(\frac{0}{0}\right)$ form

$$\lim_{x \to 0} \frac{\left[1 + \frac{x^3}{1!} + \frac{(x^3)^2}{2!} + \frac{(x^3)^3}{3!} + \cdots \infty\right] - 1 - x^3}{\left[2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots \infty\right]^6}$$

$$\lim_{x \to 0} \frac{x^3 \left[\frac{1}{2!} + \frac{x^3}{3!} + \cdots \infty\right]}{x^6 \left[2 - \frac{8x^2}{3!} + \frac{(2x)^4}{5!} - \cdots \infty\right]^6}$$

$$= \frac{\frac{1}{2!}}{2^6} = \boxed{\frac{1}{2^7}} \quad \text{Ans}$$

TAH 9 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION [JEE Mains 2024 (31 Jan)] TAH 10

If $\lim\limits_{x\to 0}\frac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}=1$, then $16(a^2+b^2+c^2)$ is equal to

Ans. 81

TAH: 10

If $\lim_{x\to 0} \frac{ax^2 e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$, then $16(a^2+b^2+c^2)$

Sol^n :-

$$\lim_{x\to 0} \frac{ax^2\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots\infty\right) - b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\infty\right) + cx\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\infty\right)}{x^3\cdot\left(\frac{\sin x}{x}\right)}$$

$$\lim_{x\to 0} \frac{x^3\left(a-\frac{b}{3}+\frac{c}{2}\right) + x^2\left(a+\frac{b}{2}-c\right) + x(c-b)}{x^3\,(\ \ )} = 1$$

$\downarrow 1$

If limit Exit.

$a+\frac{b}{2}-c=0$, $c-b=0$, $a-\frac{b}{3}+\frac{c}{2}=1$

$2a+b-2c=0$

$2a+b-2b=0$

$a=\frac{+b}{2}$

$c=b$

$\frac{b}{2}-\frac{b}{3}+\frac{b}{2}=1$

$\frac{3b-b}{3}=1$

$b=\frac{3}{2}$

$a=\frac{3}{4}$, $c=\frac{3}{2}$

Ritesh Singh, from Devbhoomi Uttarakhand

$16(a^2+b^2+c^2) = 16\left(\frac{9}{16}+\frac{9}{4}+\frac{9}{4}\right)$

$16\left(\frac{9+36+36}{16}\right) = 9+72 = 81$

Ans

Tanlo $\lim_{x\to 0} \frac{ax^2 e^x - b\lg_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$

$\lim_{x\to 0} \frac{ax^2\left(1+\frac{x}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right) - b\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right) + cx\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}\right)}{x^3}$

$\lim_{x\to 0} \frac{(cx-bx)+\left(ax^2+\frac{bx^2}{2}-cx^2\right)+\left(\frac{ax^3}{1!}-\frac{bx^3}{3}+\frac{cx^3}{2!}\right)+\cdots}{x^3}$

For limit exist : $\boxed{b=c}$

& $a+\frac{b}{2}-c=0$

$2a+b-2c=0$

$2a-b=0$

$\boxed{a=\frac{b}{2}}$

& Also $a-\frac{b}{3}+\frac{c}{2}=1$

$6a-2b+3c=1\times 6$

$6a+b=6$

$3b+b=4b=6$

$\boxed{b=\frac{3}{2}=c}$ & $\boxed{a=\frac{3}{4}}$

Now $16(a^2+b^2+c^2) \Rightarrow 16\left(\frac{9}{16}+\frac{9}{4}+\frac{9}{4}\right)$

$\Rightarrow 9+36+36$

$\Rightarrow 81$ Ans

kamran Ashraf
Muzaffarpur

QUESTION [JEE Mains 2021 (20 July)]

TAH 11

If $\lim\limits_{x\to 0} \dfrac{\alpha x e^{x} - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10, \alpha, \beta, \gamma \in R$, then the value of $\alpha + \beta + \gamma$ is

TAH 11

$$\lim_{x\to 0} \frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$$

$$\lim_{x\to 0} \frac{\alpha x\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\dots\right) - \beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\dots\right) + \gamma x^2\left(1-x+\frac{x^2}{2!}-\dots\right)}{x^3}$$

$$\lim_{x\to 0} \frac{(\alpha x - \beta x) + \left(\frac{\alpha x^2}{1} + \frac{\beta x^2}{2} + \gamma x^2\right) + \left(\frac{\alpha x^3}{2} - \frac{\beta x^3}{3} + \gamma x^3\right)}{x^3}$$

For limit x

$\alpha = \beta$ $\quad \alpha + \frac{\beta}{2} + \gamma = 0$ $\quad$ & $\quad \frac{9}{2} - \frac{\beta}{3} - \gamma = 10$

$\frac{3\beta}{2} + \gamma = 0$

$\gamma \to -\frac{3\alpha}{2}$

$\therefore 3\alpha - 2\alpha - \gamma \times 6 = 60$

$\alpha - 6\gamma = 60$

$\alpha + 9\alpha = 60$

$\boxed{\alpha \to 10}$

Now $\boxed{\beta = 10}$ & $\boxed{\gamma \to -15}$

$\therefore \alpha + \beta + \gamma = 20 - 15 = 5$ Ans

kamran Ashraf
Muzaffarpur

If $\lim_{x\to 0} \frac{\alpha x e^x - \beta \log(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$,

$\alpha, \beta, \gamma \in R$, then the value of $\alpha+\beta+\gamma$ is (2021)

$$\Rightarrow \lim_{x\to 0} \frac{\alpha x\left[1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\infty\right]-\beta\left[x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\cdots\infty\right] + \gamma x^2\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\infty\right]}{x^3\left[\frac{\sin^2 x}{x^2}\right]} = 10$$

$$\Rightarrow \lim_{x\to 0} \frac{x(\alpha-\beta)+x^2\left[\alpha+\frac{\beta}{2}+\gamma\right]+x^3\left[\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right]+\cdots\infty}{x^3\left[\frac{\sin^2 x}{x^2}\right]} = 10$$

For limit to Exist,

$\alpha-\beta=0$ & $\alpha+\frac{\beta}{2}+\gamma=0$

$\Rightarrow \alpha=\beta$

$\Rightarrow \frac{3\alpha}{2}+\gamma=0$

$\Rightarrow \gamma=-\frac{3\alpha}{2}$

Now,

$$\lim_{x\to 0} \frac{x^3\left[\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right]+x^4(\quad)+\cdots\infty}{x^3\left[\frac{\sin^2 x}{x^2}\right]} = 10$$

$\Rightarrow \frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10$

$\Rightarrow \frac{\alpha+3\alpha}{2}-\frac{\beta}{3}=10 \Rightarrow \frac{\alpha+9\alpha}{6}=10 \Rightarrow \alpha=6$

So, $\alpha=\beta=6$ & $\gamma=\frac{-3\alpha}{2}=-\frac{18}{2}=-9$

Now, $\alpha+\beta+\gamma=6+6-9=3$

TAH 11
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ODISHA
(JATNI)

QUESTION [JEE Mains 2021]

TAH 12 A

If $\lim\limits_{x\to 0} \frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of $a-2b$ is

If $\lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is Equal to b, then the value of $a-2b$ is

$$\Rightarrow \lim_{x\to 0} \frac{ax-\left[1+\frac{4x}{1!}+\frac{16x^2}{2!}+\frac{64x^3}{3!}+\cdots-1\right]}{ax\left[1+\frac{4x}{1!}+\frac{16x^2}{2!}+\frac{64x^3}{3!}+\cdots-1\right]}$$

$$\lim_{x\to 0} \frac{ax-\left[\frac{4x}{1!}+\frac{16x^2}{2!}+\frac{64x^3}{3!}+\cdots\right]}{ax^2\left[4+\frac{16x}{2!}+\frac{64x^2}{3!}+\cdots\right]}$$

$$\lim_{x\to 0} \frac{x(a-4)-x^2\left[\frac{16}{2!}+\frac{64x}{3!}+\cdots\right]}{ax^2\left[4+\frac{16x}{2!}+\frac{64}{3!}x^2+\cdots\right]}$$

For limit Exist, $a=4$

Now,

$$\lim_{x\to 0} \frac{-x^2\left[8+\frac{64x}{3!}+\cdots\right]}{4x^2\left[4+\frac{16x}{2!}+\frac{64x^2}{3!}+\cdots\right]} = b$$

$$= \frac{-8}{16} = \frac{-1}{2} = b$$

So, $a-2b = 4+1 = 5$

TAH 12(A)
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

TAH-12 :→

Q → If $\lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to 6, then the value of $a-2b$ is

1 alw

$$\lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)} = 6$$

$$\lim_{x\to 0} \frac{ax-\left[1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\cdots\infty-1\right]}{ax\left[1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\cdots\infty-1\right]} = 6$$

$$\lim_{x\to 0} \frac{\left[x(a-4)-\frac{(4x)^2}{2!}-\cdots\infty\right]}{a\left[\frac{4x^2}{1!}+\frac{16x^3}{2!}+\cdots\infty\right]} = 6$$

for limit exist coefficient of x two

$$a-4=0$$

$$\boxed{a=4}$$

$$\lim_{x\to 0} \frac{-x^1\left[\frac{16}{2!}+\frac{4^3\cdot x^4}{3!}+\cdots\infty\right]}{4\cdot x^1\left[\frac{4}{1!}+\frac{16x}{2!}+\cdots\infty\right]} = 6$$

$$-\frac{8}{16} = 6 \Rightarrow \boxed{6=-\frac{1}{2}}$$

$$\Rightarrow a-2b = 4+1 = 5$$

An

TAH 12 RAHUL DHAKAD❤️❤️ FROM AGRA UP

Tah (12)

Hm $\quad y = b = \lim_{x \to 0} \left(\frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} \right)$

$$b = \lim_{x \to 0} \left(\frac{ax + 1 - e^{4x}}{ax(e^{4x} - 1)} \right) = \left(\frac{ax + 1 - \left(1 + \frac{(4x)}{1!} + \frac{(4x)^2}{2!} + \frac{(4x)^3}{3!} - \infty\right)}{ax\left(1 + \frac{(4x)}{1!} + \frac{(4x)^2}{2!} - \infty - 1\right)} \right)$$

$$b = \lim_{x \to 0} \left(\frac{(a-4)x - \left(\frac{16x^2}{2} + \frac{(4x)^3}{3!} - \infty\right)}{ax\left(\frac{4x}{1!} + \frac{(4x)^2}{2!} - \infty\right)} \right)$$

For Existence of limit.

$(a-4) = 0$, $\boxed{a = 4}$

$$b = \lim_{x \to 0} \left(\frac{-\left(\frac{16x^2}{2} + \frac{(4x)^3}{3!} - \infty\right)}{4x^2\left(4 + \frac{4x}{2!} - \infty\right)} \right) = \frac{-8}{16}$$

$$\boxed{b = -\frac{1}{2}}$$

Now, $\quad \boxed{a - 2b = 4 - 2\left(-\frac{1}{2}\right) = 5}$

TAH 12 (A)
____hashtag_sonu____
from Bihar siwan

QUESTION [JEE Mains 2023 (13 April)]

TAH 12B

If $\lim\limits_{x\to 0} \dfrac{e^{ax} - \cos(bx) - \dfrac{cx\, e^{-cx}}{2}}{1 - \cos(2x)} = 17$, then $5a^2 + b^2$ is equal to

A 64

B 68

C 72

D 76

If $\lim_{x\to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1-\cos(2x)} = 17$ then a^2+b^2 is Equal to (2023)

(A) 64
(B) 68
(C) 72
(D) 76

$$= \lim_{x\to 0} \frac{\left(1+ax+\frac{a^2x^2}{2!}+\frac{a^3x^3}{3!}+\cdots\infty\right)-\left(1-\frac{b^2x^2}{2!}+\frac{b^4x^4}{4!}+\cdots\infty\right) - \frac{cx}{2}\left(1-\frac{cx}{1!}+\frac{c^2x^2}{2!}-\frac{c^3x^3}{3!}+\cdots\right)}{\frac{1-\cos 2x}{(2x)^2}\cdot 4x^2}$$

$\longrightarrow$ Should be 0

$$= \lim_{x\to 0} \frac{\left(a-\frac{c}{2}\right)x + x^2\left(\frac{a^2}{2!}+\frac{b^2}{2!}+\frac{c^2}{2!}\right)+x^0(\ )+\cdots\infty}{4x^2\ \frac{(1-\cos 2x)}{(2x)^2}}$$

$$a-\frac{c}{2}=0 \Rightarrow c=2a \quad \text{--- (1)}$$

$$= \lim_{x\to 0} \frac{x^2\left[\left(\frac{a^2+b^2+c^2}{2}\right)+x(\ )+\cdots\cdots\infty\right]}{4x^2\ \frac{(1-\cos 2x)}{(2x)^2}}$$

$$= \frac{\frac{a^2+b^2+c^2}{2}}{4 \cdot \frac{1}{2}} = \frac{a^2+b^2+c^2}{4}$$

TAH 12(B)
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

Betulimit = 17

$$\frac{a^2+b^2+c^2}{4} = 17$$

$$a^2+b^2+c^2 = 68 \quad — (2)$$

From (1) & (2),

$$a^2+b^2+(2a)^2 = 68$$

$$5a^2+b^2 = 68$$

QUESTION

TAH 13

Let $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin\frac{1}{x}+x}{1+x^{2n}}$ then which of the following alternative(s) is/are correct?

A $\lim_{x\to\infty} x f(x) = 2$

B $\lim_{x\to 1} f(x)$ does not exist.

C $\lim_{x\to 0} f(x)$ does not exist.

D $\lim_{x\to-\infty} f(x)$ is equal to zero.

TAH-13

(1) ⇒ Let $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin\frac{1}{x} + x}{1+x^{2n}}$ Hum which of the following alternative (s) is/are correct.

Solve $f(x) = \lim_{n\to\infty} \frac{2x^{2n}\sin\frac{1}{x} + x}{1+x^{2n}}$

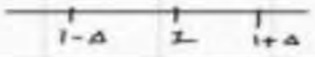
(A) $\lim_{x\to\infty} xf(x) = 2$

Solve if x is very large +ve no.

$$f(x) = \lim_{n\to\infty} \frac{2\sin\frac{1}{x} + \frac{x}{x^{2n}}}{1+\frac{1}{x^{2n}}} = 2\sin\frac{1}{x}$$

$$\lim_{x\to\infty} x\,f(x) = \lim_{x\to\infty} 2\cdot\frac{\sin\frac{1}{x}}{\frac{1}{x}} = 2$$

(B) $\lim_{x\to 1} f(x)$ does not exist



$$f(x) = \begin{cases} \lim_{n\to\infty} \frac{2\sin\frac{1}{x} + \frac{x}{x^{2n}}}{\frac{1}{x^{2n}}+1} & x\in(1,1+\Delta) \\ \lim_{n\to\infty} \frac{2\cdot x^{2n}\sin\frac{1}{x} + x}{1+x^{2n}} & x\in(1-\Delta,1) \end{cases}, \quad x>1, \quad 0<x<1$$

($\frac{x}{x^{2n}} \to 0$, $\frac{1}{x^{2n}} \to 0$, $2x^{2n} \to 0$, $x^{2n} \to 0$)

$$f(x) = \begin{cases} 2\sin\frac{1}{x}, & x\in(1,1+\Delta) \\ x, & x\in(1-\Delta,1) \end{cases}$$

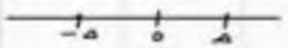
$\lim_{x\to 1} f(x)$:
- RHL: $\lim_{x\to 1^+} 2\cdot\sin\frac{1}{x} = 2\sin 1$
- LHL: $\lim_{x\to 1^-} x = 1$

$\lim_{x\to 1} f(x)$ = does not exist

PART 1

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(C) ✗ $\lim_{x\to 0} f(x)$ does not exist



$$f(x) = \begin{cases} \lim_{n\to\infty} \frac{1\cdot x^{2n}\sin\frac{1}{x} + x}{1+x^{2n}}, & x\in(0,\Delta) \text{ or } 0<x<1 \\ \lim_{n\to\infty} \frac{1\cdot x^{2n}\sin\frac{1}{x} + x}{1+x^{2n}}, & x\in(-\Delta,0) \text{ or } -1<x<0 \end{cases}$$

$$h(x) = \begin{cases} x, & x\in(0,\Delta) \\ x, & x\in(-\Delta,0) \end{cases}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} h(x) = 0$$

$\lim_{x\to 0} f(x)$ = exist

(D) ✓ $\lim_{x\to -\infty} f(x)$ is equal to Zero

Solve if x is very large -ve no.

$$\lim_{n\to\infty} \frac{2\sin\frac{1}{x} + \frac{x}{x^{2n}}}{\frac{1}{x^{2n}}+1} = 2\sin\frac{1}{x}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} 2\sin\frac{1}{x}$$

put $x = -t$, $x\to-\infty$, $t\to\infty$

$$\lim_{t\to\infty} -2\sin\frac{1}{t} = 0$$

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PART 2

(Solution to RPP)

QUESTION

(RPP 1)

If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$ is

A $(-\infty, \infty)$

B $R - \left\{\frac{1}{2}\right\}$

C $R - \left\{\frac{4}{9}, \frac{1}{2}\right\}$

D $R - \left\{\frac{3}{2}\right\}$

Ans. C

RPP → 1

Q → If $x \in R$ then range of $f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$

(m-1)

Solve

$$f(x) = \frac{x^2+2x-3}{2x^2+3x-9}$$

$$= \frac{(x+3)(x-1)}{2x^2+6x-3x-9}$$

$$= \frac{(x+3)(x-1)}{2x(x+3)-3(x+3)}$$

$$= \frac{(x+3)(x-1)}{(x+3)(2x-3)}$$

$$f(x) = \frac{x-1}{2x-3}$$

$f(-3) = \frac{-4}{-9}$

Range → $R - \left\{\frac{1}{2}, f(-3)\right\}$

Range → $R - \left\{\frac{1}{2}, \frac{4}{9}\right\}$ Ans

RPP 1 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION

(RPP 2)

If the quadratic polynomial $f(x) = (a-3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in

A $[0, 2]$

B $[3, 5]$

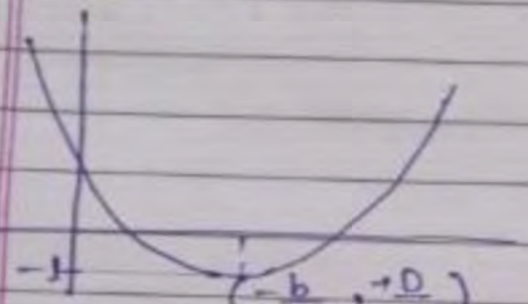
C $[4, 6)$

D $[5, 7]$

Ans. C

RPP (2)

Que If the Quadratic polynomial $f(x) = (a-3)x^2 - 2ax + 3a - 7$ ranges from $[-1, \infty)$ for every $x \in R$, then the value of a lies in.



$$-\frac{b}{2a} = \frac{+2a}{2(a-3)} = \frac{a}{a-3}$$

now $f\left(\frac{a}{a-3}\right) = (a-3)\left(\frac{a^2}{(a-3)^2}\right) - 2a\left(\frac{a}{a-3}\right) + 3a - 7$

$\because f\left(\frac{a}{a-3}\right) = -1$

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Mr. anoop Mishra (UP)
Mrs Smita Mishra

$$\frac{a^2}{(a-3)} - \frac{2a^2}{(a-3)} + 3a - 7 = -1$$

$$a^2 - 2a^2 + 3a(a-3) = 6(a-3)$$

$$-a^2 + 3a^2 - 9a = 6a - 18$$

$$2a^2 - 15a + 18 = 0$$

$$2a^2 - 12a - 3a + 18 = 0$$

$$2a(a-6) - 3(a-6) = 0$$

$$(a-6)(2a-3) = 0 \quad a = 6, \frac{3}{2}$$

for upward opening parabola

$a - 3 > 0$

$a > 3$

$a \in [3, \infty)$

$a = 6$

$a \in [5, 7]$

$F(x) = (a-3)x^3 - 2ax + (3a-7)$

Range of $F(x) \in \left[\frac{-D}{4a}, \infty\right) = [-1, \infty)$

$\Rightarrow \frac{4D}{4a} = +1$

$\Rightarrow 4a^2 - 4(3a-7)(a-3) = 4(a-3)$

$\Rightarrow 2a^2 - 15x + 18 = 0$

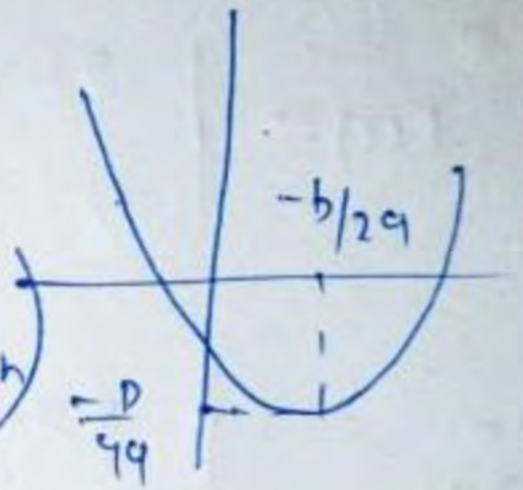
$a = 3, 6.$

But $a \neq 3, a > 3.$

Then $a \in (3, 6]$

For integral Value of $a \in [4, 6]$

($a-3 > 0$ – upward opening Graph
$a > 3$)



RPP 2
____hashtag_sonu____
from Bihar siwan

QUESTION

(RPP 3)

If the function $f: R^+ \to A$ defined by $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$ is surjective then find A.

Ans. $[6, \infty]$

RPP → 3

Q → If the func. $f: R^+ \to A$ defined by $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$ is surjective then find A

Solu → A is co-domain

$f(x)$ is surjective (onto) ⇒

Range of $f(x)$ = co-domain

Range of $f(x) = A$

$$f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$$

$$= \frac{x\left(\frac{1}{x}+x+1\right) \cdot x^2\left(\frac{1}{x^2}+x^2\right)}{x^3}$$

$$= \left(\frac{1}{x}+x+1\right)\left(x^2+\frac{1}{x^2}\right)$$

≥ 3 ≥ 2

≥ 6

RPP 3 RAHUL DHAKAD ❤❤

FROM AGRA UP

$y \in [6, \infty)$ Ans

(Solution to Home Challenge)

Home Challenge - 07

The number of value of k for which $(x^2 - (k - 2)x - 2k)(x^2 + kx + 2k - 4)$ is a perfect square is

yes ↑

no ↓

A 1

B 2

C 3

D 0

Ans. A

$(x^2-(k-2)x-2k)\cdot(x^2+kx+2k-4)$ is a perfect square if

case (i) Both are perfect squares

$\Rightarrow D_1=0$ & $D_2=0$

$(k-2)^2-4\cdot(-2k)=0$ & $k^2-4(2k-4)=0$

$k^2-4k+4+8k=0$ $\qquad$ $k^2-8k+4=0$

$k^2+4k+4=0$ $\qquad$ $\Downarrow$

$(k+2)^2=0$ $\qquad$ does not satisfy

$k=-2$

$\Rightarrow$ There exist no value of k for which both become perfect squares.

case (ii) Both roots of 1st & 2nd quad are common.

$x^2-(k-2)x-2k \lessgtr \alpha, \beta$

$x^2+kx+2k-4 \lessgtr \alpha, \beta$

$\Rightarrow \frac{1}{1}=\frac{2-k}{k}=\frac{-2k}{2k-4}$

$k=1$ only one value

Bhoolnaa Mat Yeh Batayaa thaa

Revision: Trigonometry
Till 6 Aug

If A is an Nilpotent matrix, then Trace of A =0


THANK
YOU

PRAYAS

JEE 2025

Lecture-08 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Types of Discontinuity
2. Practice Problems

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION

ASRQ+KCLS

Let $f(x) = \begin{cases} 5, & -\infty < x \le 2 \\ px, & 2 < x < 4 \\ 3x - q, & 4 \le x < \infty \end{cases}$ and $g(x) = \begin{cases} 3x + 2, & -\infty < x < 1 \\ x + 1, & 1 \le x < \infty \end{cases}$.

If $\lim_{x \to 1} f(g(x)) = 5$, then find the value of $(2p + q)$.

$\lim_{x \to 1} f(g(x))$

RHL $\lim_{h \to 0} f(g(1+h)) = \lim_{h \to 0} f(1+1+h) = \lim_{h \to 0} f(2+h)$

LHL $\lim_{h \to 0} f(g(1-h)) = \lim_{h \to 0} f(3(1-h)+2) = \lim_{h \to 0} f(5-3h)$

$= \lim_{h \to 0} 3(5-3h) - q$

$= 15 - q$

$2p = \lim_{h \to 0} p(2+h)$

Since $\lim_{x \to 1} f(g(x)) = 5$

$\Downarrow$

LHL = RHL = 5

$2p = 15 - q = 5$.

$p = 5/2, \; q = 10$

$2p + q = 15$ Ans

QUESTION

Tan 1

ASRQ+KCLS

Refer to the graph of $y = f(x)$ and $g(x) = (x-2)^2, x < 2$

$= 7 - x, x \geq 2$

then which of the following limits are non-existent.

(a) $\lim_{x \to 2} f(g(x))$

(b) $\lim_{x \to 0} g(f(x))$

(c) $\lim_{x \to 5} g(f(x))$

$\lim_{x \to 2} f(g(x)) = 2$ Ans

(A) RHL

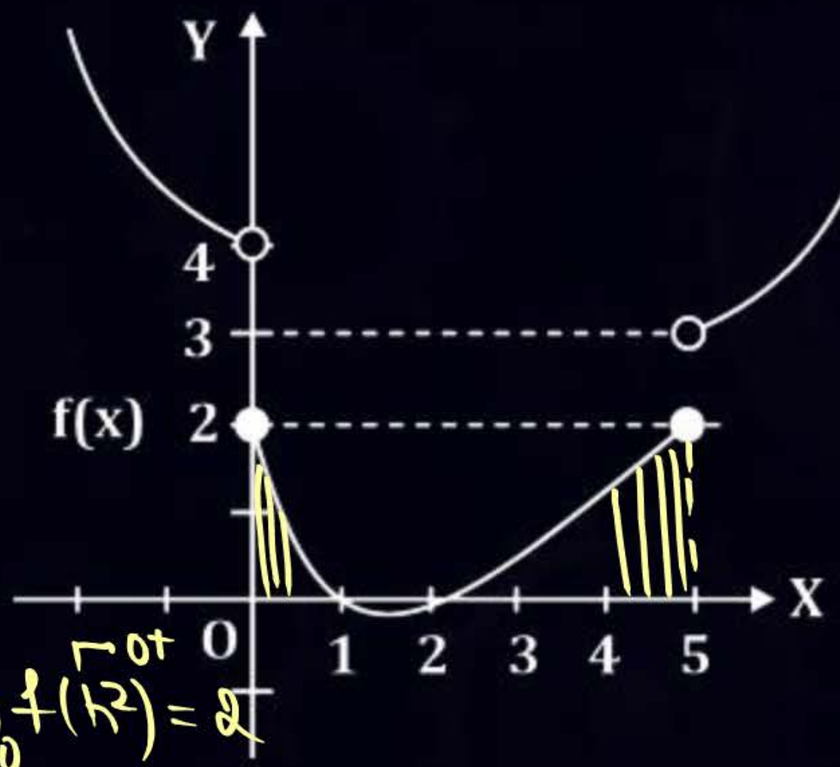
$\lim_{h \to 0} f(g(2+h))$

$= \lim_{h \to 0} f(7-(2+h))$

$= \lim_{h \to 0} f(5-h) = 2.$

LHL $\lim_{h \to 0} f(g(2-h))$

$= \lim_{h \to 0} f((2-(2-h))^2) = \lim_{h \to 0} f(h^2) = 2$



QUESTION

Tan 1

ASRQ+KCLS

Refer to the graph of $y = f(x)$ and $g(x) = (x-2)^2, x < 2$

$= 7 - x, x \geq 2$

then which of the following limits are non-existent.

(a) $\lim_{x \to 2} f(g(x))$

(b) $\lim_{x \to 0} g(f(x))$ DNE

(c) $\lim_{x \to 5} g(f(x))$

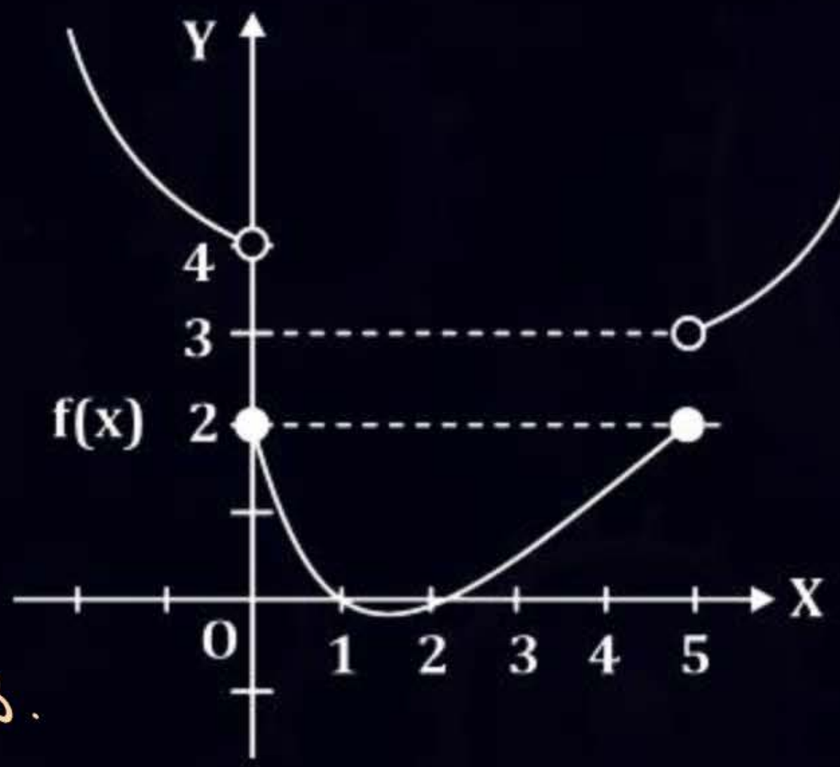
(B) RHL $\lim_{h \to 0} g(f(h))$ $f(h) \to 2^-$

$= \lim_{h \to 0} (f(h)-2)^2$ $f(h) \to 0^+$

$= 0$

LHL $\lim_{h \to 0} g(f(-h))$ $f(-h) \to 0^-$, $f(-h) \to 4^+$

$= \lim_{h \to 0} 7 - (f(-h)) = 7 - 4 = 3.$



QUESTION [JEE Mains 2022 (28 July)]

$\lim_{n\to\infty} x^{2n} = \infty, \; x>1 \text{ or } x<-1 \qquad (n\in N)$

The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ is continuous for all x in :

$\lim_{n\to\infty} x^{2n} = 0 \quad -1<x<1$

$x=1,-1$

$\lim_{n\to\infty} x^{2n} = 1$

$$f(x)=\begin{cases} \lim_{n\to\infty} \frac{\cos 2\pi x - x^{2n}\cdot \sin(x-1)}{1+x^{2n+1}-x^{2n}}, & -1<x<1 \\ \lim_{n\to\infty} \frac{\frac{\cos 2\pi x}{x^{2n}} - \sin(x-1)}{\frac{1}{x^{2n}}+x-1}, & x>1,\ x<-1 \end{cases}$$

(A) $\mathbb{R}-\{-1\}$

(B) $\mathbb{R}-\{-1,1\}$

(C) $\mathbb{R}-\{1\}$

(D) $\mathbb{R}-\{0\}$

$$f(x)=\begin{cases} \cos 2\pi x & -1<x<1 \\ \frac{-\sin(x-1)}{x-1} & x<-1 \text{ or } x>1 \end{cases}$$

$f(1) = \cos 2\pi = 1$

$f(-1) = \frac{\cos(-2\pi)+\sin 2}{} = -(1+\sin 2)$

$$f(x)=\begin{cases}\cos 2\pi x & -1<x<1\\ -\dfrac{\sin(x-1)}{x-1} & x<-1 \text{ or } x>1\\ 1 & x=1\\ -(1+\sin 2) & x=-1\end{cases}$$

AT $x=1$

LHL $= \cos 2\pi = 1$

RHL $= \lim\limits_{x\to 1^+} -\dfrac{\sin(x-1)}{(x-1)} = -1$

$\Big\rangle$ f is discontinuous at $x=1$

AT $x=-1$

LHL $= \lim\limits_{x\to -1^-} -\dfrac{\sin(x-1)}{x-1} = -\dfrac{\sin(-2)}{-2} = -\dfrac{\sin 2}{2}$

RHL $= \lim\limits_{x\to -1^+} \cos 2\pi x = \cos(-2\pi) = 1$

$\Big\rangle$ discontinuous at $x=-1$

QUESTION

TAH 2

Let $f(x) = \lim_{n\to\infty} \frac{x^{2n-1}+ax^2+bx}{x^{2n}+1}$.

If $f(x)$ is continuous for all $x \in \mathbb{R}$ then find the values of a and b. $(n \in N)$

Ans. $a = 0, b = 1$

QUESTION

Let function f be defined as $f : R^+ \to R^+$ and function g is defined as $g : R^+ \to R^+$. Function f and g are continuous in their domain. Suppose function $h(x) = \lim_{n\to\infty} \frac{x^{n+1}f(x)+x^2}{x^n+g(x)}, x > 0$. If h(x) is continuous in its domain then $f(1) \cdot g(1)$ is equal to

A 2

B 1

C 1/2

D 0

clearly h will be continuous on R^+ & hence $x=1$

$$h(x)=\begin{cases} \lim\limits_{n-\infty} \dfrac{f(x)+\frac{x^2}{x^{n+1}}}{\frac{1}{x}+\frac{g(x)}{x^{n+1}}} & (1, 1+\Delta) \\ \lim\limits_{n-\infty} \dfrac{f(x)\,x^{n+1}+x^2}{x^n+g(x)} & (1-\Delta, 1) \end{cases}$$

($x^2/x^{n+1} \to 0$; $1/x \to 0$; $g(x)/x^{n+1} \to 0$; $x^n \to 0$)

$$h(x)=\begin{cases} x f(x) & x \in (1, 1+\Delta) \\ \dfrac{x^2}{g(x)} & x \in (1-\Delta, 1) \end{cases}$$

Now h is continuous at $x=1$

RHL $\lim\limits_{x \to 1^+} x f(x) = 1 \cdot f(1)$

LHL $\lim\limits_{x \to 1^-} \dfrac{x^2}{g(x)} = \dfrac{1}{g(1)}$

$f(1) = \dfrac{1}{g(1)}$

$f(1) \cdot g(1) = 1$

Five Golden Points to remember

T-1 : Sum, difference, product and quotient of two continuous functions is always a continuous function. However $h(x) = \frac{f(x)}{g(x)}$ is continuous at $x = a$ only if $g(a) \neq 0$.

T-2 : If $f(x)$ is continuous and $g(x)$ is discontinuous then $f(x) \pm g(x)$ must be discontinuous function.

Five Golden Points to remember

T-3 : If $f(x)$ is continuous & $g(x)$ is discontinuous at $x = a$ then nothing definite can be said about the product function $\phi(x) = f(x) \cdot g(x)$

Astonishing Result (1)

If f is continuous at $x=a$ & $f(a)=0$

& g is discontinuous at $x=a$ with finite LHL & RHL & $g(a)$ is def.

Then their product fn $\phi(x)=f(x)\cdot g(x)$ will be

Continuous at $x=a$

QUESTION

The function $f(x) = [x] \cdot \cos\frac{2x-1}{2}\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at :-

(A) all x

(B) all integer points

(C) no x

(D) x which is not an integer

$f(x) = [x] \cdot \cos\left((2x-1)\frac{\pi}{2}\right)$ — continuous at all integers also 0 value=0 at $x \in I$

↓

discontinuous at all Integers. with finite LHL & RHL. & defined at all integers.

⇓

By A·R ① f is continuous at $x \in I$

Clearly f is continuous $\forall\, x \notin I$

Five Golden Points to remember

T-4 : If $f(x)$ and $g(x)$ both are discontinuous at $x = a$ then also nothing definite can be said about sum or the product $f(x)g(x)$ at $x = a$.

$f(x) = [x]$
$g(x) = \{x\}$
> discontinuous at all integers

$h(x) = f(x) + g(x) = [x] + \{x\} = x$ — continuous at all integers.

Five Golden Points to remember

T-5 : If the function 'f' is continuous at x = a and g(x) is continuous at x = f(a) then the composite function g(f(x)) is also continuous at x = a.

$g(f(x))$ — f continuous at $x=a$; g is continuous at $x=f(a)$ ⇒ then gof is continuous at $x=a$

$f(g(x))$ — g if continuous at $x=a$; f if continuous at $x=g(a)$ ⇒ then fog is continuous at $x=a$

If f is continuous at $x=a$ & g is continuous at $x=f(a)$ then gof is continuous at $x=a$

i.e $\lim_{x\to a} gof(x) = gof(a)$ ⇒ i.e $\lim_{x\to a} g(f(x)) = g(f(a))$

i.e $\lim_{x-a} g(f(x)) = g(f(a))$

Since f is continuous at $x=a$

$\Downarrow$

$\lim_{x-a} f(x) = f(a)$

$\lim_{x \to a} g(f(x)) = g(\lim_{x-a} f(x))$

$\lim_{x-a} g \circ f(x) = ?$ if $\lim_{x-a} f(x) = \ell$ & g is continuous at $x=\ell$

then $\lim_{x-a} gof(x) = g(\lim_{x-a} f(x))$

$\ell = \lim_{x-0} (1+x)^{1/x}$

$\ln \ell = \ln(\lim_{x \to 0} (1+x)^{1/x})$

$= \lim_{x-0} \ln(1+x)^{1/x}$

If f & g at both continuous at $x=a$ then gof may or may not be continuous at $x=a$

Ex: $f(x)=\begin{cases}1+x & x\geq 0\\ 1-x^2 & x<0\end{cases}$ $g(x)=\begin{cases}2+x & x\geq 1\\ 3+x & x<1\end{cases}$

Both fn are continuous at $x=0$

AT $x=0$ for $gof(x)$

RHL: $\lim_{x\to 0^+} g(f(x)) = \lim_{x\to 0^+} g(1+x) = \lim_{x\to 0^+} 2+1+x = 3$

LHL: $\lim_{x\to 0^-} g(f(x)) = \lim_{x\to 0^-} g(1-x^2) = \lim_{x\to 0^-} 3+1-x^2 = 4$ ($1-$)

Clearly gof is discontinuous at $x=0$.

If f is discontinuous at $x = a_1, a_2, a_3 \ldots$
& g is discontinuous at $x = \alpha, \beta, \gamma \ldots$

Probable points of discontinuity of $gof(x) = g(f(x))$

① $x = a_1, a_2, a_3 \ldots$

② $f(x) = \alpha, \beta, \gamma \ldots$



Ex: $\text{Sgn}(f(x))$ Probable discontinuity points $f(x) = 0$
↳ if f is continuous on R

[·]GIF Ex: $[f(x)]$ Probable points of discontinuity are those where $f(x)$ becomes Integer
↳ if is continuous on R

Ex: $\{f(x)\}$ Probable points of discontinuity are those where $f(x)$ becomes Integer

QUESTION

Let $f(x)=\frac{\sqrt{x^2+kx+1}}{x^2-k}$. The interval(s) of all possible values of k for which f is continuous for every $x \in R$, is

A $(-\infty,-2]$

B $[-2,0)$

C $R-(-2,2)$

D $(-2,2)$

$f(x)=\frac{\sqrt{x^2+kx+1}}{x^2-k}$ is continuous on R

$\Downarrow$

Should be defined on R

$x^2+kx+1 \geq 0 \quad \forall x \in R \;\&\; x^2-k \neq 0$

$D \leq 0 \Rightarrow k^2-4 \leq 0 \;\&\; x^2 \neq k$

$k \in [-2,2] \qquad x^2 \in [0,\infty)$

$\Downarrow$

$k \in (-\infty, 0)$

$k \in [-2,0)$

QUESTION

Let $f(x)$ be a continuous function such that $f\left(\frac{1}{3}\right) = 3$. Then $\lim_{x\to 0} f\left(\frac{1}{x^2} - \frac{1}{(\sin^{-1} x)^2}\right)$ is equal to

A) 1/3

B) 3

C) - 1/3

D) - 3

$$\lim_{x\to 0} f\left(\frac{1}{x^2} - \frac{1}{(\sin^{-1}x)^2}\right) = f\left(\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{(\sin^{-1}x)^2}\right)$$

Now $$\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{(\sin^{-1}x)^2} = \lim_{\theta\to 0} \frac{1}{\sin^2\theta} - \frac{1}{\theta^2}$$

put $\sin^{-1}x = \theta$

$x = \sin\theta$

$$\overset{\text{II}}{\lim_{\theta\to 0}} \frac{(\theta-\sin\theta)(\theta+\sin\theta)}{\theta^2\sin^2\theta}$$

$$\overset{\text{II}}{\lim_{\theta\to 0}} \frac{\frac{\theta-\sin\theta}{\theta^3}\cdot\frac{\theta+\sin\theta}{\theta}}{\frac{\sin^2\theta}{\theta^2}}$$

$$\frac{1}{3} = \frac{\frac{1}{6}\cdot 2}{1}$$

$$f\left(\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{(\sin^{-1}x)^2}\right) = f(1/3) = 3.$$

QUESTION

Let $f(x) = \text{sgn}(x^2 - 4|x| + k)$. If $f(x)$ is discontinuous at exactly four points, then he range of k is

A $(0, 1)$

B $(0, 2)$

C $(0, 3)$

D $(0, 4)$

$f(x) = \text{sgn}(\underbrace{x^2 - 4|x| + k}_{\text{continuous on } R})$

$x^2 - 4|x| + k = 0$ should have 4 distinct roots.

$|x| = t$

$|x|^2 - 4|x| + k = 0$

$t^2 - 4t + k = 0$

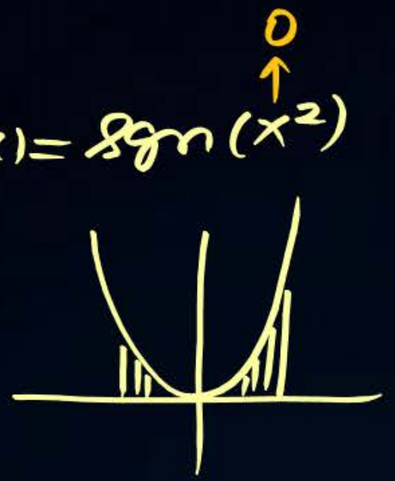
↳ Should have 2 real distinct +ve roots.

$D > 0,\ S > 0,\ P > 0$

$16 - 4k > 0,\ 4 > 0,\ k > 0$

$k < 4$ ⟵ $k \in (0, 4)$

Ex: $f(x) = \text{sgn}(x^2)$



discontinuous at $x=0$

$\downarrow$

$RHL = 1 = LHL$

$f(0) = 0$

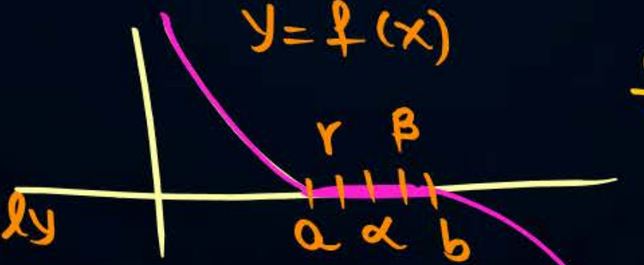
$y = \sin^2 x$.



Ex: $f(x) = \text{sgn}((x-1)^2(x-2))$

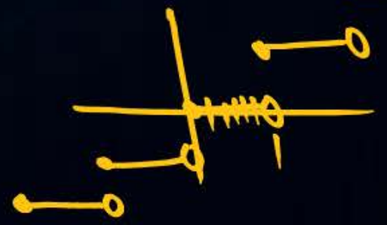
$\hookrightarrow$ discontinuous at $x = 1, 2$.

$\text{sgn}(f(x))$ is discontinuous at a, b only

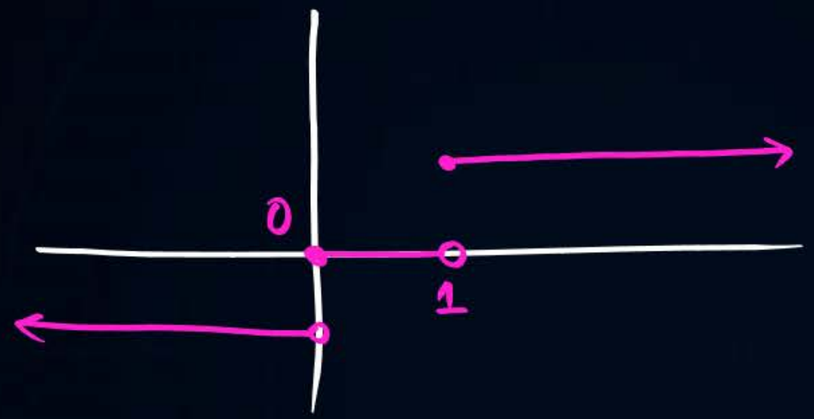


$f(x) = \text{sgn}(\sin^2 x)$ at $x = n\pi$

$f(x) = \text{sgn}([x])$ discontinuous at $x = 0, 1$.



$y = \operatorname{sgn}[x]$



AT $x=2$

★ RHL $\lim_{x\to 2^+} \operatorname{sgn}[x]$

$= \lim_{h\to 0} \operatorname{sgn}[2+h] = \operatorname{sgn} 2 = 1$

LHL $\lim_{h\to 0} \operatorname{sgn}[2-h] = \operatorname{sgn} 1 = 1$

$\operatorname{sgn}[2] = \operatorname{sgn} 2 = 1$

AT $x = 1/2$

$\lim_{h\to 0} \operatorname{sgn}[1/2+h] = \operatorname{sgn} 0 = 0$

$\lim_{h\to 0} \operatorname{sgn}[1/2-h] = \operatorname{sgn} 0 = 0$

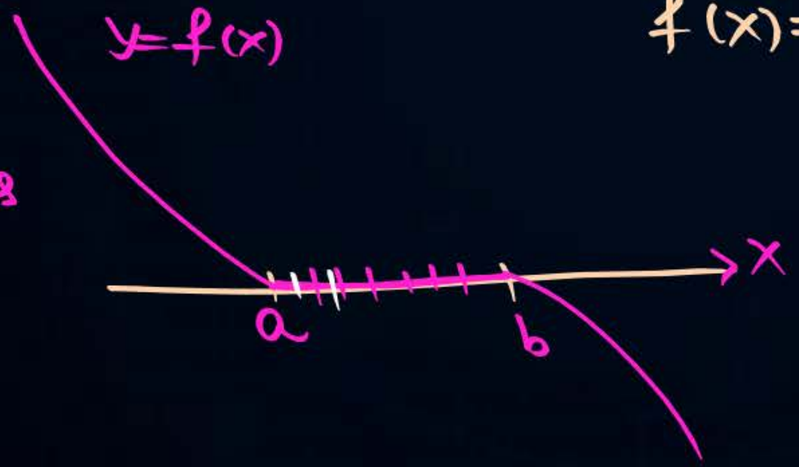
$\operatorname{sgn}[1/2] = \operatorname{sgn}(0) = 0$.

NICHOD!!

$\operatorname{sgn}(f(x))$ ← continuous

is discontinuous at points where $f(x)=0$ & does not coincide with x axis in nbd of point where $f(x)=0$.

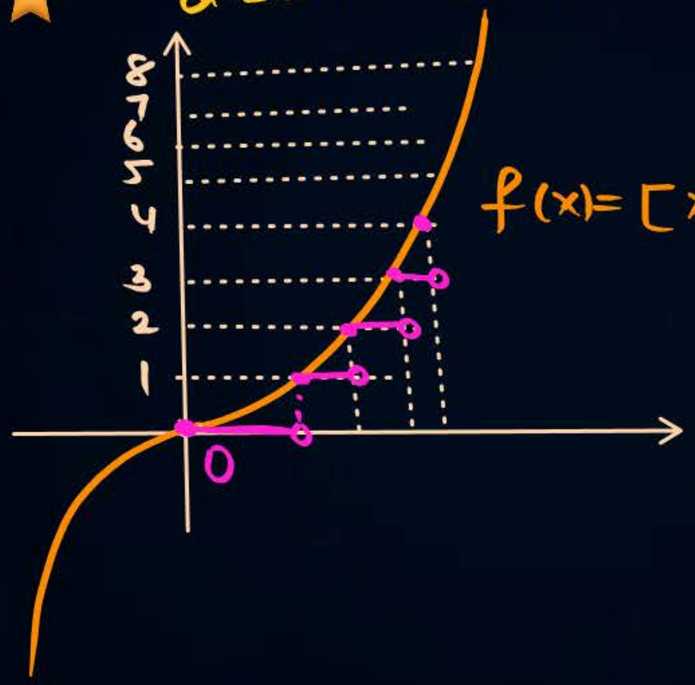
$\operatorname{sgn} f(x)$ is discontinuous at $x=a, b$.



If f is continuous on $\mathbb{R}$ then $\operatorname{sgn}(f(x))$ is discontinuous at all points where $f(x)=0$

(True / ~~False~~)

Ex:



$f(x)=[x^3]$ points of discontinuity in $[0,2]$

$x^3 \in [0,8]$

$x^3 = \cancel{0},1,2,3,4,5,6,7,8$

$x = 1, \sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{4}, \sqrt[3]{5}, \sqrt[3]{6}, \sqrt[3]{7}, 2$

Points of discontinuity.

QUESTION

Find number of Points of discontinuity of $f(x) = [x^2]$ in

(i) $[-2, 2]$ (ii) $(-1, 3]$, where [.] denotes G.I.F

① $x \in [-2, 2]$

Discontinuous @ $x^2 \in$ Integer

$x^2 \in [0, 4]$

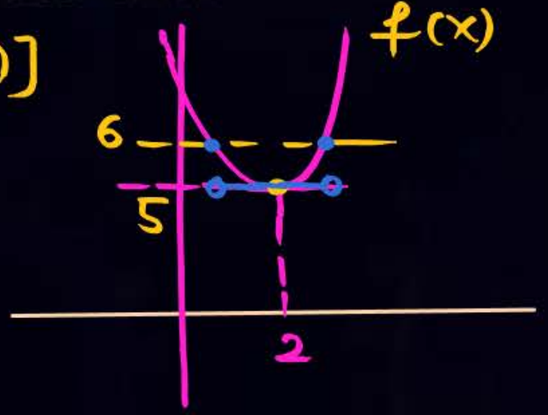
$x^2 = 0, 1, 2, 3, 4$

$x = 0, -1, 1, \sqrt{2}, -\sqrt{2}, \sqrt{3}, -\sqrt{3}, 2, -2$

⇓

8 points of discontinuity

$g(x)$ $[f(x)]$




THANK
YOU

PRAYAS

JEE 2025

Lecture-09 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Types of Discontinuity
2. Practice Problems

Recap *of previous lecture*

1. If f and g are both continuous at x = a then fog & gof are also continuous at x = a (True/False)

2. If $\lim_{x \to a} f(x) = l$ & g is continuous at x = l then $\lim_{x \to a} g(f(x))$ = $g(\lim_{x \to a} f(x)) = g(l)$

3. If f is continuous at x = a & g is continuous at x = $f(a)$ then gof(x) is continuous at x = a.

Recap *of previous lecture*

4. If f(x) is discontinuous at $x = x_1, x_2$ & x_3 while g is discontinuous at $x = \alpha, \beta, \gamma$ then probable points of discontinuity of gof (x) are x = x_1, x_2, x_3 and points where $f(x) = \alpha, \beta, \gamma$

$g''(f(x))$

6. If f (x) is a continuous function, then sgn(f(x)) is discontinuous at points where $f(x) = 0$ provided the graph of f(x) does not coincide with X axis in the neighbourhood of the point where $f(x) = 0$

Recap *of previous lecture*

7. $f(x) = [x] \cdot \sin x$ is _____ at $x = 0$.

Continuous

finite LHL & RHL at $x=0$ & $[0]=0$ i.e defined

continuous at $x=0$ $\sin(0)=0$

8. If $f(x)$ is continuous at $x = a$ & $f(a) = 0$, $g(x)$ is discontinuous at $x = a$ with finite LHL & RHL & $g(a)$ is also defined then $f(x) \cdot g(x)$ is _____

Continuous at $x=a$

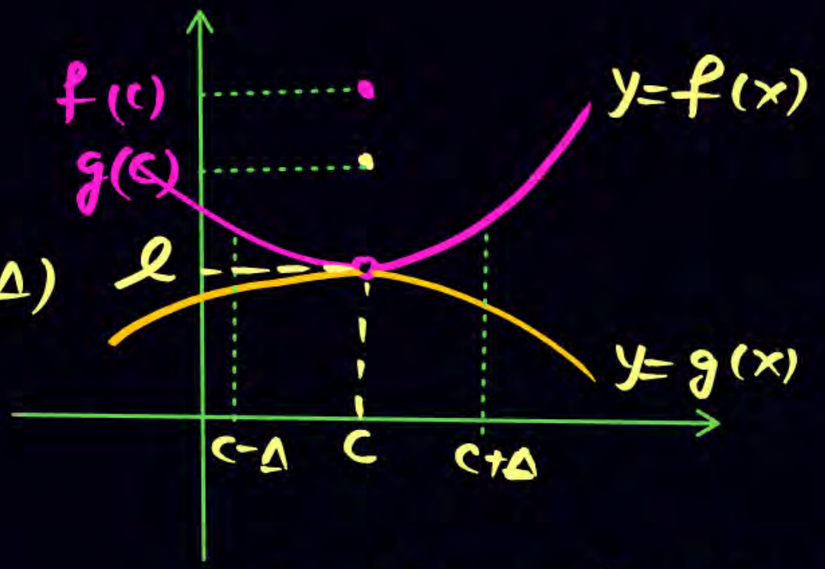
9. If $f(x) > g(x)\ \forall\ x \in (c - \Delta, c + \Delta)$ then $\lim\limits_{x\to c} f(x) > \lim\limits_{x\to c} g(x)$. (True/False)

$\Downarrow$

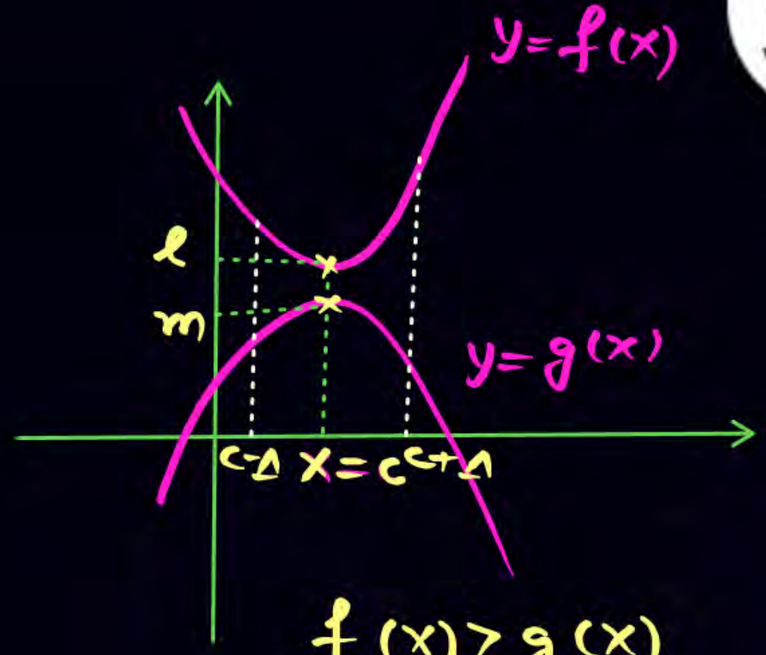
$\lim\limits_{x\to c} f(x) \geq \lim\limits_{x\to c} g(x)$

$f(x) > g(x)$

$\forall\ x \in (c-\Delta, c+\Delta)$



$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) =$$



$f(x) > g(x)$

$\forall\ x \in (c-\Delta, c+\Delta)$

& here $\lim_{x \to c} f(x) > \lim_{x \to c} g(x)$

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

Types of Discontinuity

Reasons for Discontinuity at $x=a$

R_1: $\lim_{x \to a} f(x)$ does not exist

Can not be corrected.

(2) Irremovable discontinuity

R_2: $\lim_{x \to a} f(x)$ exists finitely but is not equal to $f(a)$

R_3: $\lim_{x \to a} f(x)$ exists finitely but f is not defined at $x=a$.

Can be corrected but redefining fn at $x=a$

(1) Removable discontinuity

① Removable discontinuity is of two types.

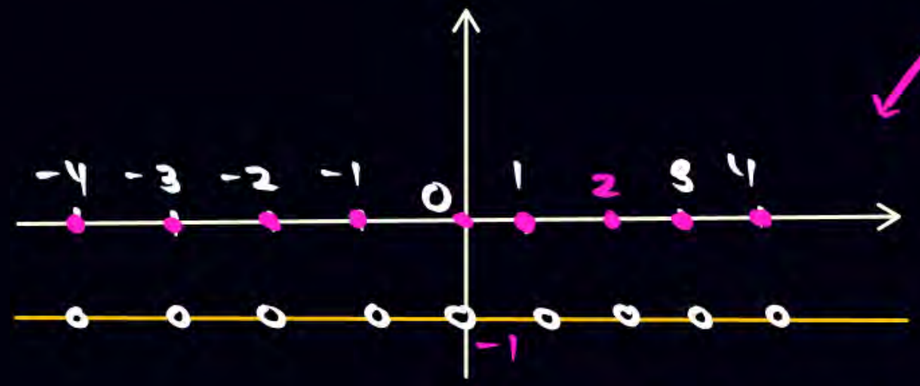
T_1: Isolated point discontinuity

Here $\lim_{x \to a} f(x)$ exist finitely but is not equal to $f(a)$

$y = f(x) = [x] + [-x]$

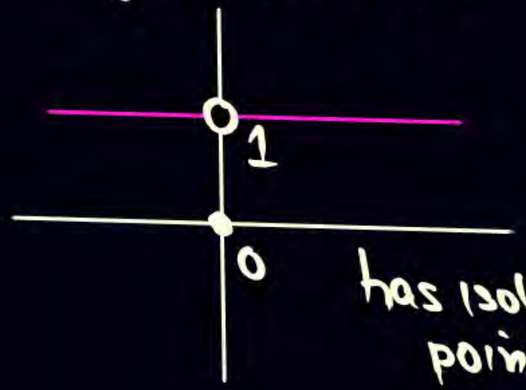
$[\cdot]$ G.I.F

$$= \begin{cases} -1 & x \notin I \\ 0 & x \in I \end{cases}$$



has isolated point discontinuity at $x \in I$

$\text{sgn}(x^2) = f(x)$

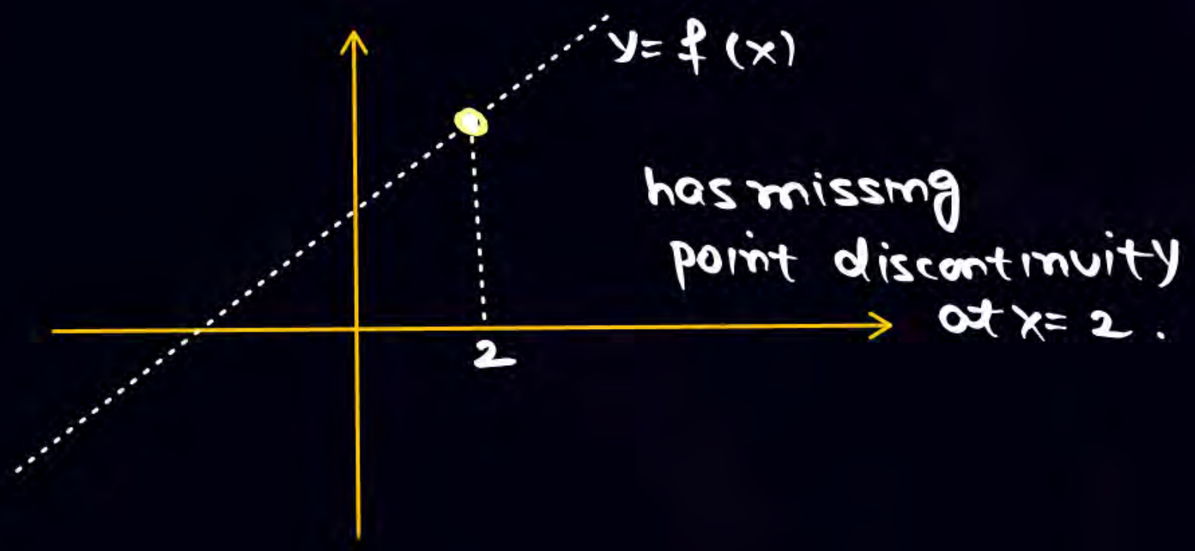


has isolated point discontinuity at $x = 0$

② Missing point Discontinuity

Here $\lim_{x \to a} f(x)$ Exist finitely but $f(a)$ is not defined.

Ex: $f(x) = \dfrac{x^2-4}{x-2} = x+2 \;,\; x \neq 2$



Types of Discontinuity

Type 2 : (Non removable discontinuity)

Here $\lim_{x \to a} f(x)$ does not exists and therefore it is not possible to redefine the function in any manner to make it continuous. Such discontinuities can be further classified into 3 fold.

(a) Finite type (both limits finite and unequal)

Ex: $f(x) = \tan^{-1}\frac{1}{x}$

$\lim_{x \to 0^+} \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$, $\lim_{x \to 0^-} \tan^{-1}x = -\frac{\pi}{2}$

limit D.N.E but both finite & unequal.

jump $= \pi$

(b) Infinite type (at least one of the two limit are infinity)

(c) Oscillatory (limits oscillates)

Finite type

Ex: $f(x) = \text{sgn}(x)$

$\lim_{x \to 0^+} \text{sgn}\, x = 1$

$\lim_{x \to 0^-} \text{sgn}\, x = -1$

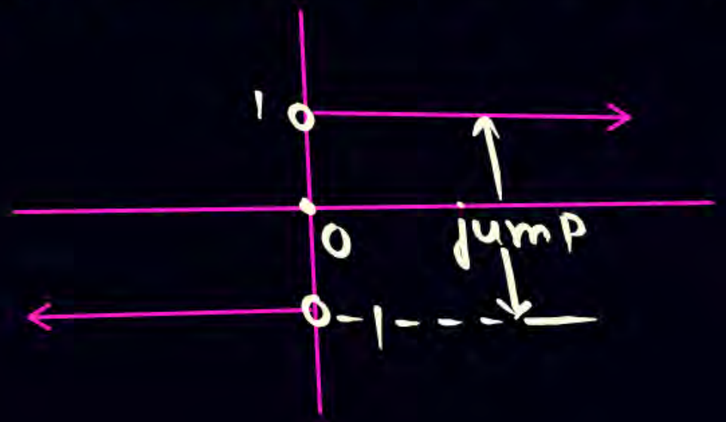
finite but unequal

$\Downarrow$

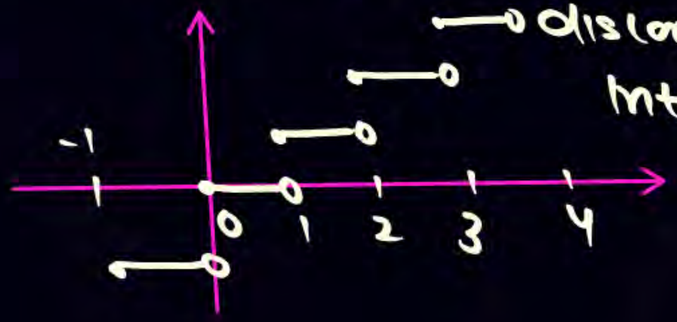
finite type non Removable Discontinuity

jump = 2.

★ Difference b/w LHL & RHL at $x = a$ is called jump of discontinuity at $x = a$



$f(x) = [x]$ has finite type discontinuity at all integer.



jump at each discontinuity = 1

Infinite type

Ex: $f(x) = 2^{\frac{1}{x}}$ AT $x = 0$

$\lim_{x \to 0^+} 2^{\frac{1}{x}} = 2^{\infty} = \infty$

$\lim_{x \to 0^-} 2^{\frac{1}{x}} = 2^{-\infty} = 0$

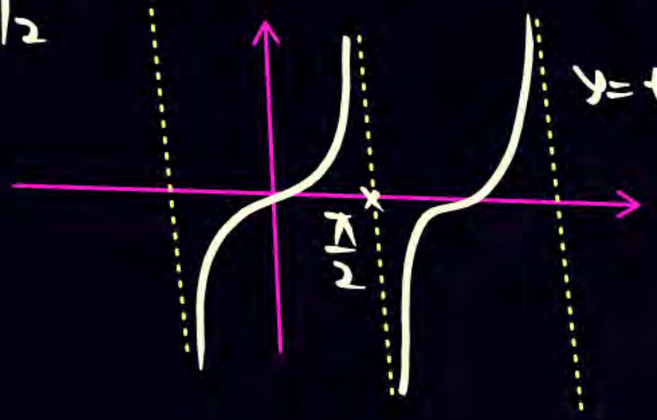
Infinite type discontinuity at $x = 0$

Ex: $f(x) = \tan x$ AT $x = \pi/2$

$\lim_{x \to \pi/2^+} \tan x = -\infty$

$\lim_{x \to \pi/2^-} \tan x = +\infty$

Infinity type discontinuity at $x = \pi/2$

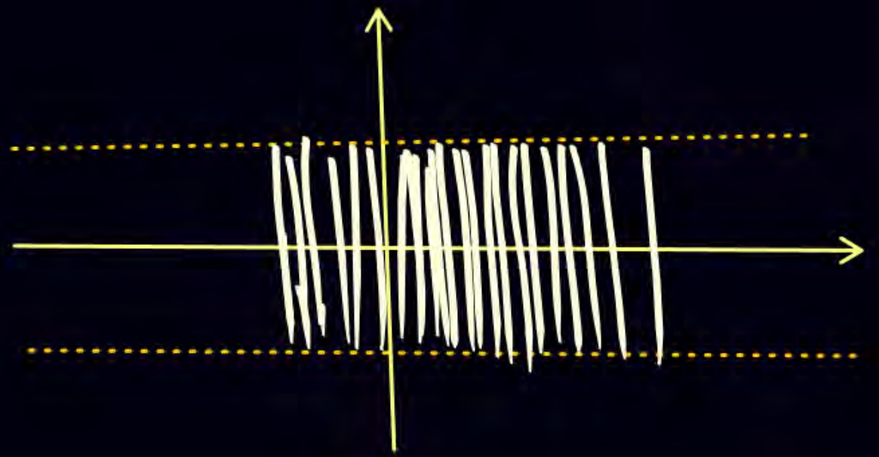


Oscillatory type

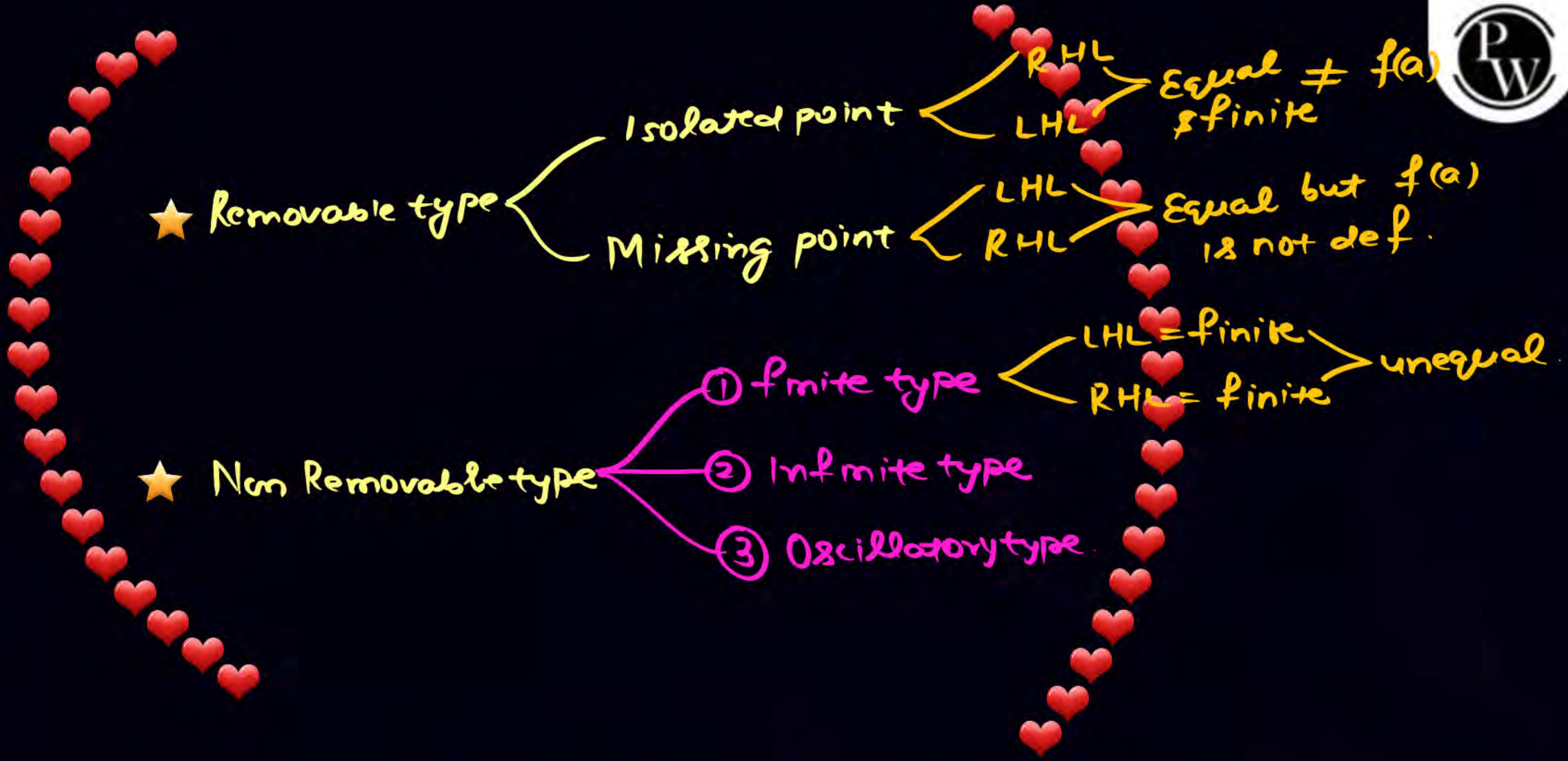
★ $f(x) = \sin\frac{1}{x}$ @ $x=0$ has an oscillatory type discontinuity

★ $f(x) = \cos\frac{1}{x}$ @ $x=0$ has an oscillatory type discontinuity

★ $f(x) = \begin{cases} 1 & x \in Q \\ -1 & x \notin Q \end{cases}$



has oscillatory type discontinuity $\forall\, x \in R$


★ Removable type
Isolated point
RHL
LHL
Equal ≠ f(a)
& finite
Missing point
LHL
RHL
Equal but f(a)
is not def.
★ Non Removable type
① finite type
LHL = finite
RHL = finite
unequal
② Infinite type
③ Oscillatory type

Types of Discontinuity

Type 1 : (Removable discontinuity)

Here $\lim_{x \to a} f(x)$ necessarily exists, but is either not equal to $f(a)$ or $f(a)$ is not defined.

In this case, therefore it is possible to redefine the function in such a manner that $\lim_{x \to a} f(x) = f(a)$ and thus making the function continuous. These discontinuities can be further classified as



(A)
Missing point discontinuity

(B)
Isolated point discontinuity

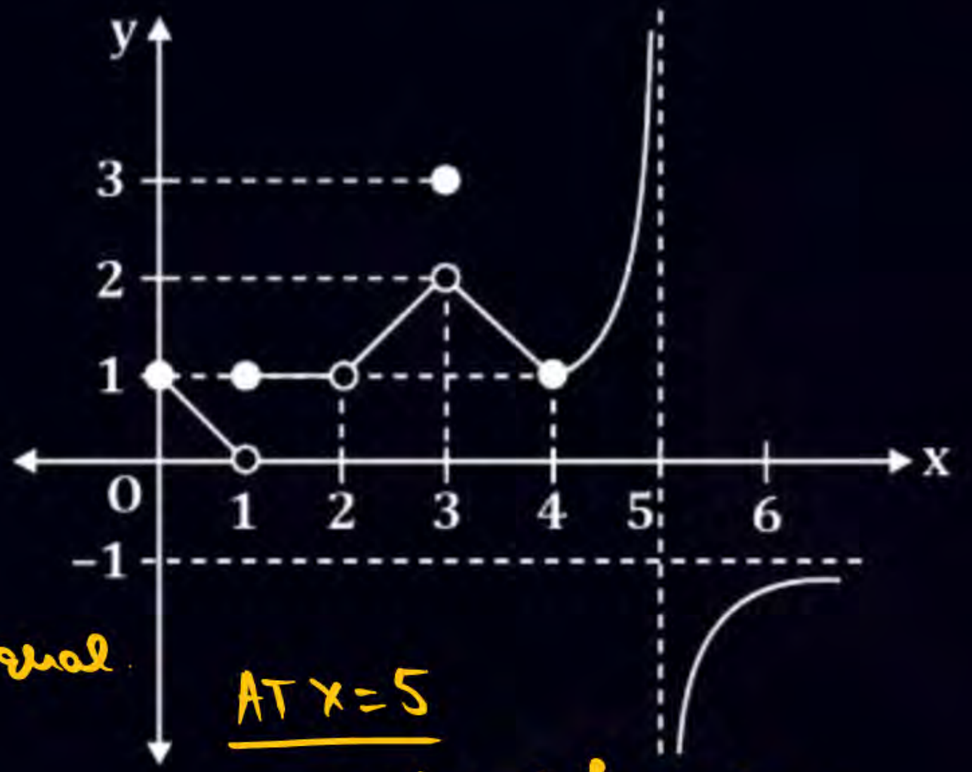
QUESTION

Refer to the following graph of the function f(x) in [0, 6] and match the entries of column-I with the entries of column-II.

	Column-I	Column-II	
(A)	f is continuous (S)	(P)	at x = 1
(B)	f has a removable discontinuity (Missing point) (Q)	(Q)	at x = 2
(C)	f has non removable discontinuity (finite) (P)	(R)	at x = 3
(D)	f has removable discontinuity (isolated point) (R)	(S)	at x = 4

LHL at x=2 is 1
RHL at x=2 is 1
But f(2) N.D.

b'coz at x=1 LHL=0, RHL=1 → unequal



AT x=5

we have infinite type

QUESTION

Tah!

Consider the function $f(x) = \begin{cases} \frac{x}{[x]} & \text{if} \quad 1 \le x < 2 \\ 1 & \text{if} \quad x = 2 \\ \sqrt{6-x} & \text{if} \quad 2 < x \le 3 \end{cases}$

Where $[x]$ denotes step up function then at $x = 2$ function-

A has missing point removable discontinuity

B has isolated point removable discontinuity

C has non removable discontinuity finite type

D is continuous

QUESTION

Consider $f(x) = \begin{cases} x[x]^2 \log_{(1+x)} 2, & \text{for } -1 < x < 0 \\ \dfrac{\ln\left(e^{x^2} + 2\sqrt{\{x\}}\right)}{\tan\sqrt{x}}, & \text{for } 0 < x < 1 \end{cases}$, where [*] & {*} are the greatest integer function & fractional part function respectively, then :

(A) $f(0) = \ln 2 \Rightarrow f$ is continuous at $x = 0$

(B) $f(0) = 2 \Rightarrow f$ is continuous at $x = 0$

(C) $f(0) = e^2 \Rightarrow f$ is continuous at $x = 0$

(D) f has an irremovable discontinuity at $x = 0$

@ $x=0$

LHL $\lim_{x\to 0^-} x[x]^2 \log_{1+x} 2$

$= \lim_{x\to 0^-} \dfrac{x[x]^2 \ln 2}{\ln(1+x)}$

$= \lim_{x\to 0^-} \dfrac{x}{\ln(1+x)} \cdot [x]^2 \cdot \ln 2$

$= 1 \cdot (-1)^2 \cdot \ln 2$

$= +\ln 2$.

RHL

$$\lim_{x\to 0^+} \frac{\ln\left(e^{x^2}+2\sqrt{[x]}\right)}{\tan\sqrt{x}}$$

$$\lim_{x\to 0^+} \frac{\ln\left(1+e^{x^2-1}+2\sqrt{x-[x]}\right)}{\tan\sqrt{x}} = \lim_{x\to 0^+} \frac{\ln\left(1+e^{x^2-1}+2\sqrt{x}\right)}{\tan\sqrt{x}}$$

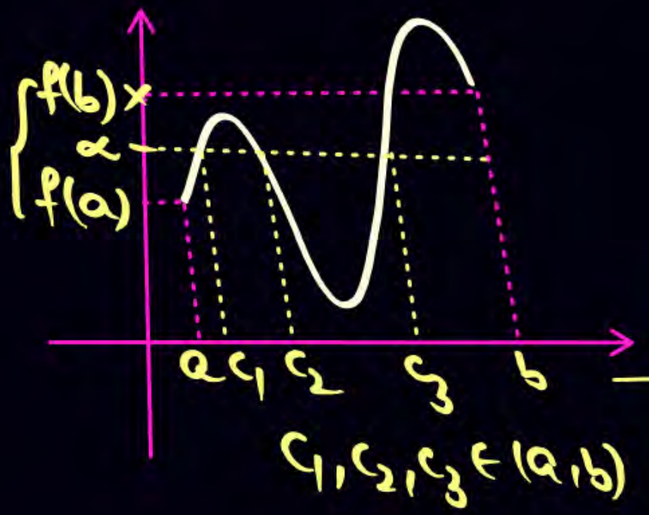
$$= \lim_{x\to 0^+} \frac{\ln\left(1+e^{x^2-1}+2\sqrt{x}\right)}{e^{x^2-1}+2\sqrt{x}} \cdot \frac{e^{x^2-1}+2\sqrt{x}}{\tan\sqrt{x}} \qquad (e^{x^2-1}\to 0)$$

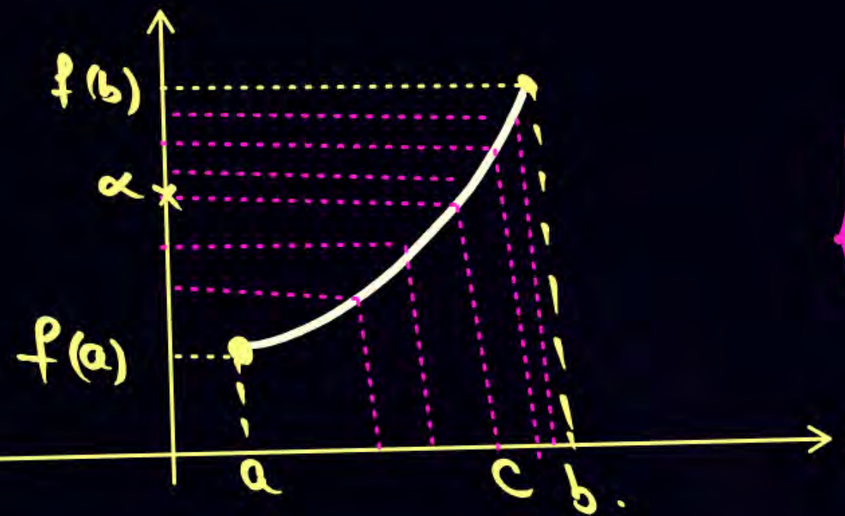
$$= \lim_{x\to 0^+} (\quad)\left(\frac{e^{x^2-1}}{x^2}\cdot\frac{x^{3/2}}{\tan\sqrt{x}}\cdot\sqrt{x} + \frac{2\sqrt{x}}{\tan\sqrt{x}}\right)$$

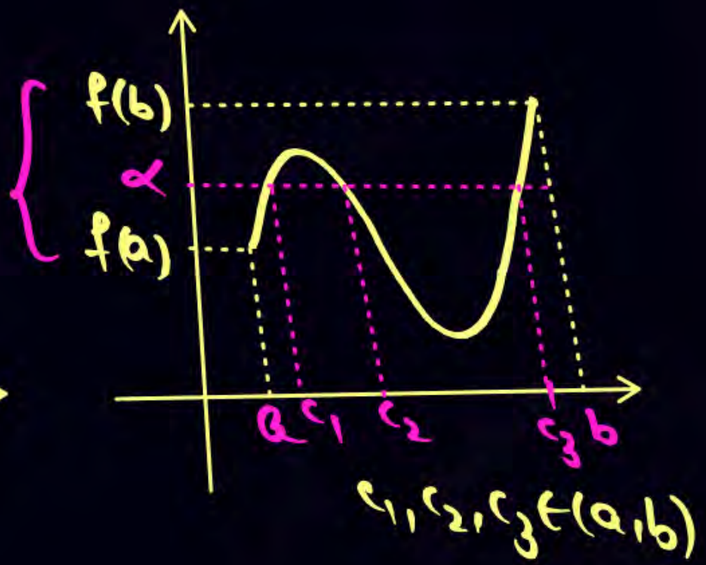
$$= 1\cdot(0+2) = 2.$$

Intermediate Value Theorem

IVT: If a fn 'f' is continuous in $[a,b]$ & $f(a) \neq f(b)$ then for any 'α' lying b/w $f(a)$ & $f(b)$ there exist at least one $x = c \in (a,b)$ s.t $f(c) = \alpha$.

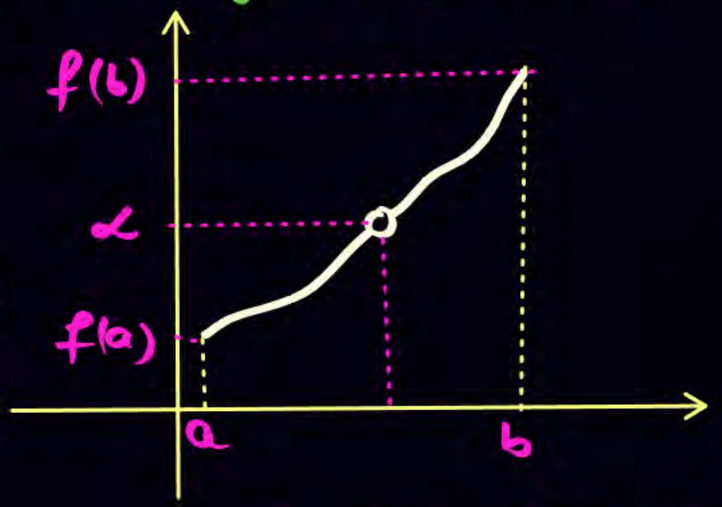




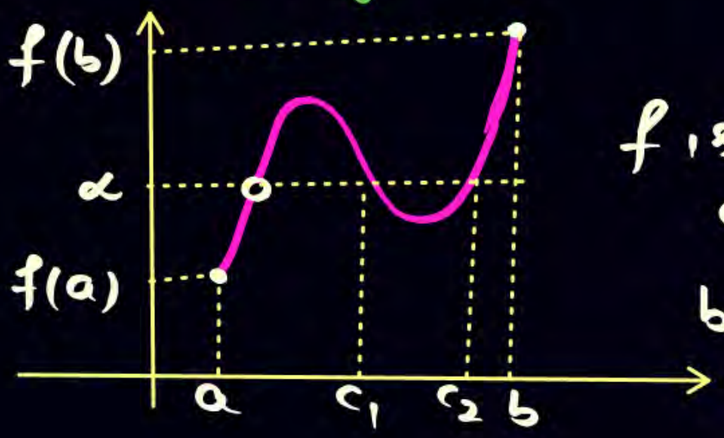


❤ Continuity throughout $[a,b]$ is necessary for IVT to hold

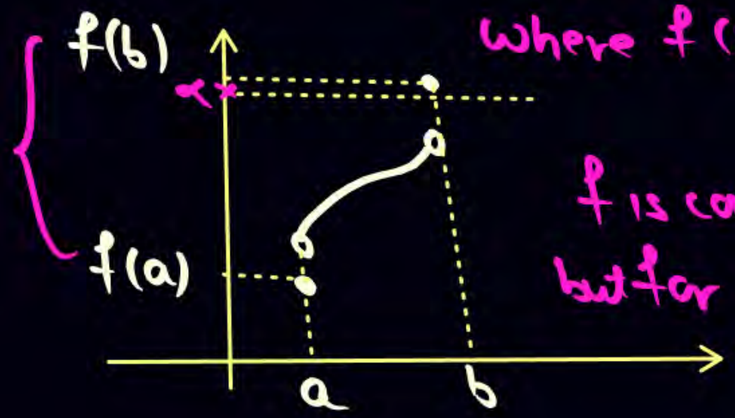
f is not continuous on $[a,b]$



$\exists$ no $x=c\in(a,b)$ where $f(c)=\alpha$

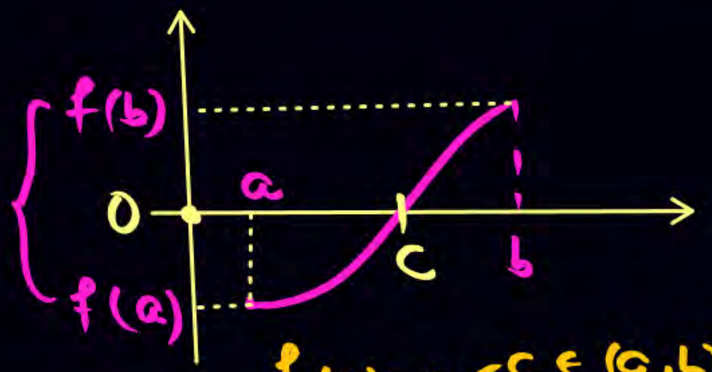


f is not continuous on $[a,b]$ but for $f(x)=\alpha$ we get $c_1, c_2 \in (a,b)$

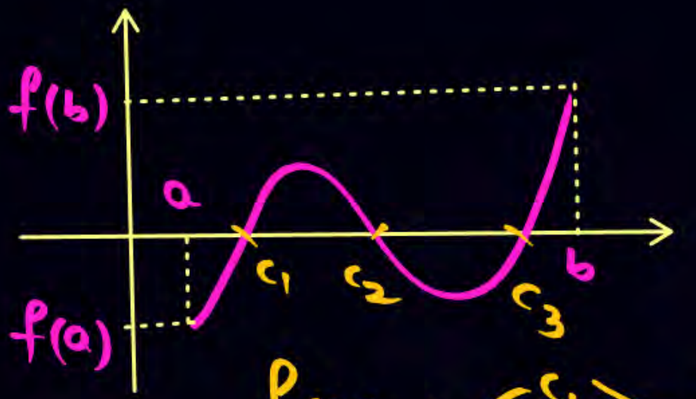


f is continuous on (a,b) but for $f(x)=\alpha$ there is no $x=c\in(a,b)$

If f is continuous on $[a,b]$ & $f(a), f(b)$ are of opposite sign i.e $f(a)\cdot f(b)<0$ then the equation $f(x)=0$ has atleast one root $x=c \in (a,b)$



$f(x)=0$ — $c \in (a,b)$



$f(x)=0 \Leftarrow c_1, c_2, c_3 \Rightarrow$ lie in (a,b)

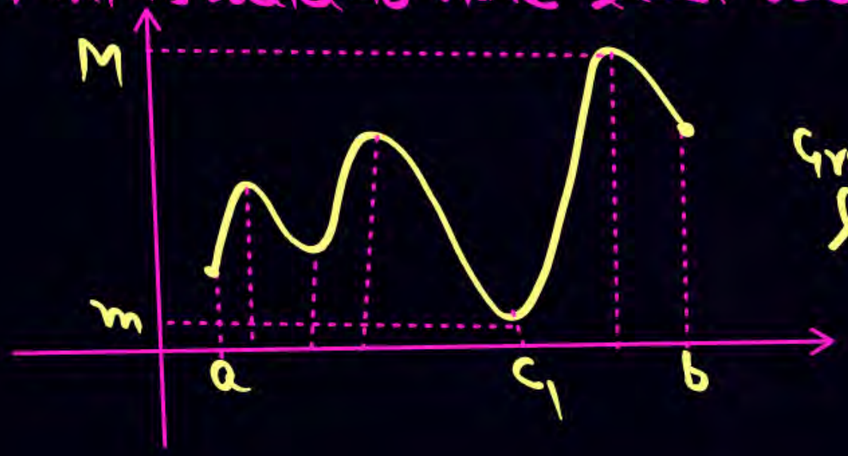
Every odd degree polynomial has atleast one real root

Global MAX / Absolute MAX / Greatest value of a function

A fn $y = f(x)$ is said to have greatest value at $x = c$ if $f(c) \geq f(x) \ \forall \ x \in D_f$

Global Min / Absolute Min / least value of fn.

A fn $y = f(x)$ is said to have least value at $x = c$ if $f(c) \leq f(x) \ \forall \ x \in D_f$



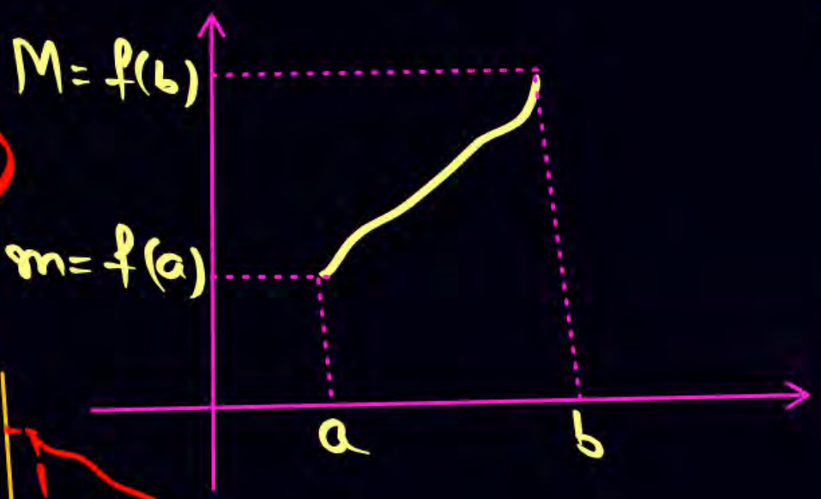
Greatest value = M
least value = m.

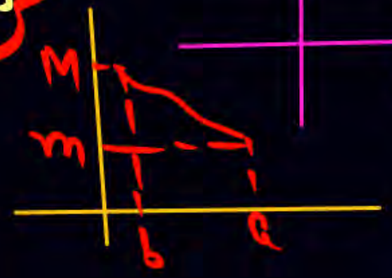
Extreme Value Theorem (EVT)

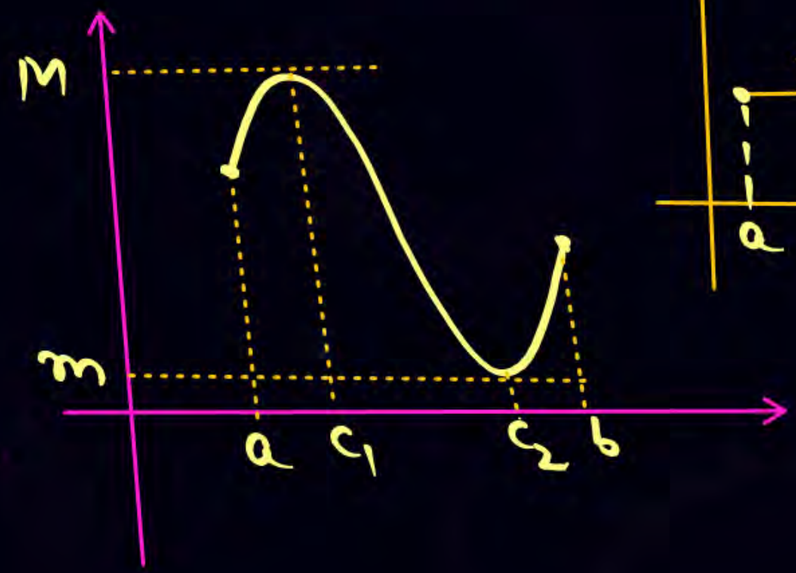
agar fun close pe defined hai to range bhi close pe hi defined hogi

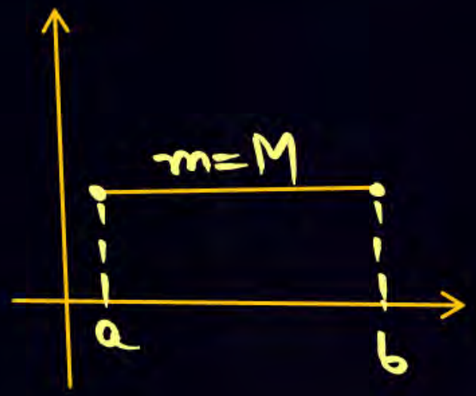
If a fn is continuous on $[a,b]$ then f shall attain its greatest value M & least value m at some points of $[a,b]$

f is continuous in $[a,b]$ then Range of f is $[m,M]$



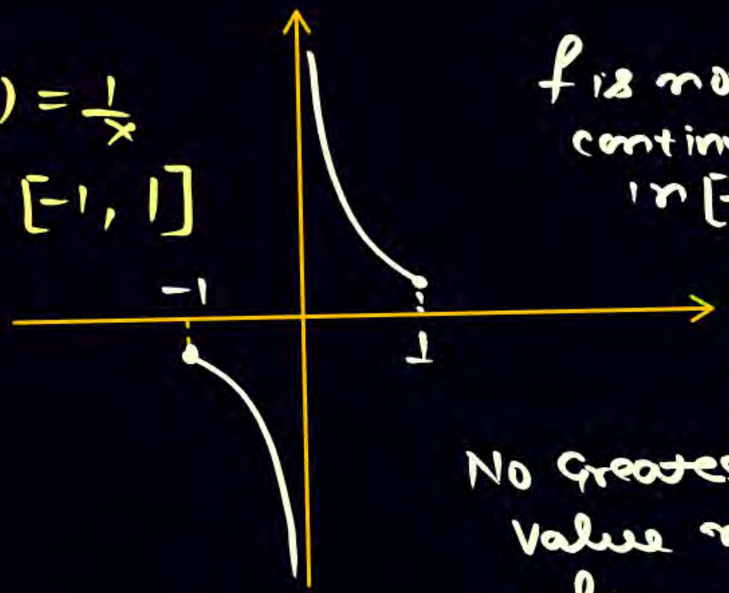






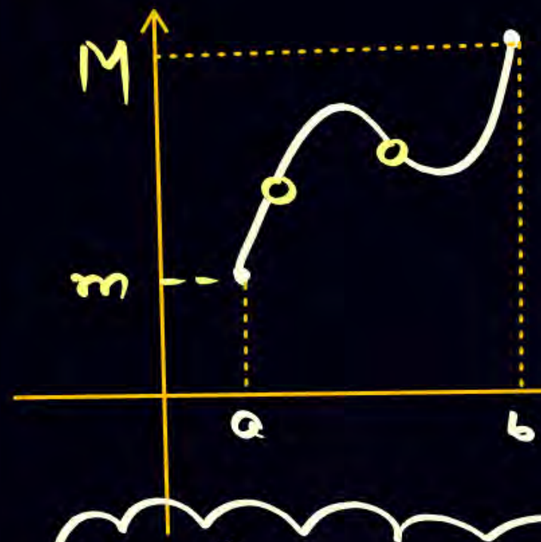
Continuity in [a, b] is necessary for EVT to Hold

$y = f(x) = \frac{1}{x}$ in $[-1, 1]$



f is not continuous in $[-1, 1]$

No Greatest Value no least value



f is not continuous on [a, b] but still greatest & least value Exist on Interval

If a fn is continuous on [a, b] then it is bounded on the interval with Range $[m, M]$

$f: [0,10] \longrightarrow (1,11)$ if f is continuous & Surjective

then no: of possible f's

A) 1 (B) 2 C) Infinite ~~D) 0~~.

since f is onto $\Rightarrow$ Range = codomain $= (1,11)$

But since f is continuous on $[0,10]$

the range will a closed interval $[m,M]$

$\Downarrow$

No such f Exists.

Golden Points

1. **Boundedness Theorem:** Let $I : [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then f is bounded on I.

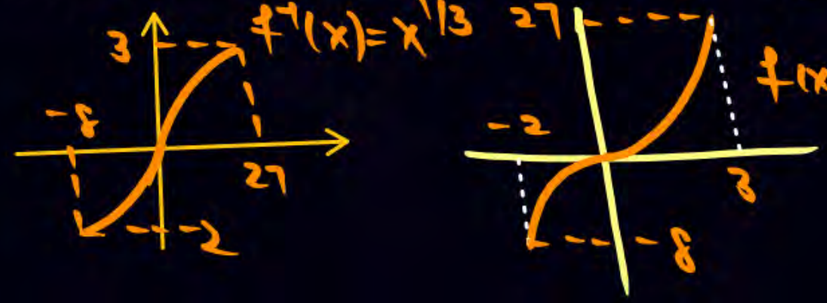
2. If $f : [a, b] \to [m, M]$ be continuous and monotonic then $f^{-1} : [m, M] \to [a, b]$ is also continuous f monotonic.

Range = codomain

Inc / Dec $\Rightarrow$ f is one-one

Ex: $f : [-2, 3] \to [-8, 27]$

$f(x) = x^3$ $f(x) = x^3$.



QUESTION



If $f : [a, b] \to \mathbb{R}$ is continuous and has only rational [respectively, irrational] values, must f be constant? Prove your assertion.

$f: [a,b] \to R$ & $\forall x \in [a,b]$, $f(x) \in \mathbb{Q}$

Proof: let on the contrary f not be a constant fn.

$\Rightarrow \exists\ x_1 \neq x_2$ s.t $f(x_1) \neq f(x_2)$ for some $x_1, x_2 \in [a,b]$

$f(x_1) = r_1$
$f(x_2) = r_2$ } Rational



But f is continuous in $[a,b] \Rightarrow f$ is continuous in $[x_1, x_2]$

& $f(x_1) \neq f(x_2)$ i.e $r_1 \neq r_2$ let i be ano: b/w $f(x_1)$ & $f(x_2)$ where $i \notin \mathbb{Q}$

then By IVT $\exists$ some $c \in (x_1, x_2)$ s.t $f(c) = i \notin \mathbb{Q}$ ✗ Contradiction.

QUESTION [IIT-JEE 1997]

Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$, then $f(1.5) =$ 10.

QUESTION

Suppose that $f(x)$ is continuous in $[0,1]$ and $f(0)=0, f(1)=0$. Prove that $f(c)=1-2c^2$ for some $c \in (0,1)$.

Consider $g(x) = f(x) - (1-2x^2)$. $\forall\ x \in [0,1]$

(f continuous, $1-2x^2$ continuous)

$\Downarrow$

g is continuous in $[0,1]$

$g(0) = f(0) - 1 = 0 - 1 = -1$

$g(1) = f(1) - (1-2\cdot 1^2) = 0 - (1-2) = 1$

$\Downarrow$

$g(0)\cdot g(1) < 0$ & g is continuous on $[0,1]$

$\Rightarrow$ By IVT $\exists$ atleast one $x = c \in (0,1)$ s.t $g(c) = 0$

i.e $f(c)-(1-2c^2)=0$

$f(c)=1-2c^2$

for some $c \in (0,1)$

QUESTION

In which of the following cases the given equations has atleast one root in the indicated interval ?

(A) $x - \cos x = 0$ in $(0, \pi/2)$

(B) $x + \sin x = 1$ in $(0, \pi/6)$

(C) $\frac{a}{x-1} + \frac{b}{x-3} = 0, a, b > 0$ in $(1, 3)$

(D) $f(x) - g(x) = 0$ in (a, b) where f and g are continuous on $[a, b]$ and $f(a) > g(a)$ & $f(b) < g(b)$

(A) $f(x) = x - \cos x$, $x \in [0, \pi/2]$ — continuous

$f(0) = -1$, $f(\pi/2) = \pi/2$

$f(0) \cdot f(\pi/2) < 0$ By IVT $\exists$ some $c \in (0, \pi/2)$ s.t $f(c) = 0$

$c - \cos c = 0$ for some $c \in (0, \pi/2)$

(B) $f(x) = x + \sin x - 1$ $x \in [0, \pi/6]$

$f(0) = -1$, $f(\pi/6) = \pi/6 + 1/2 - 1 = \pi/6 - 1/2 = \frac{\pi - 3}{6} > 0$.

$f(0) \cdot f(\pi/6) < 0 \Rightarrow \exists\, c \in (0, \pi/6)$ where $f(c) = 0$

$h(x) = f(x) - g(x)$ in $x \in [a, b]$

© Consider $f(x) = a(x-3) + b(x-1)$ $\quad a, b > 0$, $x \in [1,3]$

↳ continuous on $[1,3]$

$f(1) = -2a < 0$

$f(3) = 2b > 0$

$\Rightarrow f(1) \cdot f(3) < 0$

$\Downarrow$

By IVT $\exists$ atleast one $c \in (1,3)$

s.t $f(c) = 0$

$a(c-3) + b(c-1) = 0$

Divide by $(c-1)(c-3)$

$\frac{a}{c-1} + \frac{b}{c-3} = 0$

Single point Continuity.

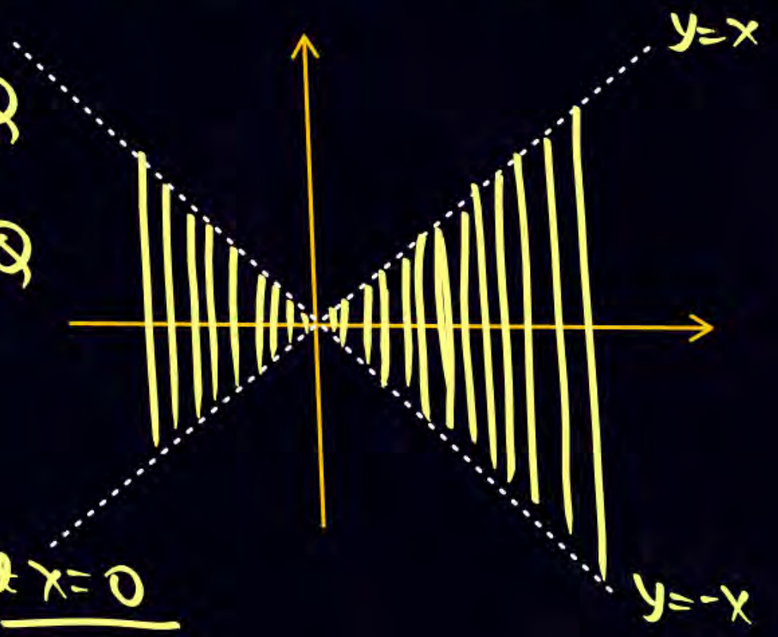
A fn which is discontinuous on entire number line except a single point where it becomes continuous is said to exhibit single point continuity.

Ex: $f(x)=\begin{cases} x & x\in Q \\ -x & x\notin Q \end{cases}$

$\lim_{x\to 0} f(x)=0=f(0)$

$\Downarrow$

f is continuous at $x=0$



Single Point Continuity

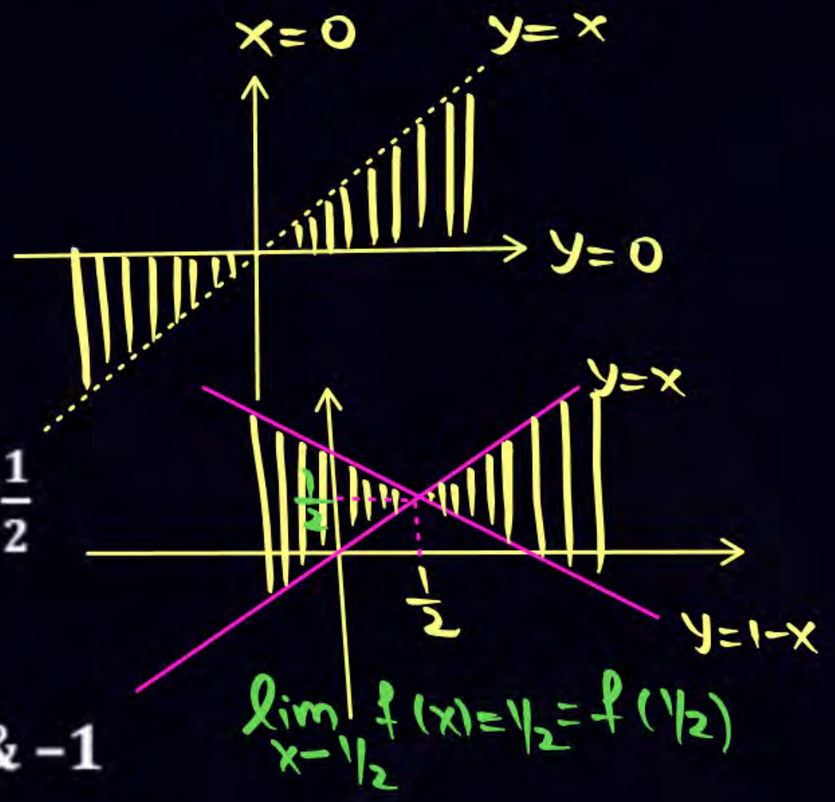
Functions which are continuous only at one point are said to exhibit single point continuity

Examples :

(1) $f(x) = \begin{cases} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$ continuous only at $x = 0$

(2) $f(x) = \begin{cases} x & \text{if } x \in Q \\ 1 - x & \text{if } x \notin Q \end{cases}$ continuous only at $x = \frac{1}{2}$

(3) $f(x) = \begin{cases} x^2 & \text{if } x \in Q \\ 1 & \text{if } x \notin Q \end{cases}$ continuous only at $x = 1$ & -1





$$\lim_{x \to 1} f(x) = 1 = f(1)$$

$$\lim_{x \to -1} f(x) = 1 = f(-1)$$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 09

$$\lim_{x \to \frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x + \sin x)}{\left(2 \sin 2x \sin \frac{3x}{2} + \cos \frac{5x}{2}\right) - \left(\sqrt{2} + \sqrt{2} \cos 2x + \cos \frac{3x}{2}\right)}$$

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

Let $f : R \to R$ be defined as $f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$

If $f(x)$ is continuous on R, then $a + b$ equals :

A -3

B -1

C 3

D 1

Ans. B

QUESTION

(KTK 2)

Let $f : R \to R$ be defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e \left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , \quad x \neq 0 \\ \alpha & , \quad x = 0 \end{cases}$

If f is continuous at $x = 0$, then α is equal to :

A 1

B 3

C 0

D 2

Ans. A

QUESTION

(KTK 3)

Let a function $f: R \to R$ be defined as $f(x) = \begin{cases} \sin x - e^x & \text{if } x \leq 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \geq 1 \end{cases}$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous on R, them $(a + b)$ is equal to :

A 4

B 3

C 2

D 5

Ans. B

QUESTION

(KTK 4)

Let $f : R \to R$ be defined as, $f(x) = \begin{cases} \frac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$ where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to:

A $e(-e+1)$

B $e(e-2)$

C 1

D $2e-1$

Ans. A

QUESTION

(KTK 5)

If the function $f(x) = \begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x} & , \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k & , \quad x = 0 \end{cases}$

is continuous at x = 0, then k is equal to :

A 1

B – 1

C e

D 0

Ans. A

QUESTION

(KTK 6)

Let $f(x) = \frac{\sqrt{\log(x^2+kx+k+1)}}{x^2+k}$.

If f(x) is continuous for all $x \in R$, then the range of k is

A $(-\infty, 0) \cup [4, \infty)$

B $(0, 4]$

C $[0, 4)$

D $[0, 4]$

Ans. B

QUESTION

(KTK 7)

Consider the function defined on $[0, 1] \to R, f(x) = \frac{\sin x - x \cos x}{x^2}$ if $x \neq 0$ and $f(0) = 0$, then the function f(x)-

A has a removable discontinuity at x = 0

B has a non removable finite discontinuity at x = 0

C has a non removable infinite discontinuity at x = 0

D is continuous at x = 0

Ans. D

QUESTION

(KTK 8)

Which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at $x = 0$?

A $f(x) = \frac{1}{1+2^{\cot x}}$

B $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

C $f(x) = x \sin\frac{\pi}{x}$

D $f(x) = \frac{1}{\ln|x|}$

Ans. B, C, D

QUESTION

(KTK 9)

Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x)-\log_e(1+\alpha x)}{x}; & \text{if } x \neq 0 \\ 10; & \text{if } x = 0 \end{cases}$ be continuous at $x = 0$.

Then α is equal to

A 10

B – 10

C 5

D – 5

Ans. D

QUESTION

(KTK 10)

Let $[x]$ be the greatest integer less than or equals to x . Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous?

A $x = -1$

B $x = 1$

C $x = 0$

D $x = 2$

Ans. A, B, D

QUESTION

(KTK 11)

If $\lim_{x\to\infty}\left[1+x\ln\left(1+b^2\right)\right]^{1/x}=2b\sin^2\theta, b>0$ and $\theta\in(-\pi,\pi]$, then the value of θ is

A $\pm\frac{\pi}{4}$

B $\pm\frac{\pi}{3}$

C $\pm\frac{\pi}{6}$

D $\pm\frac{\pi}{2}$

Ans. D

QUESTION

(KTK 12)

Let m and n be two positive integers greater than 1 . If

$$\lim_{\alpha \to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$$ then the value of $\frac{m}{n}$ is

Ans. 2

QUESTION

(KTK 13)

Let $f: R \to R$ be defined as $f(x) = \begin{cases} [e^x], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + [\sin(\pi x)], & 1 \le x < 2 \\ [e^{-x}] - c, & x \ge 2 \end{cases}$

where $a, b, c \in R$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true?

(A) There exists $a, b, c \in R$ such that f is continuous on R.

(B) If f is discontinuous at exactly one point, then $a + b + c = 1$

(C) If f is discontinuous at exactly one point, then $a + b + c \neq 1$

(D) f is discontinuous at at least two points, for any values of a, b and c

Ans. C

QUESTION

KTK 14

Let $f:(0,\pi) \to R$ be a function given by $f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$

where $a, b \in Z$. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to

Ans. 81

Homework From Module

LCD

PRARAMBH : Q1 TO Q43
PRABAL : Q1 TO Q39
PARIKSHIT : Q1 TO Q8

(Revision Practice Problems)

QUESTION

(RPP 1)

Let $f(x) = x^2 - 2x - 8$ where $x \in R$. Then number of distinct real roots of $|f(|x|)| = 8$ is

A 3

B 4

C 5

D 6

Ans. C

QUESTION

(RPP 2)

Let $f(x) = x^2 - 6x + 8$, then number of real roots of the equation $f(f(x)) = 0$ is

A 4

B 2

C 1

D 0

Ans. A

QUESTION

(RPP 3)

Paragraph

Consider, $f(x) = 3(|a^2 - 3a - 2| + 4)x^2 - 12x + 4, a \in R$. Let α, β be the zeros of the polynomial $f(x)$.

If the maximum value of $f(x)$ is $g(a)$ for $0 \leq x \leq 2$ then minimum value of $g(a)$ is

A 1

B 4

C 28

D 32

Ans. C

(Solution to TAH)

QUESTION

TAH 01

Evaluate: $\lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+4}+\frac{9}{n^3+9}+\cdots+\frac{n^2}{n^3+n^2}\right)$

Ans. 1/3

Evaluate :

$$\lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+4}+\frac{9}{n^3+9}+\cdots+\frac{n^2}{n^3+n^2}\right)$$

$\hookrightarrow f(n)$

$$\Rightarrow \frac{1}{n^3+n^2}+\frac{4}{n^3+n^2}+\cdots+\frac{n^2}{n^3+n^2} \leq f(n) \leq \frac{1}{n^3+1}+\frac{4}{n^3+1}+\frac{9}{n^3+1}+\cdots+\frac{n^2}{n^3+1}$$

$$\frac{n(n+1)(2n+1)}{6(n^3+n^2)} \leq f(n) \leq \frac{n(n+1)(2n+1)}{6(n^3+1)}$$

$$\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6(n^3+n^2)} \leq \lim_{n\to\infty} f(n) \leq \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6(n^3+1)}$$

$$\frac{1}{3} \leq \lim_{n\to\infty} f(n) \leq \frac{1}{3}$$

BY SANDWICH T.Hm

$$\lim_{n\to\infty} f(n) = \frac{1}{3} \quad \text{(Ans)}$$

TAH 1(B)
ABHIJEET KUMAR PATRA
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(JATNI)

QUESTION [JEE Mains 2024 (8 April)]

TAH 02

For $a, b > 0$, let $f(x) = \begin{cases} \frac{\tan((a+1)x)+b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$ be a continuous function at $x = 0$. Then $\frac{b}{a}$ is equal to:

(A) 4

(B) 5

(C) 8

(D) 6

Ans. D

JAH-2) ⇒ For $a, b > 0$, let $f(x) = \begin{cases} \frac{\tan(a+1)x + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$ be continuous function at $x = 0$, then $\left(\frac{b}{a}\right)$ is -

$\Rightarrow$ $\because \lim_{x \to 0} f(x) = f(0) = 3$.

RHL $\Rightarrow \lim_{x \to 0^+} \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{ax}\sqrt{x}} = 3$.

$\Rightarrow \lim_{x \to 0^+} \frac{ax + b^2x^2 - ax}{b\sqrt{ax}^{3/2}\left(\sqrt{ax+b^2x^2} + \sqrt{ax}\right)}$

$\Rightarrow \lim_{x \to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x} + \sqrt{a}\right)}$

$\Rightarrow \frac{b}{\sqrt{a} \cdot 2\sqrt{a}} = 3 \Rightarrow \frac{b}{2a} = 3$

$\therefore \boxed{\frac{b}{a} = 6}$ Ans

Nandini Mishra

Tah 2

M-29

Tah-2>

for $a, b > 0$, $f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \dfrac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{a}\, x\sqrt{x}}, & x > 0 \end{cases}$ be a continuous fn at $x=0$ Then $\frac{b}{a}$ is equal to

function is continuous at $x=0$

RHL: $\lim\limits_{x\to 0^+} \dfrac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{a}\cdot x\sqrt{x}}$

$= \lim\limits_{x\to 0^+} \dfrac{ax+b^2x^2-ax}{bx\sqrt{ax}} \dfrac{1}{\sqrt{ax+b^2x^2}+\sqrt{ax}}$

$= \lim\limits_{x\to 0^+} \dfrac{bx}{\sqrt{a}\cdot x} \cdot \dfrac{1}{(\sqrt{a^2+b^2x}+\sqrt{a})}$

$= \dfrac{b}{\sqrt{a}\cdot(2\sqrt{a})}$

$= \dfrac{b}{2a}$

tah2
kshitij from up

$$LHL: \lim_{x\to 0^-} \frac{\tan(a+1)x + b\tan x}{x} = \lim_{x\to 0^-} \frac{\tan(a+1)x}{(a+1)x}(a+1) + b\frac{\tan x}{x}$$

$$= (a+1) + b$$

$$= a+b+1$$

at $x = 0$,

$$RHL = LHL = f(0)$$

$$\frac{b}{2a} = a+b+1 = 3.$$

$$\frac{b}{a} = 6 \quad \underline{\underline{Ans}}$$

QUESTION [JEE Mains 2024 (27 Jan)]

TAH 03

Consider the function, $f(x)=\begin{cases}\frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x<3 \\ 2^{\frac{\sin(x-3)}{x-|x|}}, & x>3 \\ b, & x=3,\end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x=3$, then the number of elements in S is:

(A) Infinitely many

(B) 4

(C) 2

(D) 1

Ans. D

3) Consider the fn $f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x \leq 3 \\ 2\frac{\sin(x-3)}{x-[x]}, & x > 3 \\ b, & x = 3 \end{cases}$

Where $[x]$ denotes the greatest integer fn, If S denotes the set of all ordered pairs (a,b) such that $f(x)$ is continuous at $x=3$, then the number of Elements in S is:

TAH 3

ABHIJEET KUMAR PATRA

ODISHA

(JATNI)

$\Rightarrow$ RHL, $\lim_{x\to 3^+} 2^{\frac{\sin(x-3)}{x-[x]}} = \lim_{x\to 3^+} 2^{\frac{\sin(x-3)}{\{x\}}}$

$\Rightarrow \lim_{h\to 0} 2^{\frac{\sin(h+3-3)}{3+h-3}} = 2^1 = 2$

LHL, $\lim_{x\to 3^-} \frac{a(7x-12-x^2)}{b|x^2-7x+12|} = \frac{-a(x^2-7x+12)}{b(x^2-7x+12)}$

(+ve)

$= \frac{-a}{b}$

Now, RHL at $x=3$ = LHL at $x=3$ = $f(3)$

$\Rightarrow 2 = \frac{-a}{b} = b$

So, $b = 2$ & $a = -4$

From, $(-4, 2)$ are only one ordered pair (Answer)

QUESTION [JEE Mains 2024 (4 April)]

TAH 04

If the function $f(x)=\begin{cases}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, & x \neq 0 \\ a \log_e 2 \log_e 3, & x=0\end{cases}$ is continuous at $x=0$, then the value of a^2 is equal to

(A) 968

(B) 1250

(C) 1152

(D) 746

Ans. C

tah-4

$$f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, & x\neq 0\\ a\log_{e}2\cdot\log_{e}3, & x=0\end{cases}$$

is continuous at $x=0$ then value of a^2 is

$f(x)$ is continuous at $x=0$

tah4
kshitij from up

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to 0}\frac{9^x(8^x-1)-(8^x-1)}{(1-\cos x)}\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$=\lim_{x\to 0}\frac{\frac{(8^x-1)}{x}\frac{(9^x-1)}{x}}{\frac{1-\cos x}{x^2}}\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$=\frac{\ln 8\cdot\ln 9}{1/2}2\sqrt{2} = 4\sqrt{2}\ln 2^3\cdot\ln 3^2$$

$$= 24\sqrt{2}\ln 2\ln 3$$

If the fn $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}} & , x \neq 0 \\ a \log_e 2 \cdot \log_e 3 & , x = 0 \end{cases}$

continuous at $x = 0$, then the value of a^2 is Equal to

$$\lim_{x \to 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}$$

$$\lim_{x \to 0} \frac{9^x(8^x - 1) - (8^x - 1)}{(1 - \cos x)} \cdot (\sqrt{2} + \sqrt{1+\cos x})$$

$$\lim_{x \to 0} \frac{\frac{(8^x - 1)}{x} \cdot \frac{(9^x - 1)}{x}}{\frac{(1 - \cos x)}{x^2}} (\sqrt{2} + \sqrt{1+\cos x})$$

$$= \frac{\log 8 \log 9}{\frac{1}{2}} 2\sqrt{2} \Rightarrow 24\sqrt{2} \log 2 \cdot \log 3$$

TAH 4
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

Now, $\lim_{x \to 0} f(x) = f(0)$

$24\sqrt{2} \log_e 2 \log_e 3 = a \log_e 2 \log_e 3$

$\Rightarrow a = 24\sqrt{2}$

So, $a^2 = 576 \times 2 = 1152$

QUESTION [JEE Mains 2024 (4 April)]

TAH 05

Let $f: R \to R$ be a function given by $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$

where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

(A) 48

(B) 6

(C) 3

(D) 12

Ans. D

Let $f: R \to R$ be a fn given by $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x<0 \\ \alpha, & x=0 \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x>0 \end{cases}$

where $\alpha, \beta \in R$. If f is continuous at $x=0$, then $\alpha^2+\beta^2$ is equal to:

$\Rightarrow$ LHL,

$$\lim_{x\to 0^-} \frac{1-\cos 2x}{4x^2} \cdot 4 = \frac{1}{2} \cdot 4 = 2$$

RHL,

$$\lim_{x\to 0^+} \frac{\beta\sqrt{2}\sin\frac{x}{2}}{2\cdot\frac{x}{2}} = \frac{\beta}{\sqrt{2}}$$

Now, $\underset{at\,x=0}{LHL} = \underset{at\,x=0}{RHL} = f(0)$

$\Rightarrow 2 = \frac{\beta}{\sqrt{2}} = \alpha$

So, $\alpha = 2$ & $\beta = 2\sqrt{2}$

Therefore, $\alpha^2+\beta^2 = 4+8 = 12$ (Ans)

TAH 5
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

TAH-5

Q Let $f: R \to R$ be a function given by $f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x<0 \\ \alpha, & x=0 \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x>0 \end{cases}$

where $\alpha, \beta \in R$. If f is continuous at $x=0$, then $\alpha^2+\beta^2$ is equal to

Solve → $f(x)$ is continuous at $x=0$

RHL $\lim_{x\to 0^+} \frac{\beta\sqrt{1-\cos x}}{x}$

$\lim_{x\to 0^+} \frac{\beta\sqrt{2}\sin\frac{x}{2}}{2\cdot\frac{x}{2}} = \frac{\beta}{\sqrt{2}}$

LHL $\lim_{x\to 0^-} \frac{1-\cos 2x}{4x^2}\cdot 4 = 2$

RHL at $x=0$ = LHL at $x=0$ = $f(0)$

$\frac{\beta}{\sqrt{2}} = 2 = \alpha$

$\alpha = 2$, $\beta = 2\sqrt{2}$

$\alpha^2+\beta^2$
$= 4+8$
$= 12$ Ans

TAH 5 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION [JEE Mains 2023 (25 Jan)]

TAH 06

If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} & , 0 < x < \frac{\pi}{2} \\ \mu & , x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}} & , \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(A) 11

(B) 10

(C) 8

(D) $2e^4 + 8$

TAH-6

Q If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0 < x < \frac{\pi}{2} \\ \mu & x = \frac{\pi}{2} \\ e^{\frac{\cot 4x}{\cot 2x}} & \frac{\pi}{2} < x < \pi \end{cases}$

is continuous at $x = \frac{\pi}{2}$

then find $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

Solve → $f(x)$ is continuous at $x = \frac{\pi}{2}$

RHL $\lim_{x \to \frac{\pi}{2}^+} e^{\frac{\cot 4x}{\cot 2x}} = \lim_{h \to 0} e^{\frac{\cot 4\left(\frac{\pi}{2}+h\right)}{\cot 2\left(\frac{\pi}{2}+h\right)}} = \lim_{h \to 0} e^{\frac{\cot 4h}{\cot 2h}}$

$= \lim_{h \to 0} e^{\frac{\tan 4h}{4h} \cdot 4 \cdot \frac{2h}{\tan 2h} \cdot \frac{1}{2}} = e^{2/3}$

LHL $\lim_{x \to \frac{\pi}{2}^-} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}$

$= \lim_{x \to \frac{\pi}{2}^-} \left((1+|\cos x|)^{\frac{1}{|\cos x|}} \right)^{\lambda} = e^{\lambda}$

RHL at $x = \frac{\pi}{2}$ = LHL at $x = \frac{\pi}{2}$ = $f\left(\frac{\pi}{2}\right)$

$e^{2/3} = e^{\lambda} = \mu$

$\boxed{\lambda = \frac{2}{3}}$, $\boxed{\mu = e^{2/3}}$

$\Rightarrow 9 \times \frac{2}{3} + 6 \log_e e^{2/3} + (e^{2/3})^6 - e^{6 \times \frac{2}{3}}$

$\Rightarrow 6 + 4 = \boxed{10}$ Ans

TAH 6 RAHUL DHAKAD ❤❤ FROM AGRA UP

6) If the function $f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0 < x < \pi/2 \\ \mu, & x = \pi/2 \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$

is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to : (2023)

(A) 11
(B) 10
(C) 8
(D) $2e^4 + 8$

⇒ Since f is continuous at $x = \pi/2 \Rightarrow LHL = RHL = f(\pi/2) = \alpha$

$$\text{AT } x = \pi/2,$$

$$LHL = \lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}$$

$$= e^{\lim_{x \to \pi/2^-} \frac{\lambda}{|\cos x|}(1+|\cos x|-1)}$$

$$= e^{\lim_{x \to \frac{\pi}{2}^-} \frac{\lambda |\cos x|}{|\cos x|}}$$

$$= e^{\lambda}$$

TAH 6
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

$$RHL = \lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} e^{\frac{\cot 6x}{\cot 4x}}$$

$$= \lim_{x \to \frac{\pi}{2}+} e^{\frac{\tan 4x}{\tan 6x}}$$

$$= \lim_{h \to 0} e^{\frac{\tan 4(\pi/2+h)}{\tan 6(\pi/2+h)}}$$

$$= \lim_{h \to 0} e^{\frac{\tan(2\pi+4h)}{\tan(3\pi+6h)}}$$

$$= e^{\lim_{h \to 0} \frac{\tan 4h}{\tan 6h}}$$

$$= e^{\lim_{h \to 0} \frac{\tan 4h}{4h} \cdot \frac{6h}{\tan 6h} \cdot \frac{4}{6} \frac{2}{3}}$$

$$= e^{2/3}$$

$\Rightarrow LHL = RHL = f(\pi/2)$

So, $e^{\lambda} = e^{2/3} = \mu \Rightarrow \lambda = 2/3, \mu = e^{2/3}$

Now,

$$9\lambda + 6\log\mu + \mu^6 - e^{6\lambda}$$

$$= \frac{9 \times 2}{3} + 6\log e^{2/3} + e^{2/3 \cdot 6} - e^{6 \cdot 2/3}$$

$$= 6 + \frac{6^2 \cdot 2}{6}\log e^{1}$$

$$= 6 + 4$$

$$= 10$$

QUESTION [JEE Mains 2021 (March)]

TAH 7

If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is

If the fn $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain & $f(0) = \frac{1}{k}$ then k is :

$$\Rightarrow \lim_{x\to 0} \frac{2\sin\left(\frac{\sin x + x}{2}\right)\cdot \sin\left(\frac{x - \sin x}{2}\right)}{4\left(\frac{\sin x + x}{2}\right)\cdot\left(\frac{x - \sin x}{2}\right)} \cdot \left(\frac{\sin x + x}{2}\right)\left(\frac{x - \sin x}{x^3}\right)$$

$$= \frac{2}{4x} \cdot 2 \cdot \frac{1}{6} = \frac{1}{6}$$

So, $\lim_{x\to 0} f(x) = f(0) = \frac{1}{k} = \frac{1}{6}$

Therefore, $\{k = 6\}$

TAH 7
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ODISHA
(JATNI)

TAH-7

Q If the function $f(x) = \dfrac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$ then find k.

Solution →

$$f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$$

at $x = 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$$

$$= \lim_{x \to 0} \frac{2 \sin\left(\frac{\sin x + x}{2}\right) \cdot \sin\left(\frac{x - \sin x}{2}\right)}{4 \cdot \left(\frac{\sin x + x}{2}\right) \cdot \left(\frac{x - \sin x}{2}\right)} \cdot \frac{(\sin x + x)}{x} \cdot \frac{(x - \sin x)}{x^3}$$

$$= \frac{2}{4} \cdot (1 + 1) \cdot \frac{1}{6} = \frac{1}{6}$$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\frac{1}{6} = \frac{1}{K}$$

$$\boxed{K = 6}$$ Ans

TAH 7 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION [JEE Mains 2021 (July)] TAH 8

Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to R$ be defined as, $f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}}, & -\frac{\pi}{4}<x<0\\ b, & x=0\\ e^{\cot 4x/\cot 2x}, & 0<x<\frac{\pi}{4}\end{cases}$

If f is continuous at $x=0$, then the value of $6a+b^2$ is equal to:

A $1-e$

B $e-1$

C $1+e$

D e

Let $f:\left(\frac{-\pi}{4},\frac{\pi}{4}\right)\to R$ be defined as $f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}}, & \frac{-\pi}{4}<x<0\\ e^{\cot 4x/\cot 2x}, & 0<x<\frac{\pi}{4}\\ b, & x=0\end{cases}$

If f is continous at $x=0$, then the value of $6a+b^2$ is:

TAH 8
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

$\Rightarrow$ RHL, $\lim_{x\to 0^+} e^{\frac{\tan 2x}{\tan 4x}} = e^{\frac{2\cdot\frac{\tan 2x}{2x}}{\frac{\tan 4x\cdot 4}{4x}}} = e^{1/2}$

LHL, $\lim_{x\to 0^-} (1+|\sin x|)^{\frac{3a}{|\sin x|}}$ ↳ 1^∞ $= e^{\lim_{x\to 0^-}\frac{3a}{|\sin x|}(|\sin x|)} = e^{3a}$

So, $\underset{\text{at } x=0}{\text{LHL}} = \underset{\text{at } x=0}{\text{RHL}} = f(0)$

$\Rightarrow e^{3a} = e^{1/2} = f(0) = b$

$\Rightarrow b = e^{1/2}$ & $a = \frac{1}{6}$

From, $6a+b^2 = 1+e$ (Ans)

Tah-8 $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to R$ be defined as $f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & -\frac{\pi}{4}<x<0\\ b & x=0\\ e^{\cot 4x/\cot 2x} & 0<x<\frac{\pi}{4}\end{cases}$

If f is continuous at $x=0$ then the value of $6a+b^2$.

$f(x)$ is continuous at $x=0$.

RHL: $\lim_{x\to 0^+} e^{\frac{\cot 4x}{\cot 2x}}$

$= \lim_{x\to 0^+} e^{\frac{\tan 2x}{2x}\cdot 2\frac{4x}{\tan 4x}\cdot\frac{1}{4}}$

$= e^{\frac{1}{2}}$

LHL: $\lim_{x\to 0^-}\left((1+\beta|\sin x|)^{\frac{1}{|\sin x|}}\right)^{3a}$

$= e^{3a}$

tah8
kshitij from up

at $x=0$

LHL = RHL = $f(0)$

$e^{3a}=e^{\frac{1}{2}}=b$

$a=\frac{1}{6}, b=e^{\frac{1}{2}}$

$6a+b^2=1+e$. Ans

QUESTION

ASRQ+KCLS

TAH 01

Refer to the graph of $y = f(x)$ and $g(x) = (x-2)^2, x < 2$

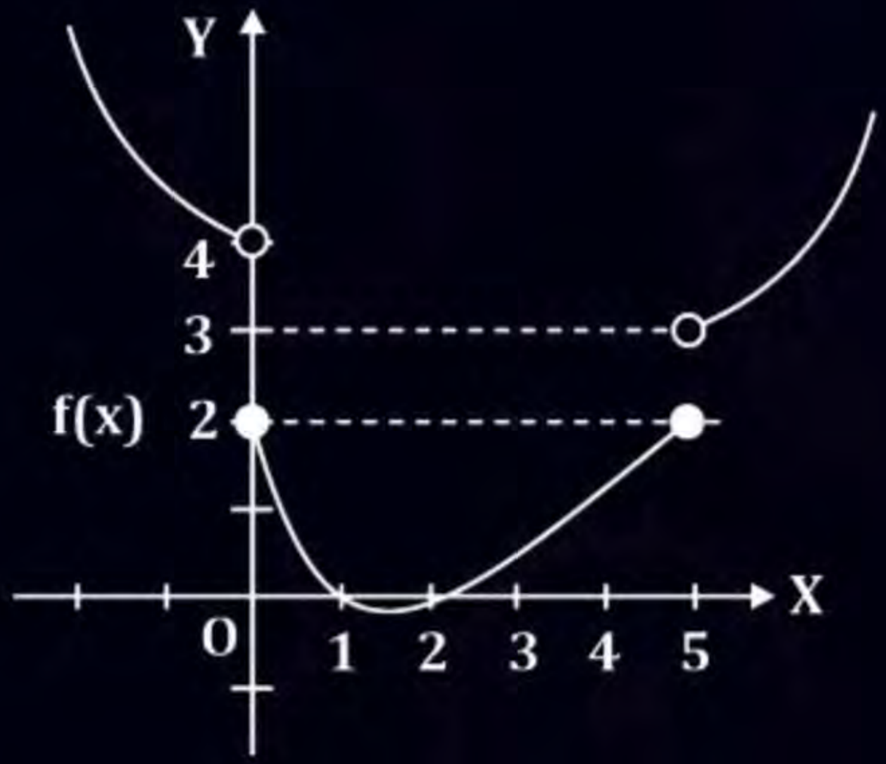
$= 7 - x, x \geq 2$

then which of the following limits are non-existent.

(a) $\lim_{x \to 2} f(g(x))$

(b) $\lim_{x \to 0} g(f(x))$

(c) $\lim_{x \to 5} g(f(x))$

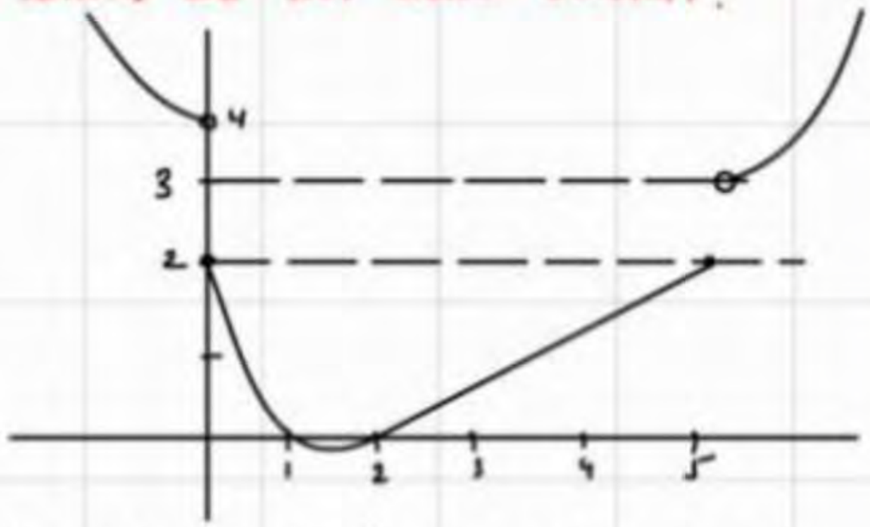


TAH-1

Q AIRQ

Refer to the graph of $y = f(x)$ and $g(x) = (x-2)^2, \; x < 2$
$= 7 - x, \; x \geq 2$

Then which of the following limits are not non-existent.



Solve → (a) $\lim_{x \to 2} f(g(x))$

(A) RHL $\lim_{h \to 0} f(g(2+h)) = \lim_{h \to 0} f(7-(2+h))$

$= \lim_{h \to 0} f(5-h) = 2$

LHL $\lim_{h \to 0} f(g(2-h))$

$\lim_{h \to 0} f((2-h-2)^2)$

$\lim_{h \to 0} f(h^2) = 2$ (root)

$\Rightarrow \lim_{x \to 2} f(g(x)) = 2$ Ans

(b) $\lim_{x\to 0} g(f(x)) \Rightarrow$ RHL $\lim_{h\to 0} g(f(0+h)) = \lim_{h\to 0} g(f(h))$ ($f(h) \to 1^-$)

$= \lim_{h\to 0} (f(h)-2)^2$ ($f(h) \to 0^+$)

$= 0$

LHL $\lim_{h\to 0} g(f(-h))$ ($f(-h) \to 0^-$, $\to 4^+$)

$= \lim_{h\to 0} 7 - (f(-h))$ ($\to 4^+$) $= 7-4 = 3$

TAH 1RAHUL DHAKAD



FROM AGRA UP

$\lim_{x\to 0} g(f(x))$ DNE

(C) $\lim_{x\to 5} g(f(x))$

RHL $\lim_{h\to 0} g(f(5+h))$ ($f(5+h) \to 3^+$)

$= \lim_{h\to 0} 7 - f(5+h)$

$= 7-3 = 4$

LHL $\lim_{x\to 5^-} g(f(x))$

$\lim_{h\to 0} g(f(5-h))$ ($f(5-h) \to 2^-$)

$\lim_{h\to 0} (f(5-h)-2)^2$ ($f(5-h) \to 2^-$)

$= 0$

$\lim_{x\to 5} g(f(x))$ DNE

QUESTION

TAH 02

Let $f(x) = \lim_{n\to\infty} \frac{x^{2n-1}+ax^2+bx}{x^{2n}+1}$.

If $f(x)$ is continuous for all $x \in \mathbb{R}$ then find the values of a and b. $(n \in N)$

Ans. $a = 0, b = 1$

TAH-2

Q Let $f(x) = \lim_{n\to\infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n}+1}$ If $f(x)$ is continuous $\forall\, x \in R$ then find the value of a and b (NCERT)

Solution $f(x) = \lim_{n\to\infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n}+1}$

$f(x) = \lim_{n\to\infty} \frac{\frac{1}{x} + \frac{ax^2}{x^{2n}} + \frac{bx}{x^{2n}}}{1 + \frac{1}{x^{2n}}}$, $x > 1$ or $x < -1$

$\lim_{n\to\infty} \frac{\frac{x^{2n}}{x} + ax^2 + bx}{x^{2n}+1}$, $-1 < x < 1$

$\frac{1+a+b}{2}$, $x = 1$

$\frac{-1+a-b}{2}$, $x = -1$

if $x > 1$ or $x < -1$
$n \to \infty$ $x^{2n} \to \infty$

if $-1 < x < 1$
$n \to \infty$ $x^{2n} \to 0$

if $x = 1$
$n \to \infty$, $x^{2n} = 1$

if $x = -1$
$n \to \infty$, $x^{2n} = 1$

$f(x) = \begin{cases} \frac{1}{x} & , x > 1 \text{ or } x < -1 \\ ax^2 + bx & , -1 < x < 1 \\ \frac{1+a+b}{2} & , x = 1 \\ \frac{a-1-b}{2} & , x = -1 \end{cases}$

$f(x)$ is continuous $\forall\, x \in R$

continuous at $x = 1$ & $x = -1$

at $x = 1$ RHL $\lim_{x\to 1^+} \frac{1}{x} = 1$

TAH 2 RAHUL DHAKAD ❤️❤️ FROM AGRA UP

PART 1

LHL $\lim_{x\to 1^-} ax^2 + bx = a + b$

RHL at $x=1$ = LHL at $x=1$ = $f(1)$

$1 = a + b = \frac{1+a+b}{2}$

$a + b = 1$, $\boxed{a+b=1}$

continuous at $x = -1$

RHL $\lim_{x\to -1^+} ax^2 + bx = a - b$

LHL $\lim_{x\to -1^-} \frac{1}{x} = -1$

RHL at $x=-1$ = LHL at $x=-1$ = $f(-1)$

$a - b = -1 = \frac{a-1-b}{2}$

$\boxed{a-b=-1}$

$2a = 0$

$\boxed{a = 0}$

$\boxed{b = 1}$

Ans

TAH 2 RAHUL DHAKAD ❤️❤️ FROM AGRA UP

PART 2

(Solution to KTK)

QUESTION

(KTK 1)

$$\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)}$$

Ans. $\frac{1}{\sqrt{2}}$

KTK-1

Archita Dwivedi
(from UP)

Q. $\lim_{x \to \pi} \frac{1 - \sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)}$

Soln. $\lim_{x \to \pi} \frac{1 - 2\sin\frac{x}{4}\cos\frac{x}{4}}{\left(\cos^2\frac{x}{4} - \sin^2\frac{x}{4}\right)\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)}$

$\lim_{x \to \pi} \frac{\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)^2}{\left(\cos\frac{x}{4} + \sin\frac{x}{4}\right)\left(\cos\frac{x}{4} - \sin\frac{x}{4}\right)^2}$

$= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$ Answer

QUESTION

(KTK 2)

$$\lim_{x \to \frac{\pi}{2}} (2x \tan x - \pi \sec x)$$

Ans. –2

Anshika Dwivedi (From UP)

KTK-2

Q. $\lim_{x \to \frac{\pi}{2}} (2x\tan x - \pi\sec x)$

Sol: $\lim_{x \to \frac{\pi}{2}} (2x\tan x - \pi\sec x)$

Let $x = \frac{\pi}{2} + t$.

as $x \to \frac{\pi}{2}$, $t \to 0$

$\Rightarrow \lim_{t \to 0} \left((\pi + 2t)\tan\left(\frac{\pi}{2} + t\right) - \pi\sec\left(\frac{\pi}{2} + t\right)\right)$

$\Rightarrow \lim_{t \to 0} (-(\pi + 2t)\cot t + \pi\operatorname{cosec} t)$

$= \lim_{t \to 0} \left(-\pi\frac{\cos t}{\sin t} - 2t\frac{\cos t}{\sin t} + \frac{\pi}{\sin t}\right)$

$= \lim_{t \to 0} \left(\frac{\pi(1 - \cos t)}{\sin t} - \frac{2\cos t \cdot t}{\sin t}\right)$

$= \lim_{t \to 0} \left(\pi \cdot \frac{(1 - \cos t)}{t^2} \cdot \frac{t}{\sin t} \cdot t - 2\cos t \cdot \frac{t}{\sin t}\right)$

($\frac{(1 - \cos t)}{t^2} \to \frac{1}{2}$, $\frac{t}{\sin t} \to 1$, $t \to 0$, $\cos t \to 1$, $\frac{t}{\sin t} \to 1$)

$= 0 - 2$

$= -2$ Answer

QUESTION

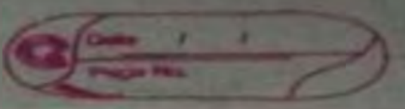
(KTK 3)

$$\lim_{x \to -1} \frac{\sqrt{\pi} - \sqrt{\arccos x}}{\sqrt{x+1}}$$

Ans. $\frac{1}{\sqrt{2\pi}}$

KTK-3

Ayushita Dwivedi
(From UP)



Q. $\lim_{x \to -1} \frac{\sqrt{\pi} - \sqrt{\arccos x}}{\sqrt{x+1}}$

Soln. Let $\cos^{-1} x = t$

$x = \cos t$.

as $t \to -1 \quad t \to \pi$

$$\lim_{t \to \pi} \frac{\sqrt{\pi} - \sqrt{t}}{\sqrt{1+\cos t}} = \lim_{t \to \pi} \frac{(\pi - t)\sqrt{1-\cos t}}{\sqrt{1-\cos^2 t}\,(\sqrt{\pi}+\sqrt{t})}$$

$$\lim_{t \to \pi} \frac{(\pi - t)\sqrt{1-\cos t}}{\sin t} \cdot \frac{\sqrt{1-\cos t}}{(\sqrt{\pi}+\sqrt{t})}$$

$$\lim_{t \to \pi} \frac{(\pi - t)}{\sin(\pi - t)} \times \frac{\sqrt{1-\cos t}}{\sqrt{1-\cos t}}.$$

$$1 \cdot \frac{\sqrt{2}}{2\sqrt{\pi}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2\pi}}$$ Answer.

QUESTION

(KTK 4)

If $\lim_{x\to\infty}\left(\ln\left(x^2+5x\right)-2\ln(cx+1)\right)=-2$ then

A $c=e^{-1}$

B $c=e^{-2}$

C $c=e$

D $c=e^{\frac{-1}{2}}$

Ans. D

If $\lim_{x\to\infty} (\log(x^2+5x) - 2\log(cx+1)) = -2$ then

$$\Rightarrow \lim_{x\to\infty} \log\left(\frac{x^2+5x}{c^2x^2+1+2cx}\right) = -2$$

$$\lim_{x\to\infty} \log\left(\frac{x^2\left(1+\frac{5}{x}\right)}{x^2\left(c^2+\frac{1}{x^2}+\frac{2c}{x}\right)}\right) = -2$$

$$\Rightarrow \log\left(\frac{1}{c^2}\right) = -2$$

$$\Rightarrow \frac{1}{c^2} = e^{-2}$$

$$\Rightarrow c = e, -e$$

KTK 4
ABHIJEET KUMAR PATRA
ODISHA
(JATNI)

RTK 4

$$\lim_{x\to\infty} \left(\ln(x^2+5x) - 2\ln(cx+1)\right) = -2$$

$$\lim_{x\to\infty} \ln\left(\frac{x^2+5x}{(cx+1)^2}\right) = -2$$

$$\lim_{x\to\infty} \ln\left(\frac{x^2\left(1+\frac{5}{x}\right)}{x^2\left(c^2+\frac{1}{x^2}+\frac{2c}{x}\right)}\right) = -2$$

$$\Rightarrow \lim_{x\to\infty} \ln\left(\frac{\left(1+\frac{5}{x}\right)}{\left(c^2+\frac{1}{x^2}+\frac{2c}{x}\right)}\right) = \ln\left(\frac{1}{c^2}\right)$$

$$\ln\left(\frac{1}{c^2}\right) = -2 \Rightarrow \frac{1}{c^2} = e^{-2}$$

$$e^{+2} = c^2$$

$$\boxed{c = e^{+1}}$$

Deepika Dubey
From UP

QUESTION

(KTK 5)

The value of $\lim_{x \to 3} \frac{\sqrt{x+6}-\sin(x-3)-3}{(x-3)\cos(x-3)}$ is equal to

A $\frac{1}{5}$

B 5

C $\frac{-3}{4}$

D $\frac{-5}{6}$

Ans. C

KTK 53

$$y = \lim_{x \to 3} \left(\frac{\sqrt{x+6} - \sin(x-3) - 3}{(x-3)\cos(x-3)} \right)$$

$$y = \lim_{x \to 3} \left(\frac{\sqrt{x+6}-3}{(x-3)\cos(x-3)} - \frac{\tan(x-3)}{(x-3)} \right)$$

$$y = \lim_{x \to 3} \left(\frac{(x-3)}{(x-3)\cos(x-3)} \cdot \frac{1}{\sqrt{x+6}+3} - \frac{\tan(x-3)}{(x-3)} \right)$$

$$y = \frac{1}{6} - 1$$

$$\boxed{y = \frac{-5}{6}}$$ Ans

KTK 5
_____hashtag_sonu_____
from Bihar siwan

The value of $\lim_{x \to 3} \frac{\sqrt{x+6} - \sin(x-3) - 3}{(x-3)\cos(x-3)}$ is Equal to :

KTK 5
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ODISHA
(JATNI)

$\Rightarrow$ $x = 3+h$

As $x \to 3$, $h \to 0$

$$\lim_{h \to 0} \frac{\sqrt{3+h+6} - \sin(3+h-3) - 3}{(3+h-3)\cos(3+h-3)}$$

$$\lim_{h \to 0} \frac{\sqrt{9+h} - \sin h - 3}{h\cos(h)}$$

$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h\cos(h)} - \frac{\sin h}{h\cos(h)}$$

$$\lim_{h \to 0} \frac{9+h-9}{h\cos h(\sqrt{9+h}+3)} - \frac{\sin h}{h\cos(h)} = \frac{1}{6} - 1 = \frac{-5}{6}$$

(Solution to RPP)

QUESTION

(RPP 1)

If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is

A $x^2 + 10x - 11 = 0$

B $11x^2 + 10x + 1 = 0$

C $x^2 + 10x + 11 = 0$

D $11x^2 - 10x + 1 = 0$

Ans. B

RPP-1 α, β are roots of the eq^n $2x^2+4x-5=0$, the eq^n whose roots are the reciprocal of $2\alpha-3$ & $2\beta-3$ is.

$$2x^2+4x-5=0 \begin{matrix}\alpha \\ \beta\end{matrix} \quad \alpha\beta = \frac{-5}{2}, \ \alpha+\beta=-2$$

$$P = \frac{1}{(2\alpha-3)(2\beta-3)} = \frac{1}{4\alpha\beta-6(\alpha+\beta)+9} = \frac{1}{-10+12+9} = \frac{1}{11}$$

$$S = \frac{1}{2\alpha-3}+\frac{1}{2\beta-3} = \frac{2\beta-3+2\alpha-3}{(2\alpha-3)(2\beta-3)} = \left(2(\alpha+\beta)-6\right)\times\frac{1}{11}$$

$$= \frac{-10}{11}.$$

· eq^n : $x^2 - Sx + P = 0$

$$x^2+\frac{10}{11}x+\frac{1}{11}=0$$

$$11x^2+10x+1=0. \quad \text{Ans}$$

rpp1
kshitij from up

QUESTION

(RPP 2)

If α, β be the roots of $x^2 + x + 2 = 0$ and γ, δ be the roots of $x^2 + 3x + 4 = 0$, then $(\alpha + \gamma)(\alpha + \delta)(\beta + \gamma)(\beta + \delta)$ is equal to

A -18

B 18

C 24

D 44

Ans. D

RPP-2 α, β are roots of $x^2+x+2=0$ & γ, δ be the roots of $x^2+3x+4=0$, then $(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$ is equal to:

$x^2+x+2=0 < \alpha, \beta$

$\alpha\beta = 2, \alpha+\beta = -1$

$x^2+3x+4=0 < \gamma, \delta$

$\gamma+\delta = -3, \gamma\delta = 4$

$\gamma^2+3\gamma+4=0$

$\gamma^2 = -4(\gamma+1)$

Similarly

$\beta^2 = -4(\beta+1)$

$(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$

$= (\alpha\beta + (\alpha+\beta)\gamma + \gamma^2)(\alpha\beta + (\alpha+\beta)\delta + \delta^2)$

$= (2-\gamma+\gamma^2)(2-\delta+\delta^2)$

$= (2-4\gamma-4)(2-4\delta-4)$

$= 4(1+2\gamma)(1+2\delta)$

$= 4(1+2(\gamma+\delta)+4\gamma\delta)$

$= 4(1+2(-3)+16)$

$= 4(1+10)$

$= 44$ Ans

rpp2

kshitij from up

RPP-2

$x^2+x+2 \begin{cases} \gamma \\ \beta \end{cases}$ $\quad \alpha+\beta=-1$, $\alpha\beta=2$

and $x^2+3x+4 \begin{cases} \delta \\ \xi \end{cases}$ $\Rightarrow \delta+\xi=-3$, $\delta\xi=4$

Now,

$$\Rightarrow (\alpha+\gamma)(\alpha+\xi)(\beta+\gamma)(\beta+\xi) = \left(\alpha^2+\alpha(\delta+\xi)+\delta\xi\right)\left(\beta^2+\beta(\delta+\xi)+\delta\xi\right)$$

$$\Rightarrow (\alpha+\gamma)(\alpha+\xi)(\beta+\gamma)(\beta+\xi) = (\alpha^2-3\alpha+4)(\beta^2-3\alpha+4)$$

$$\alpha^2-3\alpha+4 = 2-4\alpha,$$

$$\beta^2-3\alpha+4 = 2-4\beta$$

RPP 2
____hashtag_sonu____
from Bihar siwan

$$\Rightarrow (\alpha+\gamma)(\alpha+\xi)(\beta+\gamma)(\beta+\xi) = (2-4\alpha)(2-4\beta)$$

$$= 4-8(\alpha+\beta)+16\alpha\beta$$

$$= 4+8+32$$

$$= 44 \quad \text{Ans.}$$

If α, β be the roots of $x^2+x+2=0$ and γ, δ be the roots of $x^2+3x+4=0$, then $(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$

$\Rightarrow x^2+x+2=0 \begin{cases}\to \alpha \\ \to \beta\end{cases} \Rightarrow \alpha+\beta=-1$, $\alpha\beta=2$

$x^2+3x+4=0 \begin{cases}\to \gamma \\ \to \delta\end{cases} \Rightarrow \gamma+\delta=-3$, $\gamma\delta=4$

RPP 2
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Now,

$(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$

$= (\alpha^2+\alpha(\gamma+\delta)+\gamma\delta)(\beta^2+\beta(\gamma+\delta)+\gamma\delta)$

$= (\alpha^2+\alpha(-3)+4)(\beta^2+\beta(-3)+4)$

$= (\alpha^2-3\alpha+4)(\beta^2-3\beta+4)$

$= (\alpha\beta)^2-3\alpha^2\beta+4\alpha^2-3\alpha\beta^2+9\alpha\beta-12\alpha+4\beta^2-12\beta+16$

$= (\alpha\beta)^2-3\alpha\beta(\alpha+\beta)+4(\alpha^2+\beta^2)+9\alpha\beta-12(\alpha+\beta)+16$

$= (\alpha\beta)^2-3\alpha\beta(\alpha+\beta)+4((\alpha+\beta)^2-2\alpha\beta)+9\alpha\beta-12(\alpha+\beta)+16$

$= (2)^2-3(2)(-1)+4((-1)^2-2\cdot2)+9\cdot2-12(-1)+16$

$= 4+6+4(1-4)+18+12+16$

$= 4+6-12+18+12+16$

$= 10+18+16 = 44$

$= 44$ (Ans)

QUESTION

(RPP 3)

The value of 'a' for which the quadratic expression $ax^2 + |2a - 3|x - 6$ is positive for exactly three integral values of x is

(A) $\left(-\frac{3}{5}, -\frac{1}{2}\right]$

(B) $\left[-\frac{3}{5}, -\frac{1}{2}\right]$

(C) $\left[-\frac{3}{5}, -\frac{1}{6}\right]$

(D) None of these

Ans. A

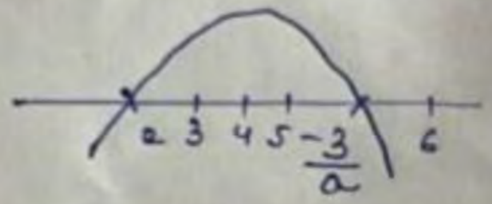
Clearly $a<0 \Rightarrow$ $f(x) = ax^2-(2a-3)x-6$

Since f is +ve for only three integral value of x.

$= ax^2-2ax+3x-6$

$= ax(x-2)+3(x-2)$

$= (ax+3)(x-2)$



Clearly $-\frac{3}{a}$ = +ve

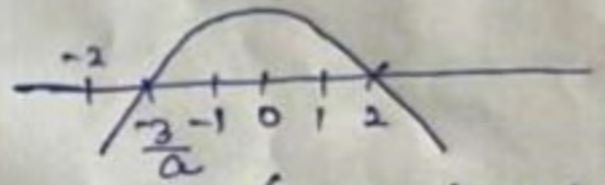
$5 < -\frac{3}{a} \leq 6$ → +ve

$5/3 < -1/a \leq 2$

$\Rightarrow \frac{1}{2} \leq -a < 3/5$

$-\frac{3}{5} < a \leq -\frac{1}{2}$

$\Rightarrow a \in (-3/5, -1/2]$



(case is not possible)

$\because -\frac{3}{a}$ = +ve.

(Solution to Home Challenge)

Home Challenge - 08

Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0, 2\pi)$, holds.

If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to:

A $\frac{\pi}{16}$

B $\frac{\pi}{48}$

C $\frac{\pi}{8}$

D $\frac{\pi}{12}$

Home challenge → 08

Q Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0$, $\theta \in (0, 2\pi)$ holds. If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$ then α is equal to

Solution :

$$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \quad \theta \in (0, 2\pi)$$

$$\sin^{-1}(\sin\theta) > \cos^{-1}(\sin\theta)$$

$\frac{\pi}{2} + \sin^{-1}(\sin\theta)$

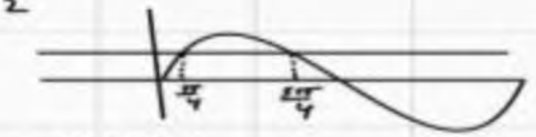
$$2\sin^{-1}(\sin\theta) > \frac{\pi}{2}$$

$$\sin^{-1}(\sin\theta) > \frac{\pi}{4}$$

But $-\frac{\pi}{2} \le \sin^{-1}(\sin\theta) \le \frac{\pi}{2}$

$$\frac{\pi}{4} < \sin^{-1}(\sin\theta) \le \frac{\pi}{2}$$

$$\frac{1}{\sqrt{2}} < \sin\theta \le 1$$



$$\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) = (a, b)$$

$$\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$$

$$\alpha x^2 + \beta x + \sin^{-1}\underbrace{((x-3)^2 + 1)}_{\ge 0} + \cos^{-1}\underbrace{((x-3)^2 + 1)}_{\ge 0} = 0$$

at $x = 3$

$$(3)^2\alpha + 3\beta + \frac{\pi}{2} = 0$$

$$9\alpha + 3\beta = -\frac{\pi}{2}$$

$$\alpha - \beta = \frac{\pi}{2}$$

$$\boxed{\alpha = \frac{\pi}{12}}$$ Ans

HOME CHALLENGE 08

RAHUL DHAKAD ❤❤
FROM AGRA UP


THANK
YOU

PRAYAS

JEE 2025

Lecture-10 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

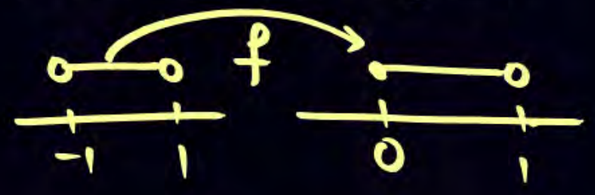
1. Introduction to Differentiability.
2. Practice Problems

Recap *of previous lecture*

1. A continuous function always maps a closed interval on to a closed interval. (True/False) $f:[a,b] \longrightarrow [m,M]$

2. A continuous function always maps an open interval on to an open interval. (True/False) Ex: $f(x)=x^2 \quad x \in (-1,1)$

Range : $[0,1)$



3. If f is continuous on R & f(1) = 3, f(5) = 100 then there exists a $x_0 \in (1, 5)$ such that $f(x_0) = 50\sqrt{3}$. (True/False)

f is continuous on R

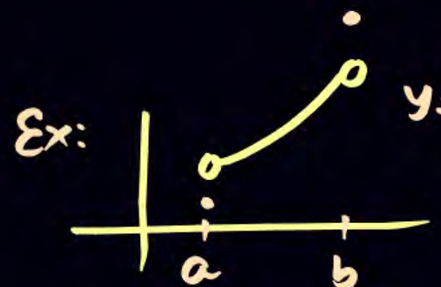
f is continuous on [1,5]

$f(1)=3 < 50\sqrt{3} < 100 = f(5)$

f(1)=3, f(5)=100

By IVT $\exists\, c \in (3,5) \quad f(c)=50\sqrt{3}$.

Recap *of previous lecture*



└ continuous on (a, b)

But greatest & least value do not Exist.

4. If f is continuous on (a, b) then f will take its greatest as well as least value at some point of the interval. (True/False)

5. If f is a continuous function on [a, b] then it is bounded on this interval. (True/False)

7. If f is a continuous function then the points of discontinuity of [f(x)] where [.] denotes GIF are those where f(x) = 0. (False)

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION

Let f be a continuous function defined on [a, b] with range [a, b].
Show that there is a x = c, c ∈ [a, b] such that f(c) = c.

for f: Domain : [a,b] Range = [a,b]

$[a,b] \xrightarrow{f} [a,b]$

Consider $g(x) = f(x) - x \quad \forall x \in [a,b]$

$f(x)$ and x continuous on $[a,b]$ ⇓ g is continuous on $[a,b]$

$g(a) = f(a) - a$

$g(b) = f(b) - b$

Case ① if $f(a) = a$ or $f(b) = b$ ⇒ result hold trivially ①

⇓ $c = a$ or $c = b$.

⇒ so let $f(a) \neq a$ & $f(b) \neq b$. then

$\Rightarrow f(a) > a, \; f(b) < b$

$\because \quad g(a) = f(a) - a > 0$

$g(b) = f(b) - b < 0$

$\Downarrow$

$g(a) \cdot g(b) < 0$

$\Downarrow$ By IVT

$\exists$ atleast one $x = c \in (a,b)$ s.t $g(c) = 0$

$\Downarrow$

$f(c) - c = 0$

$f(c) = c$ for atleast one $c \in (a,b)$ (II)

from (I) & (II) $f(c) = c$ for some $c \in [a,b]$

QUESTION

Let f(x) be continuous on the interval $[0, 1]$ such that f(0) = f(1). Prove that there exists a point c in $\left[0, \frac{1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$.

f is continuous on [0,1], f(0) = f(1) (given)

Consider $g(x) = f(x) - f(x+\frac{1}{2})$ on $x \in [0, \frac{1}{2}]$

$g(0) = f(0) - f(1/2) = f(1) - f(1/2)$

$g(1/2) = f(1/2) - f(1)$

Case① if $f(1) = f(1/2)$

$\Rightarrow f(0) = f(1) = f(1/2)$

$f(0) = f(0+1/2) = f(1/2) = f(1/2+1/2)$

$\Rightarrow f(c) = f(c+1/2)$

$c = 0, 1/2$ — ①

Case (II) if $f(1) \neq f(\frac{1}{2})$

$g(0) = f(1) - f(\frac{1}{2})$

$g(\frac{1}{2}) = f(\frac{1}{2}) - f(1)$ } opp sign

$\Downarrow$

$g(0) \cdot g(\frac{1}{2}) < 0$

$\Downarrow$ By IVT

$\exists$ atleast one $c \in (0, \frac{1}{2})$ s.t $g(c) = 0$

i.e $f(c) - f(c + 1/2) = 0$

$f(c) = f(c + 1/2)$ for some $c \in (0, 1/2)$ — (II)

from (I) & (II) $f(c) = f(c + 1/2)$ for some $c \in [0, 1/2]$

Continuity of function defined by some Functional Rule

If $f(x+y) = f(x) \cdot f(y)$ for all x & y & $f(x) = 1 + g(x) \cdot G(x)$ where

$\lim_{x\to 0} g(x) = 0$ & $\lim_{x\to 0} G(x)$ exist. Prove that f(x) is continuous for all x.

$f(x+y) = f(x)\cdot f(y)$ $\quad$ $f(x) = 1+g(x)\cdot G(x)$

To show: f is continuous $\forall\, x \in R$

Let $x = a \in R$ then

RHL $\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a)\cdot f(h) = \lim_{h\to 0} (1+g(h)\cdot G(h))\, f(a) = 1\cdot f(a) = f(a)$ ($g(h)\to 0$, $G(h)$ exist)

LHL $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+(-h)) = \lim_{h\to 0} f(a)\cdot f(-h) = \lim_{h\to 0} f(a)\cdot(1+g(-h)\cdot G(-h)) = f(a)$ ($g(-h)\to 0$, $G(-h)$ exist)

RHL = LHL = f(a)

⇓

f is continuous at x = a

But since 'a' was arbitary

$\Downarrow$

f is continuous on R

$g(x) = [f(x)]$ (where $f(x)$ is a continuous fn)

then probable points of discontinuity are where

$f(x)$ becomes an Integer.

QUESTION

Find number of Points of discontinuity of $f(x) = [x^2]$ in

(i) $[-2, 2]$ (ii) $(-1, 3]$, where [.] denotes G.I.F

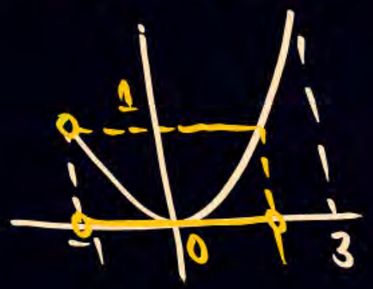
$x \in (-1, 3]$

$g(x) = x^2 \in [0, 9]$

$f(x) = [x^2]$

$x^2 = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$x = \cancel{0}, 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3.$



⇓

9 Points of Discontinuity.

find no: of points of discontinuity

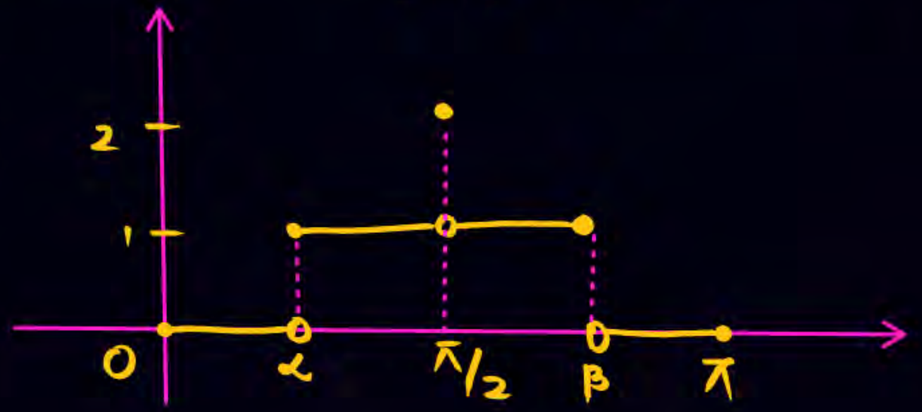
Ex: $f(x)=[2\sin x]$ in a) $[0,\pi]$ (b) $[\pi,2\pi]$

$y=[2\sin x]$

$y=2\sin x$



$\Rightarrow$



a) $2\sin x \in [0,2]$

$2\sin x = 0, 1, 2$.

$x = 0, \alpha, \pi/2, \beta, \pi$

$\Downarrow$

3 points of discontuity

If n is no: of points of discontinuity in $[0,\pi]$ & S represents the sum of all values of x for which f is discontinuous n+S.

$n=3$, $S=\alpha+\beta+\pi/2$

$= \sin^{-1}1/2 + \pi - \sin^{-1}1/2 + \pi/2$

$= 3\pi/2$

$n+S = 3\pi/2+3$

$f(x) = [2\sin x]$

(b) $x \in [\pi, 2\pi]$

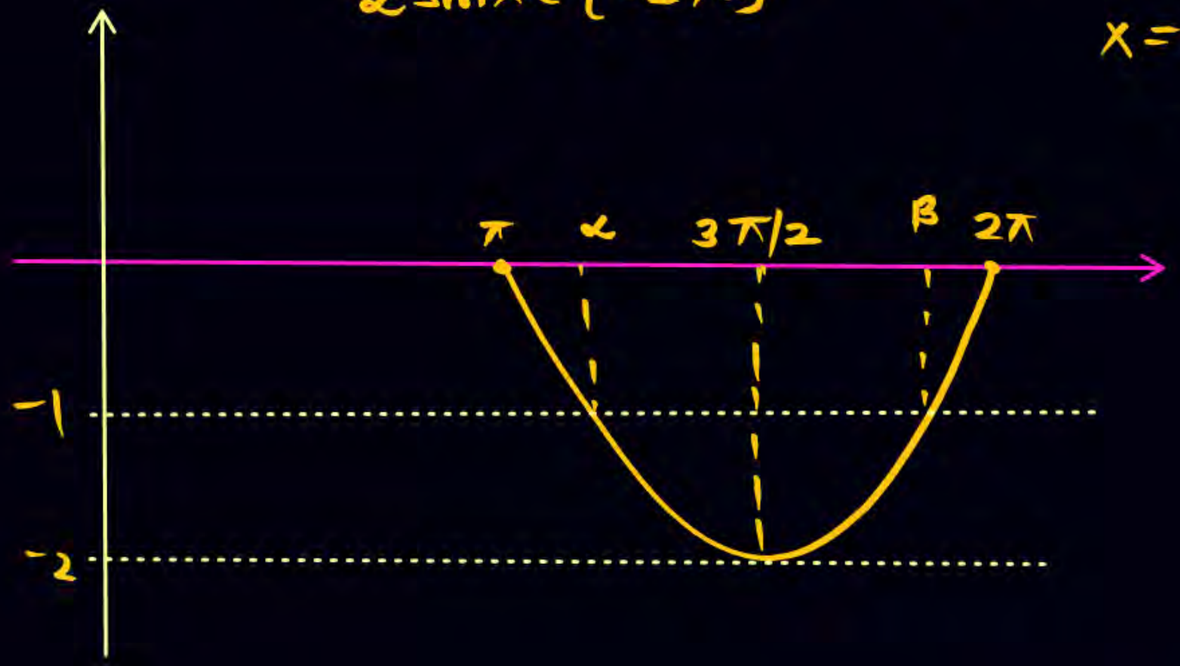
$2\sin x \in [-2, 0]$

$2\sin x = -2, -1, 0$

$x = 3\pi/2, \alpha, \beta, \pi, 2\pi$

$\Downarrow$

4 points.



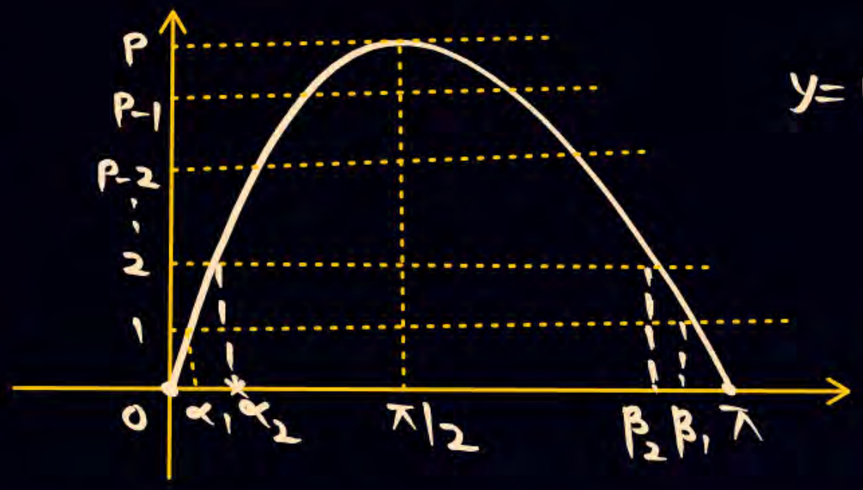
Find no: of points of discontinuity of

$f(x) = [n + p\sin x]$, $n \in I$, $p \in N$. in

(i) $[0, \pi]$ (ii) $(0, \pi)$ (iii) $[\pi, 2\pi]$ (iv) $(\pi, 2\pi)$

$f(x) = n + [p\sin x]$

(i) $[0, \pi]$



$y = p\sin x$

$p\sin x = 0, 1, 2, 3, \ldots, P-1, P$

$x = 0, \pi, \alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_{P-1}, \beta_{P-1}, \pi/2$

$\Downarrow$

(2p-1 points)

(ii) $(0, \pi)$ still (2p-1) points of discontinuity

$x \in [0, \pi]$

$f(x) = [n + p\sin x]$

n = no. of points of discontinuity

S = Sum of all values of x corresponding to points of discontinuity.

$p\sin x = 1, 2, 3, \ldots, p-1, p$

$\sin x = \frac{1}{p}, \frac{2}{p}, \frac{3}{p}, \ldots \frac{p-1}{p}, 1$

$x = \sin^{-1}\frac{1}{p}, \pi - \sin^{-1}\frac{1}{p}$

$x = \sin^{-1}\frac{2}{p}, \pi - \sin^{-1}\frac{2}{p}$

$x = \sin^{-1}\frac{3}{p}, \pi - \sin^{-1}\frac{3}{p}$

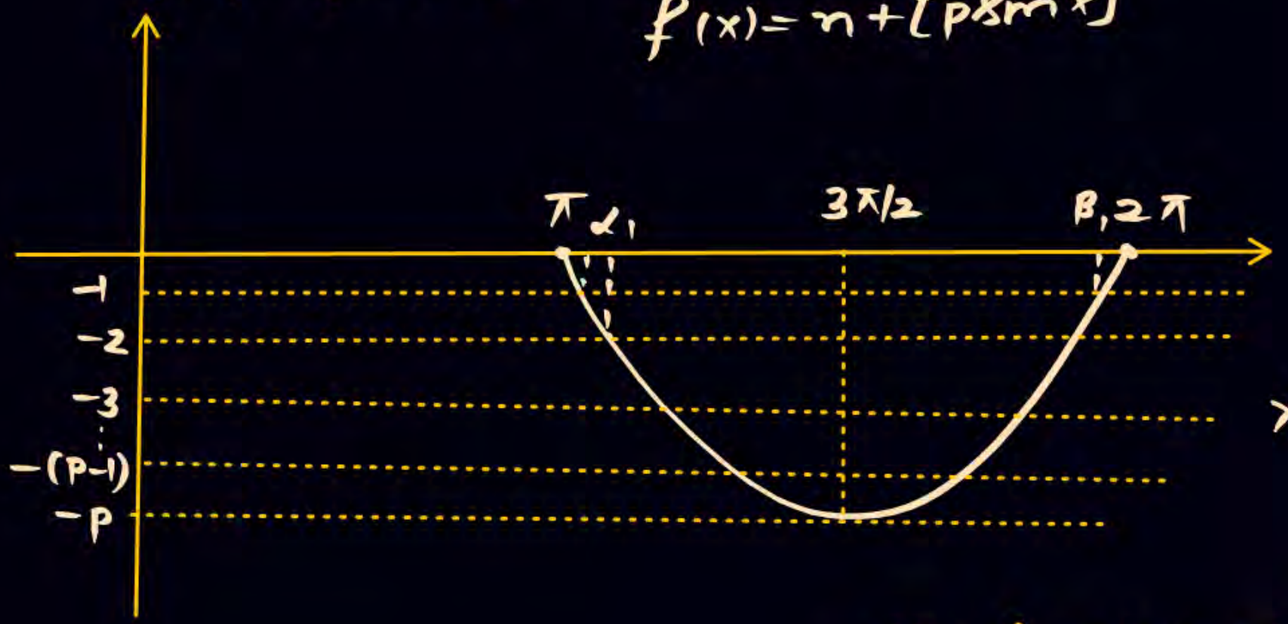
$x = \sin^{-1}\frac{p-1}{p}, \pi - \sin^{-1}\frac{p-1}{p}$

$x = \pi/2$

$n + S = 2p - 1 + (p-1)\pi + \pi/2$.

(iii) $[\pi, 2\pi]$

$f(x) = n + [p\sin x]$



$p\sin x \in [-p, 0]$

$p\sin x = 0, -1, -2, \ldots -(p-1), -p$

$x = \pi, 2\pi ; \alpha_1, \beta_1, \alpha_2, \beta_2 \ldots \alpha_{p-1}, \beta_{p-1}, \frac{3\pi}{2}$

NO: of points of discontinuity $= 2p$

(iv) $(\pi, 2\pi) \rightarrow$ NO: of points of discontinuity $= 2p-2$

QUESTION [JEE Mains 2023 (6 April)]

Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$. Then the number of point, where the function $f(x) = [a + 13 \sin x], x \in (0, \pi)$ is not Continuous, is _____

$f(x) = a + [13 \sin x]$

No: of points of discontinuity $= 13 \times 2 - 1 = 25$ Ans

QUESTION

Tan 1

Find number of points of discontinuity of $f(x) = [3 \sin x]$ in

(i) $[0, \pi]$ (ii) $(\pi, 2\pi)$, where [.] denotes G.I.F.

QUESTION

Tan 2

In [1, 3], the function $[x^2 + 1]$, [.] denoting the greatest integer function, is continuous-

$f(x) = [x^2] + 1$

A For all x

B For all x except at nine points

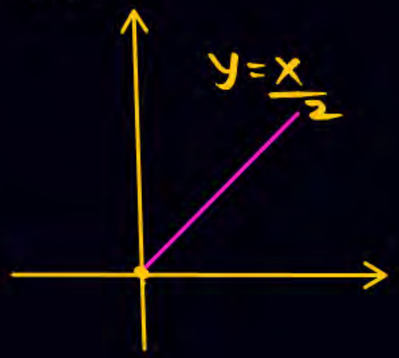
C For all x except at seven points

D For all x except at eight points

QUESTION [JEE Mains 2024 (6 April)]

Let [t] denote the greatest integer less than or equal to t. Let $f: [0, \infty) \to R$ be a function defined by $f(x) = \left[\frac{x}{2} + 3\right] - [\sqrt{x}]$. Let S be the set of all points in the interval [0, 8] at which f is not continuous.

Then $\sum_{a \in S} a$ is equal to



$f(x) = \left[\frac{x}{2}\right] - [\sqrt{x}] + 3 \qquad [0, 8]$

$x \in [0, 8]$

$\frac{x}{2} \in [0, 4]$

$\frac{x}{2} = 0, 1, 2, 3, 4$

$x = 0, 2, 4, 6, 8$

$\sqrt{x} \in [0, 2\sqrt{2}]$

$\sqrt{x} = 0, 1, 2$

$x = 0, 1, 4$

$y = \sqrt{x}$

$y^2 = x$



$f(x)$ = continuous at $x = a$

$g(x)$ = discontinuous at $x = a$

$\Downarrow$

$f(x) \pm g(x)$ is discontinuous at $x = a$

Ans. 17

	$\left[\frac{x}{2}\right]$	$[\sqrt{x}]$
1	Cont	Disc.
2	Dis	cont
4	Dis	Dis.
6	Dis.	cont
8	Dis	cont

$f(x)$ is discontinuous at $x=2, 6, 8, 1$

we need to check at $x=4$

at $x=4$

RHL $\lim\limits_{x\to 4^+} f(x) = \lim\limits_{x\to 4^+} [x/2] - [\sqrt{x}] + 3$

$= \lim\limits_{h\to 0} \left[\frac{4+h}{2}\right] - [\sqrt{4+h}] + 3$

$= \lim\limits_{h\to 0} [2+h/2] - [\sqrt{4+h}]$

$= 2^- - 2 + 3 = 3.$

LHL $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{h\to 0} \left[\frac{4-h}{2}\right] - [\sqrt{4-h}] + 3$

$= \lim\limits_{h\to 0} \left[2-\frac{h}{2}\right] - [\sqrt{4-h}]$

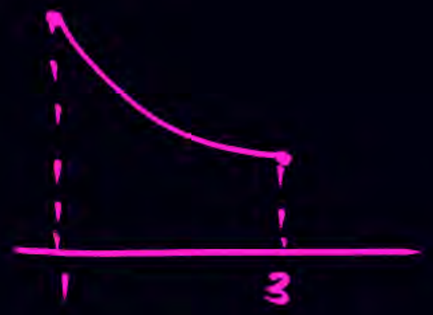
$= 1 - 1 + 3 = 3$

$f(4) = [4/2] - [\sqrt{4/2}] + 3 = 3$

f is continuous at $x=4$

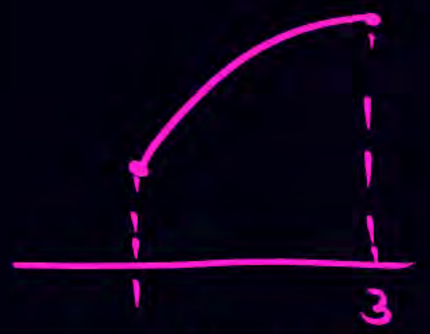
$S = \{2, 6, 8, 1\}$

$$\sum_{a \in S} a = 2+6+8+1 = 17 \text{ Ans}$$



MAX at $x=1$

Min at $x=3$



Min at $x=1$

MAX at $x=3$



Min at $x=a, c_2$

MAX at $x=c_1, b$.

QUESTION [JEE Mains 2024 (5 April)]

Tan3

Let, $f:[-1,2] \rightarrow R$ be given by $f(x)=2x^2+x+\left[x^2\right]-[x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is

A 5

B 6

C 4

D 3

Ans. C

QUESTION [JEE Mains 2022 (26 July)]

(KCLS)

Let $f : R \to R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to :

(A) 4

(B) 10

(C) 11

(D) 16

$f(3x) - f(x) = x$, $f(8) = 7$

$x \to \frac{x}{3}$ $\quad f(x) - f(x/3) = \frac{x}{3}$

$x \to x/3$ $\quad f(x/3) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$

$x \to x/3^2$ $\quad f(x/3^2) - f(x/3^3) = \frac{x}{3^3}$

$x \to x/3$ $\quad f(x/3^{n-1}) - f(x/3^n) = \frac{x}{3^n}$

$f(x) - f(x/3^n) = (x/3 + x/3^2 + \cdots + x/3^n)$

$$f(x) - f\left(\frac{x}{3^n}\right) = \frac{x}{3} + \frac{x}{3^2} + \frac{x}{3^3} + \cdots + \frac{x}{3^n}$$

$$\lim_{n\to\infty} f(x) - f(x/3^n) = \lim_{n\to\infty} \left(x/3 + \frac{x}{3^2} + \cdots + \frac{x}{3^n}\right)$$

$$f(x) - f(0) = \frac{x/3}{1 - 1/3}$$

$$f(x) = f(0) + \frac{x}{2}$$

Now put $x = 8$

$$f(8) = f(0) + 4$$

$$7 = f(0) + 4$$

$$f(0) = 3$$

$$f(x) = 3 + x/2$$

$$f(14) = 3 + 7 = 10.$$

QUESTION

Tan 4

Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$, is equal to (Where $[x]$ denotes greatest integer less than or equal to x)

A 14

B 13

C 10

D 8

QUESTION [JEE Mains 2023 (12 April)]

Let [x] be the greatest integer ≤ x. Then the number of points in the interval (–2, 1), where the function $f(x) = |[x]| + \sqrt{x - [x]}$ is discontinuous, is 2

$$f(x) = |[x]| + \sqrt{x-[x]}$$

$$= \begin{cases} |-2| + \sqrt{x+2} & -2 < x < -1 \\ |-1| + \sqrt{x+1} & -1 \le x < 0 \\ |0| + \sqrt{x-0} & 0 \le x < 1 \end{cases} = \begin{cases} \sqrt{x+2} + 2 & -2 < x < -1 \\ \sqrt{x+1} + 1 & -1 \le x < 0 \\ \sqrt{x} & 0 \le x < 1 \end{cases}$$

AT x=0

RHL $\lim_{x \to 0^+} \sqrt{x} = 0$

LHL $\lim_{x \to 0^-} \sqrt{x+1} + 1 = 2$

→ Discontinuous at x=0

AT x=-1

LHL $\lim_{x \to -1^-} \sqrt{x+2} + 2 = 3$

RHL $\lim_{x \to -1^+} \sqrt{x+1} + 1 = 1$

→ Discont at x=-1

QUESTION [JEE Mains 2023 (11 April)]

Tan5

Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is :

A continuous at $x = 0$, but not continuous at $x = 1$

B continuous at $x = 0$ and $x = 1$

C continuous at $x = 1$, but not continuous at $x = 0$

D not continuous at $x = 0$ and $x = 1$

QUESTION [JEE Mains 2022 (25 June)]

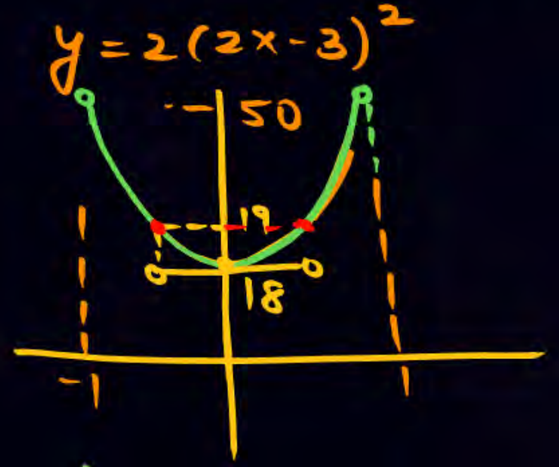
$2(2\times(-.001)-3)^2$
$2(-.002-3)^2$
$2(-3.002)^2$ $2\times 9+$

Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x-3, & x<0 \\ 2x+3, & x\geq 0 \end{cases}$, where $[t]$ is the greatest integer $\leq t$.

Then, in the open interval $(-1, 1)$, the number of point where fog is discontinuous is equal to 62

$f(x)=[2x^2]+1$ $\quad g(x)=\begin{cases} 2x-3 & x<0 \\ 2x+3 & x\geq 0 \end{cases}$

$fog(x)=f(g(x))=\begin{cases} [2(2x-3)^2]+1 & x<0 \\ [2(2x+3)^2]+1 & x\geq 0 \end{cases}$



AT $x=0$

RHL $\lim_{x\to 0^+}[2(2x+3)^2]+1 = 18+1=19$

LHL $\lim_{x\to 0^-}[2(2x-3)^2]+1 = 18+1=19$

$2(2x-3)^2 = 18, 19, 20, 21, \ldots 49$ — 31 points

$2(2x+3)^2 = 18, 19, 20 \ldots 49$ — 31 points.

62 points of discontinuity

$2(2x+3)^2 = 19, 20, 21, \ldots, 49$

$2x+3 = \sqrt{\frac{19}{2}}, \sqrt{10}, \sqrt{\frac{21}{2}}, \ldots \sqrt{\frac{49}{2}}$

$x = \frac{1}{2}\left(-3+\sqrt{\frac{19}{2}}\right), \frac{1}{2}(-3+\sqrt{10})$

$\ldots, \frac{1}{2}\left(-3+\sqrt{\frac{21}{2}}\right), \ldots \frac{1}{2}\left(-3+\sqrt{\frac{49}{2}}\right)$

31 points

$2(2x-3)^2 = 19, 20, \ldots, 49$

$(2x-3)^2 = \frac{19}{2}, 10, \frac{21}{2}, \ldots, \frac{49}{2}$

$2x-3 = -\sqrt{\frac{19}{2}}, -\sqrt{10}, -\sqrt{\frac{21}{2}} \ldots, -\sqrt{\frac{49}{2}}$

$x = \frac{1}{2}\left(3-\sqrt{\frac{19}{2}}\right), \frac{1}{2}(3-\sqrt{10}), \frac{1}{2}\left(3-\sqrt{\frac{21}{2}}\right) \ldots$

$\ldots, \frac{1}{2}\left(3-\sqrt{\frac{49}{2}}\right)$

31 points

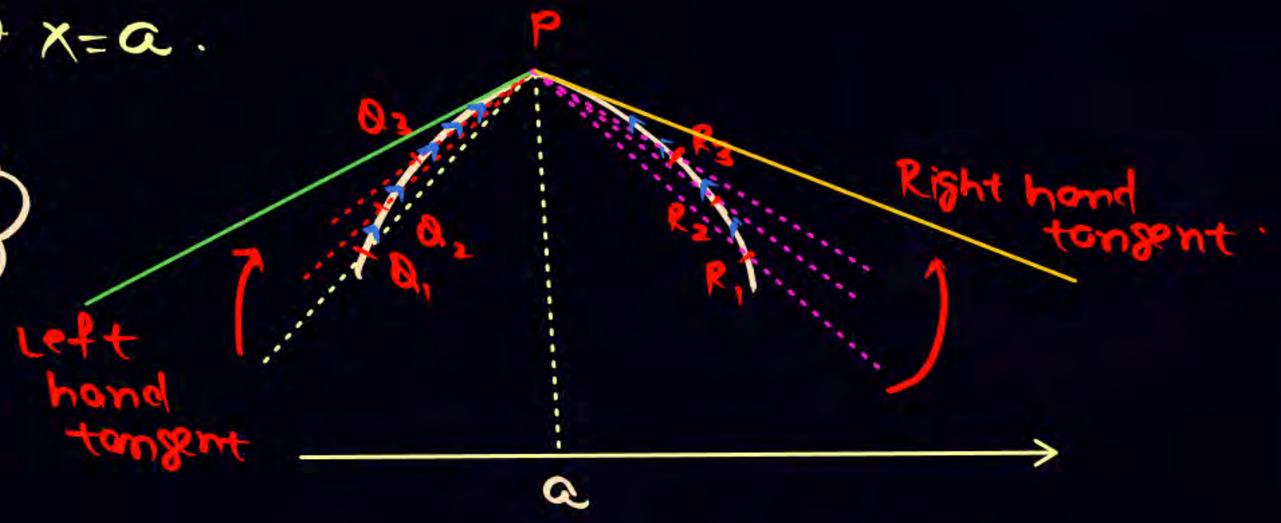
62 points

Meaning of Differentiability

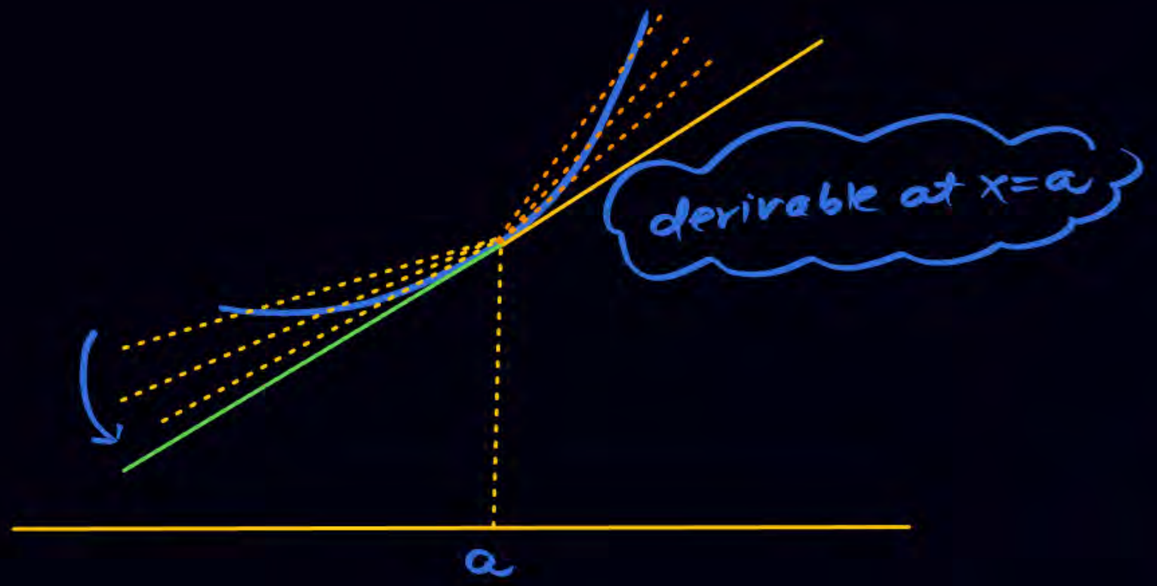
A fn is said to differentiable / derivable at $x=a$ if we can draw a unique tangent of finite slope at $x=a$.

Tangent is limiting case of secant



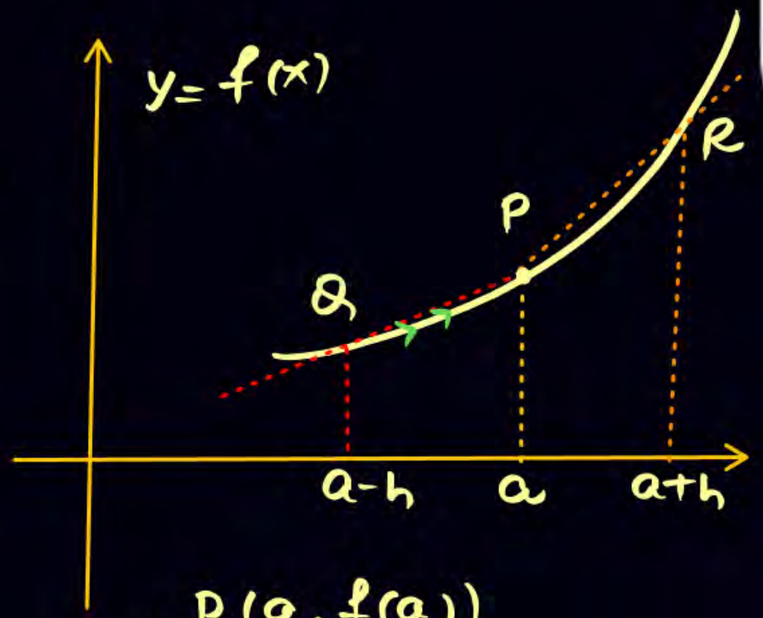


non derivable at $x=a$


derivable at x=a
a

$$m_{PR} = \frac{f(a+h)-f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

$$m_{PQ} = \frac{f(a-h)-f(a)}{a-h-a} = \frac{f(a-h)-f(a)}{-h}$$



$P(a, f(a))$

$Q(a-h, f(a-h))$

$R(a+h, f(a+h))$

$$LHD = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h} = \text{Slope of left hand tangent at } x=a$$

$$RHD = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \text{Slope of Right hand tangent at } x=a.$$

for f to be derivable at $x=a$

$LHD = RHD = a\,fq.$

$y = f(x)$ is said to be derivable at $x = a$

If $LHD = RHD = a$ fq

i.e $\lim_{h \to 0} \frac{f(a-h) - f(a)}{-h} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = a$ fq.

$$f'(a^-) = f'(a^+) = a \text{ fq}.$$

Golden Point

RHD = LHD = afq

★ If f is derivable at $x=a$ then it is continuous at $x=a$

Proof:

$$\text{RHL} = \lim_{h\to 0} f(a+h)$$

$$= \lim_{h\to 0} f(a+h) - f(a) + f(a)$$

$$= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \times h + f(a) = f(a)$$

($\frac{f(a+h)-f(a)}{h}$ → RHD = afq; $h \to 0$)

$$\text{LHL} \lim_{h\to 0} f(a-h) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h} \times (-h) + f(a) = f(a)$$

($\frac{f(a-h)-f(a)}{-h}$ → LHD = afq; $h \to 0$)

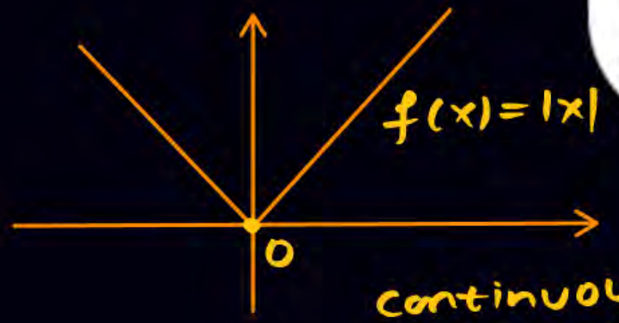
LHL = RHL = f(a)

⇓

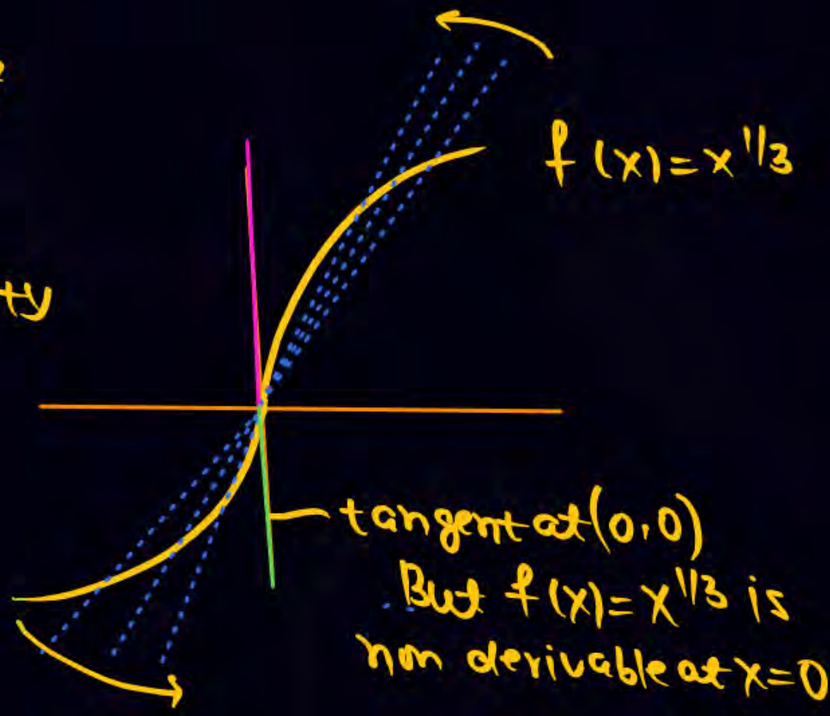
continuous at $x=a$

Golden Points

- ★ Continuity $\not\Rightarrow$ Differentiability
- ★ Discontinuity $\Rightarrow$ non Differentiable
- ★ Differentiability $\Rightarrow$ Tangentability
- ★ Tangentability $\not\Rightarrow$ Differentiability



continuous at $x=0$ but non derivable at $x=0$



tangent at (0,0)
But $f(x)=x^{1/3}$ is non derivable at $x=0$

f is derivable in (a,b) if it is derivable at each & every point of (a,b)



① derivable in $[a,b]$

② Non derivable at $x=a, b$ in $x \in (0,c)$

f is derivable in $[a,b]$ if

① f is derivable in (a,b)

② R.H.D at $x=a$ exists finitely

$$f'(a^+) = a\text{f}q = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

③ LHD at $x=b$ exists finitely

$$f'(b^-) = a\text{f}q = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$$

QUESTION

(KTK 6)

Let $f(x) = \frac{\sqrt{\log(x^2+kx+k+1)}}{x^2+k}$.

If f(x) is continuous for all $x \in R$, then the range of k is

A $(-\infty, 0) \cup [4, \infty)$

B $(0, 4]$

C $[0, 4)$

D $[0, 4]$

for continuity on R

$\log(x^2+kx+k+1) \geq 0 \quad \forall x \in R$

$\Rightarrow x^2+kx+k+1 \geq 1$ (No need $x^2+kx+k+1 > 0 \ \forall x \in R$)

$x^2+kx+k \geq 0 \quad \forall x \in R$

$D \leq 0 \Rightarrow k^2-4k \leq 0$

$k \in [0,4]$

$\forall x \in R, \ x^2+k \neq 0$

$x^2 \neq -k$.

$[0, \infty)$

$-k \in (-\infty, 0)$

$k \in (0, \infty)$

$k \in (0, 4]$

Ans. B

Home Challenge - 10

Which of the following is the solution set of the equation

$2\cos^{-1}(x) = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$ **?**

A $(0, 1)$

B $(-1, 1) - \{0\}$

C $(-1, 0)$

D $[-1, 1]$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

The following functions are not defined at $x = 0$. Find $f(0)$ so that f is continuous at $x = 0$.

(a) $f(x) = \frac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}$

(b) $f(x) = \frac{2-\sqrt[4]{x^2+16}}{\cos 2x-1}$

(c) $f(x) = \frac{1}{x} - \frac{2}{(e^{2x}-1)} (x \neq 0)$

(d) $f(x) = \frac{a-\sqrt{x+a^2}}{\sin bx}$ $(a > 0$ and $b \neq 0)$

Ans. (a) $(\ln 3)^2$; (b) $\frac{1}{64}$; (c) 1; (d) $-\frac{1}{2ab}$

QUESTION

(KTK 2)

The function $f(x) = \frac{4-x^2}{4x-x^3}$, is

A discontinuous at only one point in its domain.

B discontinuous at two points in its domain.

C discontinuous at three points in its domain.

D continuous everywhere in its domain.

Ans. D

QUESTION

(KTK 3)

If $f(x) = \begin{cases} \frac{\cos \pi x + \sin\left(\frac{\pi x}{2}\right)}{(x-1)(3x^2-2x-1)} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$

is continuous at $x = 1$, then find the value of k.

Ans. $\frac{3\pi^2}{32}$

QUESTION

(KTK 4)

For the function $f(x) = \frac{1}{x+2^{\frac{1}{(x-2)}}}$, $x \neq 2$ which of the following holds?

A $f(2) = 1/2$ and f is continuous at $x = 2$

B $f(2) \neq 0, 1/2$ and f is continuous at $x = 2$

C f can not be continuous at $x = 2$

D $f(2) = 0$ and f is continuous at $x = 2$.

Ans. C

QUESTION

(KTK 5)

If $f(x) = \begin{cases} \sin x, & x \neq n\pi, n = 0, \pm 1, \pm 2, \pm 3 \ldots \\ 2, & \text{otherwise} \end{cases}$ and $g(x) = \begin{cases} x^2 + 1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$

then $\lim_{x \to 0} g(f(x))$ is equal to

A 1

B 4

C 5

D non existent

Ans. A

QUESTION

(KTK 6)

$$\lim_{n\to\infty}\left(\cos\frac{x}{2}\cdot\cos\frac{x}{4}\ldots\ldots\cos\frac{x}{2^n}\right)$$

Ans. $\frac{\sin x}{x}$

QUESTION

(KTK 7)

$$\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2} - \arctan\frac{x}{x+2}\right)$$

Ans. 1/2

QUESTION

(KTK 8)

$$\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2}-\frac{\pi}{4}\right)$$

Ans. $-1/2$

QUESTION

(KTK 9)

$$\lim_{x \to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

Ans. $\frac{\alpha}{\beta}$

QUESTION

(KTK 10)

The value of the limit $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$ is:

A 1

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

Ans. D

Homework From Module

LCD

PRARAMBH : Q1 TO Q43
PRABAL : Q1 TO Q39
PARIKSHIT : Q1 TO Q8

(Revision Practice Problems)

QUESTION [JEE Mains 2024 (6 April)] (RPP 1)

Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0, t \in \mathbb{R}$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

(A) $-1/2$

(B) $-1/4$

(C) $1/4$

(D) $1/2$

Ans. B

QUESTION [JEE Advanced 2021 (Paper 1)] **(RPP 2)**

For $x \in R$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is

Ans. 4

QUESTION [JEE Mains 2024 (27 Jan)] (RPP 3)

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then:

(A) $2S_{12} = S_{11} + S_{10}$

(B) $S_{12} = S_{11} + S_{10}$

(C) $S_{11} = S_{10} + S_{12}$

(D) $2S_{11} = S_{12} + S_{10}$

Ans. B

(Solution to TAH)

QUESTION

Consider the function $f(x) = \begin{cases} \frac{x}{[x]} & \text{if} \quad 1 \le x < 2 \\ 1 & \text{if} \quad x = 2 \\ \sqrt{6-x} & \text{if} \quad 2 < x \le 3 \end{cases}$

Where $[x]$ denotes step up function then at $x = 2$ function-

A has missing point removable discontinuity

B has isolated point removable discontinuity

C has non removable discontinuity finite type

D is continuous

6 August Homework :→

TAH→1

Q consider the function $f(x) = \begin{cases} \frac{x}{[x]} & ; \text{ if } 1 \le x < 2 \\ 1 & ; \text{ if } x = 2 \\ \sqrt{3-x} & ; \text{ if } 2 < x \le 3 \end{cases}$

where $[x]$ denote greatest integer function then at $x=2$ func.

Sol. → at $x = 2$

$$\text{RHL} \quad \lim_{x \to 2^+} \sqrt{3-x} = 2$$

$$\text{LHL} \quad \lim_{x \to 2^-} \frac{x}{[x]} = \lim_{h \to 0} \frac{2-h}{[2-h]} \quad (2-h \to 2^-)$$

$$= \frac{2}{1} = 2$$

$$f(2) = 1$$

$$\text{RHL} = \text{LHL} \neq f(a)$$

Ans → has isolated point removable discontinuity

TAH 1RAHUL DHAKAD

❤❤

FROM AGRA UP

(Solution to KTK)

QUESTION

(KTK 1)

Let $f : R \to R$ be defined as $f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$

If $f(x)$ is continuous on R, then $a + b$ equals :

A -3

B -1

C 3

D 1

Ans. B

KTK-1

$$f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & x<-1 \\ |ax^2+x+b|, & -1\le x\le 1 \\ \sin\pi x, & x>1 \end{cases}$$

$\because$ $f(x)$ is continuous at $x=R$, so, LHL = RHL = $f(a)$

At $x=-1$

LHL :- $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} 2\sin\left(-\frac{\pi x}{2}\right) = 2$

RHL :- $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} |ax^2+x+b|$

$= |a+b-1|$

$\therefore \quad |a+b-1| = 2$

$a+b-1 = 2 \quad$ or $\quad a+b-1 = -2$

$a+b = 3 \quad$ or $\quad a+b = -1$ ——— (1)

At $x=1$

LHL :- $\lim\limits_{x\to 1^-} f(x) = |ax^2+x+b| = |a+b+1|$

RHL :- $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \sin\pi x = 0$

$\therefore \quad |a+b+1| = 0$

$a+b = -1$ ——— (2)

from (1) & (2), $\boxed{a+b=-1}$

Shivam Harshwardhan
From - Haridwar

PART 1

KTK 1 RAHUL DHAKAD ❤❤ FROM AGRA UP

KTK→1

Q Let $f:R\to R$ be defined as $f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1 \\ |ax^2+x+b|, & \text{if } -1\le x\le 1 \\ \sin\pi x, & \text{if } x>1 \end{cases}$

if $f(x)$ is continuous on R, then $a+b$ is equal to

Solu → function is continuous on R

at $x=1$ $f(x)$ is continuous

RHL $\lim\limits_{x\to 1^+} \sin(\pi x)$

$= \lim\limits_{h\to 0} \sin(\pi(1+h))$

$= \lim\limits_{h\to 0} \sin(\pi+\pi h) = 0$

LHL $\lim\limits_{x\to 1^-} |ax^2+x+b| = \lim\limits_{h\to 0} |a(1-h)^2+(1-h)+b|$

$= \lim\limits_{h\to 0} |a(1+h^2-2h)+(1-h)+b|$

$= \lim\limits_{h\to 0} |ah^2-h(2a+1)+a+b+1|$

$= |a+b+1|$

$f(1) = |a+b+1|$

RHL at $x=1$ = LHL at $x=1$ = $f(1)$

$0 = |a+b+1| = |a+b+1|$

$a+b+1=0$

$\boxed{a+b=-1}$

at $x=-1$ function is continuous

RHL $\lim_{x\to -1^+} |ax^2+x+b| = \lim_{h\to 0} |a(-1+h)^2+(-1+h)+b|$

$= \lim_{h\to 0} |a(1+h^2-2h)+(-1+h)+b|$

$= \lim_{h\to 0} |ah^2-h(2a-1)+a+b-1|$

$= |a+b-1|$

LHL $\lim_{x\to -1^-} 2\sin\left(\frac{-\pi x}{2}\right) = \lim_{h\to 0} 2\sin\left(\frac{-\pi(-1-h)}{2}\right)$

$= \lim_{h\to 0} 2\sin\left(\frac{\pi}{2}+\frac{\pi h}{2}\right)$

$= 2$

$f(-1) = |a+b-1|$

RHL = LHL = $f(-1)$

at $x=-1$ at $x=-1$

$|a+b-1| = 2 = |a+b-1|$

$a+b-1 = -2, +2$

$\boxed{a+b = -1, 3}$ $\boxed{a+b=-1}$

Ans

KTK 1 RAHUL DHAKAD ❤️❤️ FROM AGRA UP

PART 2

QUESTION

(KTK 2)

Let $f : R \to R$ be defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e \left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , \quad x \neq 0 \\ \alpha & , \quad x = 0 \end{cases}$

If f is continuous at $x = 0$, then α is equal to :

A 1

B 3

C 0

D 2

Ans. A

Let $f: R \to R$ be defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

If f is continuous at $x=0$, then α is equal to:

=> Now,

$$\lim_{x\to 0} \frac{x^3}{(1-\cos 2x)^2} \log\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$$

KTK 2
ABHIJEET KUMAR PATRA
ODISHA

$$\lim_{x\to 0} \frac{x^3}{(1-\cos 2x)^2} \left[\log(1+2xe^{-2x}) - \log(1-xe^{-x})^2\right]$$

$$\lim_{x\to 0} \frac{x^3}{(1-\cos 2x)^2} \left[\frac{\log(1+2xe^{-2x})}{2xe^{-2x}} \cdot 2xe^{-2x} - \frac{2\log(1-xe^{-x})}{-xe^{-x}} \cdot (-xe^{-x})\right]$$

$$\lim_{x\to 0} \frac{16x^4}{(1-\cos 2x)^2 16x} \left[\frac{\log(1+2xe^{-2x})}{2xe^{-2x}} \cdot 2xe^{-2x} + 2\frac{\log(1-xe^{-x})}{-xe^{-x}} (-xe^{-x})\right]$$

$$= \frac{4}{16}[2+2]$$

$$= \frac{1}{4} \times 4 = 1$$

So, $\lim_{x\to 0} f(x) = \alpha$

$\therefore \alpha = 1$ (Answer)

KTK-2

$$f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & ; \quad x\neq 0\\ \alpha & ; \quad x=0\end{cases}$$

$\because f(x)$ is continuous at $x=0$, $\therefore LHL = RHL = f(0)$

So, LHL at $x=0$:- $\lim\limits_{x\to 0^-}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$

$$\Rightarrow \lim_{x\to 0^-}\frac{x^3}{\frac{(1-\cos 2x)^2}{(2x)^4}\times(2x)^4}\frac{\log_e(1+2xe^{-2x})(2xe^{-2x})-2\log_e(1-xe^{-x})-xe^{-x}}{2xe^{-2x}}$$

$$\Rightarrow \lim_{x\to 0^-}\frac{x^3}{16x^4}(\ )\times\{(\ )(2xe^{-2x})-2(\ )(-xe^{-x})\}$$

($(\pm)^2$ ↳ 1 ↳ 1)

$$\Rightarrow \frac{4}{16}\times(2+2)$$

$$\Rightarrow 1$$

$\therefore LHL = RHL \Longrightarrow \boxed{\alpha = 1}$

Shivam Harshwardhan
From - Haridwar

QUESTION [JEE Mains 2021 (Feb)] **(KTK 3)**

Let a function $f: R \to R$ be defined as $f(x) = \begin{cases} \sin x - e^x & \text{if } x \leq 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \geq 1 \end{cases}$

where $[x]$ is the greatest integer less than or equal to x. If f is continuous on R, them $(a + b)$ is equal to :

(A) 4

(B) 3

(C) 2

(D) 5

Ans. B

KTK03. $f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$

Shivani
From bihar

$\lim_{x \to 1^-} = \lim_{h \to 0} a + [-(1-h)] = \lim_{h \to 0} a + [+h-1] = a-1$

$\lim_{x \to 1^+} = \lim_{h \to 0} 2x - b = \lim_{h \to 0} 2(1+h) - b = \lim_{h \to 0} 2 + 2h - b = 2 - b$

$\Rightarrow f$ is continuous on R

$a - 1 = 2 - b$

$a + b = 3$

KTK-3

$$f(x)=\begin{cases}\sin x-e^{x} & x\le 0\\ a+[-x] & 0<x<1\\ 2x-b & x\ge 1\end{cases}$$

$\because$ $f(x)$ is continous on R, then LHL = RHL

At $x=0$

LHL :- $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-} \sin x-e^{x}=-1$

RHL :- $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+} a+[-x]=a$

$\therefore$ $\boxed{a=-1}$

At $x=1$

LHL :- $\lim\limits_{x\to 1^-} a+[-x]=a-1$

RHL :- $\lim\limits_{x\to 1^+} 2x-b=2-b$

$\therefore$ $2-b=a-1$

$2-b=-2$

$\boxed{b=4}$

$\therefore$ $(a+b)=4-1=3$

Shivam Harshwardhan
From - Haridwar

QUESTION

(KTK 4)

Let $f : R \to R$ be defined as, $f(x) = \begin{cases} \frac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$ where $[x]$ is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to:

A $e(-e+1)$

B $e(e-2)$

C 1

D $2e-1$

Ans. A

KTK04. $f(x) = \begin{cases} \frac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)} & , x<2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & , x>2 \\ \mu & , x=2 \end{cases}$

$\lambda+\mu = ?$

↳ $-e^2+e$

$= e(-e+1)$ Ans:

$\lim_{x\to 2^-} = \frac{\lambda|(x-2)(x-3)|}{-\mu(x-2)(x-3)} = \lim_{x\to 2^-} \frac{\lambda(x-2)(x-3)}{-\mu(x-2)(x-3)} = -\frac{\lambda}{\mu}$

$\frac{-\lambda}{\mu} = e$

$-\lambda = e^2$

$\lim_{x\to 2^+} = \lim_{h\to 0} e^{\frac{\tan(x-2)}{x-[x]}} = \lim_{h\to 0} e^{\frac{\tan h}{h}} = e$

↳ μ

Shivani
From bihar

QUESTION [JEE Mains 2022 (26 July)] **(KTK 5)**

If the function $f(x) = \begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x} & , \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k & , \quad x = 0 \end{cases}$

is continuous at $x = 0$, then k is equal to :

(A) 1

(B) - 1

(C) e

(D) 0

Ans. A

KTKos. $f(x) = \begin{cases} \dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{0\} \\ k, & x=0 \end{cases}$

$\lim\limits_{x\to 0^-} \dfrac{\log_e\{(1-x+x^2)(1+x+x^2)\}}{1-\frac{\cos^2 x}{\cos x}=\frac{\sin^2 x}{\cos x}} \Rightarrow \lim\limits_{x\to 0^-} \dfrac{\ln((1+x^2)^2-x^2)}{\sin x\cdot\tan x} \Rightarrow \lim\limits_{x\to 0^-} \dfrac{\ln(1+x^4+x^2)}{\frac{\sin x}{x}\cdot\frac{\tan x}{x}\cdot}$

$\lim\limits_{x\to 0^-} = \dfrac{\ln(1+x^4+x^2)}{x^4+x^2}\cdot\dfrac{x^2(x^2+1)}{(\ )(\ )}$ ~~$\times 2$~~ $\Rightarrow$ (1)

L_1 L_1

LHL = RHL = f(0)

1 = K Ans:-

Shivani
From Bihar

If the $f(x) = \begin{cases} \frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x - \cos x}, & x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k, & x = 0 \end{cases}$

is continuous at $x=0$, then k is Equal to : (2022)

(A) 1
(B) -1
(C) e
(D) 0

$\Rightarrow \lim\limits_{x\to 0} \frac{\log(1-x+x^2)+\log(1+x+x^2)}{\left(\frac{1}{\cos x} - \cos x\right)} = k$

$\Rightarrow \lim\limits_{x\to 0} \frac{\log(1+x^2+x^4)}{1-\cos^2 x} \cdot \cos x = k$

$\Rightarrow \lim\limits_{x\to 0} \frac{\log(1+(x^2+x^4))}{(x^2+x^4)} \cdot \frac{x^2(1+x^2)}{\sin^2 x} \cdot \cos x = k$

$\Rightarrow 1 = k$

$\Rightarrow k = 1$

KTK 5
ABHIJEET KUMAR PATRA
ODISHA

QUESTION

(KTK 6)

Let $f(x) = \frac{\sqrt{\log(x^2+kx+k+1)}}{x^2+k}$.

If f(x) is continuous for all $x \in R$, then the range of k is

A $(-\infty, 0) \cup [4, \infty)$

B $(0, 4]$

C $[0, 4)$

D $[0, 4]$

Ans. B

KTK-6

Q Let $f(x) = \dfrac{\sqrt{\log(x^2+kx+k+1)}}{x^2+k}$ If $f(x)$ is continuous $\forall\ x \in R$ then range of k is

Value if f is continuous $\Rightarrow$ Henry should be define

$\log(x^2+kx+k+1) \geq 0 \ \wedge\ x^2+kx+k+1 > 0 \ \wedge\ x^2+k \neq 0$

(Simplified)

$x^2+kx+k+1 \geq 1$

$x^2+kx+k \geq 0 \quad \forall\ x \in R$

$a=1>0$, $D \leq 0$

$k^2-4\cdot k \leq 0$

$k(k-4) \leq 0$

$k \in [0,4]$

$x^2+k \neq 0$

$x^2 \neq -k$

$\forall\ x \in R$

$x^2 \neq -k$

$x^2 \in [0,\infty)$

$-k \in (-\infty, 0)$

$k \in (0,\infty)$

$k \in (0,4]$ Ans

KTK 6 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION

(KTK 7)

Consider the function defined on $[0, 1] \to R, f(x) = \frac{\sin x - x \cos x}{x^2}$ if $x \neq 0$ and $f(0) = 0$, then the function f(x)-

A has a removable discontinuity at x = 0

B has a non removable finite discontinuity at x = 0

C has a non removable infinite discontinuity at x = 0

D is continuous at x = 0

Ans. D

Consider the fn defined on $[0,1] \to R$,

$f(x) = \frac{\sin x - x\cos x}{x^2}$. If $x \neq 0$ & $f(0) = 0$, then the fn $f(x)$.

$\Rightarrow$ At $x = 0$,

RHL,

$$\lim_{x \to 0^+} \frac{\sin x - x\cos x}{x^2}$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \infty\right) - x\left(1 - \frac{x^2}{2} + \frac{x^4}{4} + \cdots \infty\right)}{x^2}$$

$$\Rightarrow \lim_{x \to 0^+} \left(\frac{-x}{3!} + \frac{x^3}{5!} - \ldots\right) + \left(\frac{x}{2} - \frac{x^2}{4} + \ldots \infty\right) = 0$$

LHL,

$$\lim_{x \to 0^-} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \infty\right) - x\left(\frac{-x}{2} - \frac{x^2}{4} + \cdots \infty\right)}{x^2}$$

$= 0$

And, $f(0) = 0$

Since, $\underset{\text{At } x=0}{\text{LHL}} = \underset{\text{At } x=0}{\text{RHL}} = f(0) = 0$

Therefore, $f(x)$ is continuous at $x = 0$.

KTK 7
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ODISHA

KTK-7

Q → Consider the func. defined on $[0,1] \longrightarrow R$

$f(x) = \frac{\sin x - x\cos x}{x^2}$; if $x \neq 0$ and, $f(0) = 0$,

then then the function $f(x)$ -

Sol:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x - x\cos x}{x^2}$$

$$= \lim_{x \to 0} \left[\frac{\tan x - x}{x^2 \cdot x}\right] \cdot x \cdot \cos x$$

$$= \frac{1}{3} \cdot 0 \cdot 1 = 0$$

$\lim_{x \to 0} f(x) = f(0) \Rightarrow$ is continuous at $x = 0$

Ans

KTK 7 RAHUL DHAKAD

❤️❤️

FROM AGRA UP

QUESTION

(KTK 8)

Which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at $x = 0$?

A $f(x) = \frac{1}{1+2^{\cot x}}$

B $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

C $f(x) = x \sin\frac{\pi}{x}$

D $f(x) = \frac{1}{\ln|x|}$

Ans. B, C, D

KTK-8 Part-I Archita Dwivedi (Team UP)

Q. Which of the following function(s) not defined at $x=0$ has/have removable discontinuity at $x=0$?

(A) $f(x) = \frac{1}{1+2^{\cot x}}$

Sol: LHL $\lim_{x\to 0^-} f(x) = \frac{1}{1+2^{\cot x}}$

$= \frac{1}{1+2^{\cot(0^-)}}$ ($\cot(0^-) \to -\infty$)

$= \frac{1}{1+2^{-\infty}} = \boxed{1}$

RHL $\lim_{x\to 0^+} f(x) = \frac{1}{1+2^{\cot x}}$

$= \frac{1}{1+2^{\cot(0^+)}}$ ($\cot(0^+) \to +\infty$)

$= \frac{1}{1+2^{\infty}} = 0 = \frac{1}{\infty}$

$\boxed{LHL \neq RHL}$

(B) $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

Sol: LHL $= \lim_{x\to 0} \cos\left(\frac{\sin(0^-)}{0_{-}}\right)$

$\cos(-\infty) = \cos\infty$

RHL $\lim_{x\to 0^+} \cos\left(\frac{\sin 0}{0}\right) = \cos(1) = \boxed{\cos 1}$

$\boxed{LHL = RHL}$

Part-I Archita Dwivedi (Team UP)

(C) $f(x) = x \sin\frac{\pi}{x}$

Sol: LHL $= \lim_{x\to 0^-} -0\left(\sin\frac{\pi}{0}\right)$ ($\sin\frac{\pi}{0} \to [-1,1]$)

$= 0 \times [-1,1]$

$= \boxed{0}$

RHL $= \lim_{x\to 0^+} 0\left(\sin\frac{\pi}{0}\right)$

$0\,[-1,1]$

$= \boxed{0}$

$\boxed{LHL = RHL}$

(D) $f(x) = \frac{1}{\sin(x)}$

Sol: LHL $= \lim_{x\to 0^-} \frac{1}{\sin(-0)} = \frac{1}{\infty} = 0$

RHL $\lim_{x\to 0^+} \frac{1}{\sin(0)} = \frac{1}{\infty} = 0$

$\boxed{LHL = RHL}$

Condition for removable discontinuity

$\boxed{LHL = RHL}$

Answer: (B), (C), (D).

KTK-8

Q which of the following function is not defined at $x=0$ has / have removable discontinuity at $x=0$?

Solve → X (i) $f(x) = \frac{1}{1+2^{\frac{1}{\tan x}}}$

$$\lim_{x\to 0} \frac{1}{1+2^{\frac{1}{\tan x}}} = \lim_{x\to 0} \frac{1}{2^{\frac{1}{\tan x}}+1}$$

$2^{\frac{1}{0^+}} \to \infty$, $2^{\frac{1}{0^-}} \to 0$

$= 0$

$f(0) = 0$

⇒ continuous at $x=0$

(ii) $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

$$\lim_{x\to 0} \cos\left(\frac{|\sin x|}{x}\right) = 1$$

$f(0)$ = not defined

⇒ RHL = LHL = finite but $f(0)$ not defined

⇒ missing point removable discontinuity

(iii) $f(x) = x \sin\frac{\pi}{x}$

$$\lim_{x\to 0} \frac{\sin \pi/x}{\pi/x} \cdot \pi = \pi$$

$f(0) = 0$

RHL = LHL ≠ $f(0)$ at $x=0$

⇒ isolated removable discontinuity

KTK 8 RAHUL DHAKAD ❤❤ FROM AGRA UP

Ans.



(iv) $f(x) = \frac{1}{\ln|x|}$

$$\lim_{x\to 0^+} \frac{1}{\ln|x|} = \lim_{h\to 0} \frac{1}{\ln h} = 0$$

$$\lim_{x\to 0^-} \frac{1}{\ln|x|} = \lim_{h\to 0} \frac{1}{\ln h} = 0$$

$f(0)$ = N.D

⇒ missing point removable discontinuity

QUESTION [JEE Mains 2022 (29 July)] (KTK 9)

Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x)-\log_e(1+\alpha x)}{x}; & \text{if } x \neq 0 \\ 10; & \text{if } x = 0 \end{cases}$ be continuous at $x = 0$.

Then α is equal to

A 10

B -10

C 5

D -5

Ans. D

Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x) - \log_e(1+\alpha x)}{x} & ; \text{if } x \neq 0 \\ 10 & ; \text{if } x = 0 \end{cases}$

be continuous at $x = 0$, Then α is Equal to : (2022)

(A) 10
(B) -10
(C) 5
(D) ~~-5~~

Since f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0) = \text{afq}$

$\Rightarrow \lim_{x \to 0} \frac{\log(1+5x).5}{5.x} - \frac{\log(1+\alpha x).\alpha}{\alpha x}$

$\Rightarrow 5 - \alpha = 10$

$\Rightarrow \alpha = -5$

KTK 9
ABHIJEET KUMAR PATRA
ODISHA

QUESTION [JEE Advanced 2017 (Paper 1)] **(KTK 10)**

Let $[x]$ be the greatest integer less than or equals to x . Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous?

A $x = -1$

B $x = 1$

C $x = 0$

D $x = 2$

Ans. A, B, D

KTK-10

Q → Let [x] G.I.F. Then at which of the following points (s) the function $f(x) = x\cos(\pi(x+[x]))$ is discontinuous ?

Ans: (A) at $x=-1$

$\lim_{x\to -1} x\cos(\pi(x+[x]))$

$\lim_{x\to -1} x\cos(\pi(x-1)) = -1$

$f(-1) = -1\cdot\cos(\pi(-1-2)) = +1$

$\lim_{x\to -1} f(x) \neq f(-1)$

$\Rightarrow$ at $x=-1$ $f(x)$ is discontinuous

(B) at $x=1$

$\lim_{x\to 1} x\cos(\pi(x+[x]))$

$\lim_{x\to 1} x\cos(\pi x) = -1$

$f(1) = 1\cdot\cos(2\pi) = 1$

$\lim_{x\to 1} f(x) \neq f(1)$

$\Rightarrow$ at $x=1$ $f(x)$ is discontinuous

(C) at $x=0$

$\lim_{x\to 0} x\cos(\pi(x+[x]))$

$\lim_{x\to 0} x\cos(\pi x)$

$= 0$

$f(0) = 0\cdot 1 = 0$

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow$ at $x=0$ $f(x)$ is continuous

PART 1

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(D)

at $x=2$

$\lim_{x\to 2} x\cdot\cos(\pi(x+[x]))$

$\lim_{x\to 2} x\cos(\pi(x+1))$

$= -2$

$f(2) = 2\cdot\cos(\pi(2+2))$

$= 2$

$\lim_{x\to 2} f(x) \neq f(2)$

$\Rightarrow$ at $x=2$ func. is discontinuous

KTK 10 RAHUL DHAKAD❤️❤️ FROM AGRA UP

PART 2

QUESTION [IIT-JEE 2011 (Paper 2)] (KTK 11)

If $\lim_{x\to\infty}\left[1+x\ln\left(1+b^2\right)\right]^{1/x}=2b\sin^2\theta, b>0$ and $\theta\in(-\pi,\pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$

(B) $\pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{6}$

(D) $\pm\frac{\pi}{2}$

Ans. D

If $\lim_{x\to 0} [1+x\ln(1+b^2)]^{1/x} = 2b\sin^2\theta$, $b>0$ and $\theta \in (-\pi,\pi]$,

(1^∞)

then the value of θ is

$\Rightarrow e^{\lim_{x\to 0} \frac{1}{x}(1+x\ln(1+b^2)-1)} = 2b\sin^2\theta$

$\Rightarrow e^{\ln(1+b^2)} = 2b\sin^2\theta$

$\Rightarrow (1+b^2) = 2b\sin^2\theta$

$\Rightarrow \frac{1+b^2}{2b} = \sin^2\theta$

$\Rightarrow \underbrace{\sin^2\theta}_{0\le x\le 1} = \underbrace{\frac{1}{2}\left(b+\frac{1}{b}\right)}_{\ge 2} \ge 1$

$\Rightarrow \sin^2\theta = 1$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$

If $n=0$,

$\theta = \pm\frac{\pi}{2}$ (Answer)

KTK 11
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ODISHA

QUESTION [JEE Advanced 2015 (Paper 2)] **(KTK 12)**

Let m and n be two positive integers greater than 1 . If

$$\lim_{\alpha \to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$$ then the value of $\frac{m}{n}$ is

Ans. 2

Let m and n be two positive integer greater than 1. If

$$\lim_{x\to 0}\left(\frac{e^{\cos(x^n)}-e}{x^m}\right)=-\left(\frac{e}{2}\right)$$ then the value of $\frac{m}{n}$ is

$$\Rightarrow \lim_{x\to 0} e\left(\frac{e^{\cos(x^n)-1}-1}{\cos x^n-1}\right)\left(\frac{\cos x^n-1}{x^m}\right)=-\left(\frac{e}{2}\right)$$

$$\lim_{x\to 0} -e\left(\frac{e^{\cos(x^n)-1}-1}{\cos x^n-1}\right)\left(\frac{1-\cos x^n}{(x^n)^2}\right)\left(\frac{x^{2n}}{x^m}\right)=-\frac{e}{2}$$

$$\Rightarrow +\frac{e}{2}\cdot\frac{x^{2n}}{x^m}=+\frac{e}{2}$$

$$\Rightarrow x^{2n-m}=1=x^0$$

$$\Rightarrow 2n-m=0$$

$$\Rightarrow \frac{m}{n}=\frac{2}{1}$$

KTK 12
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ODISHA

QUESTION [JEE Advanced 2017 (Paper 1)] **(KTK 13)**

Let $f: R \to R$ be defined as $f(x) = \begin{cases} [e^x], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + [\sin(\pi x)], & 1 \le x < 2 \\ [e^{-x}] - c, & x \ge 2 \end{cases}$

where $a, b, c \in R$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true?

(A) There exists $a, b, c \in R$ such that f is continuous on R.

(B) If f is discontinuous at exactly one point, then $a + b + c = 1$

(C) If f is discontinuous at exactly one point, then $a + b + c \neq 1$

(D) f is discontinuous at at least two points, for any values of a, b and c

Ans. C

KTK-13

Q Let $f: R \to R$ be defined as $f(x) = \begin{cases} [e^x] & ; x < 0 \\ ae^x + [x-1] & ; 0 \le x < 1 \\ b + [\sin(\pi x)] & ; 1 \le x < 2 \\ [e^{-x}] - c & ; x \ge 2 \end{cases}$

where $a, b, c \in R$ and $[.]$ G.I.F. Then which of the following is True.

Solve → $f(x) = \begin{cases} 0 & ; x < 0 \\ ae^x - 1 & ; 0 \le x < 1 \\ b & ; x = 1 \\ b-1 & ; 1 < x < 2 \\ 0 - c & ; x \ge 2 \end{cases}$ → $x \in [\pi, 2\pi)$



$f(x) = \begin{cases} 0 & ; x < 0 \\ ae^x - 1 & ; 0 \le x < 1 \\ b & ; x = 1 \\ b-1 & ; 1 < x < 2 \\ -c & ; x \ge 2 \end{cases}$

at $x = 0$ → RHL $a - 1$, LHL 0, $f(0) = a - 1$

for continuity at $x = 0$ → $a - 1 = 0$ → $a = 1$

$a \neq 1$

RHL $b-1$, LHL $a - 1 = 1 - 1 = 0$

$f(1) = b$

clearly $b - 1 \neq b$

↓ for any real value of b

f is discontinuous at $x = 1$

at $x = 2$ RHL $-c$ LHL $b-1$

$f(2) = -c$

⇓

for continuity at $x = 2$

$b - 1 = -c$

$b + c = 1$

Ans

⇨ If f is discontinuous at exactly one point, then $a + b + c \neq 1$

discontinuous $a \neq 1$ & $a = 1$

$b + c = 1$

KTK 13 RAHUL DHAKAD❤❤ FROM AGRA UP

QUESTION [JEE Mains 2024 (9 April)]

KTK 14

Let $f:(0,\pi) \to R$ be a function given by $f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$

where $a, b \in Z$. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to

Ans. 81

Let $f:(0,\pi)\to R$ be a fn given by $f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2}<x<\pi\end{cases}$

where $a,b\in Z$. If f is continuous at $x=\frac{\pi}{2}$, then a^2+b^2 is Equal to.

KTK 14
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$\Rightarrow$ At $x=\frac{\pi}{2}$,

RHL,

$$\lim_{x\to\frac{\pi}{2}^+}(1+|\cot x|)^{\frac{b}{a}|\tan x|}$$

$$\Rightarrow e^{\lim_{x\to\frac{\pi}{2}}\frac{b}{a}|\cot x|(|\cot x|)}=e^{b/a}$$

LHL,

$$\lim_{x\to\frac{\pi}{2}^-}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}$$

$$\lim_{h\to 0}\left(\frac{8}{7}\right)^{\frac{\tan(4\pi+8t)}{\tan(\frac{7\pi}{2}+7t)}}$$

$$\lim_{h\to 0}\left(\frac{8}{7}\right)^{\frac{\tan 8t}{-\cot 7t}}$$

$$\lim_{h\to 0}\left(\frac{8}{7}\right)^{-\tan 8t\tan 7t}$$

$$\Rightarrow \left(\frac{8}{7}\right)^0=1$$

Now, LHL = RHL = $f(\pi/2)$

$\Rightarrow 1=e^{b/a}=a-8$

$\Rightarrow a=9$ & $b=0$

So, $a^2+b^2=81$

KTK-13

$$f(x)=\begin{cases}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} & 0<x<\frac{\pi}{2}\\ a-8 & x=\frac{\pi}{2}\\ (1+|\cot x|)^{\frac{b}{a}|\tan x|} & \frac{\pi}{2}<x<\pi\end{cases}$$

$\because f(x)$ is continous at $x=\frac{\pi}{2}$, so LHL = RHL at $x=\frac{\pi}{2}$

LHL at $x=\frac{\pi}{2}$:- $\lim_{h\to 0}\left(\frac{8}{7}\right)^{\frac{\tan 8(\frac{\pi}{2}-h)}{\tan 7(\frac{\pi}{2}-h)}}$

$$\Rightarrow \lim_{h\to 0}\left(\frac{8}{7}\right)^{-\frac{\tan 8h}{\cot 7h}}$$

$$\Rightarrow \lim_{h\to 0}\left(\frac{8}{7}\right)^{-\tan 8h\cdot\tan 7h}=1$$

RHL at $x=\frac{\pi}{2}$:- $\lim_{x\to\frac{\pi}{2}^+}(1+|\cot x|)^{\frac{b}{a}|\tan x|}$

$$\lim_{x\to\frac{\pi}{2}^+}\left((1+|\cot x|)^{\frac{1}{|\cot x|}}\right)^{\frac{b}{a}}=e^{\frac{b}{a}}$$

$\because$ LHL = RHL = $f(\frac{\pi}{2})$

Shivam Harshwardhan
From - Haridwar

$e^{\frac{b}{a}}=1=a-8$

$\boxed{a=9}$ $\qquad \frac{b}{a}=0\Rightarrow \boxed{b=0}$

$\therefore a^2+b^2=81$

(Solution to RPP)

QUESTION

(RPP 1)

Let $f(x) = x^2 - 2x - 8$ where $x \in R$. Then number of distinct real roots of $|f(|x|)| = 8$ is

A 3

B 4

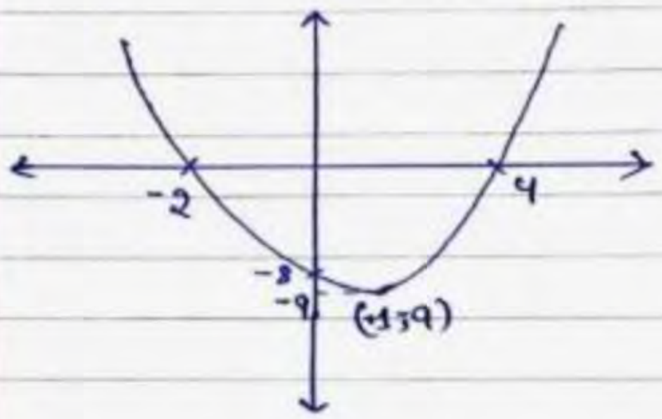
C 5

D 6

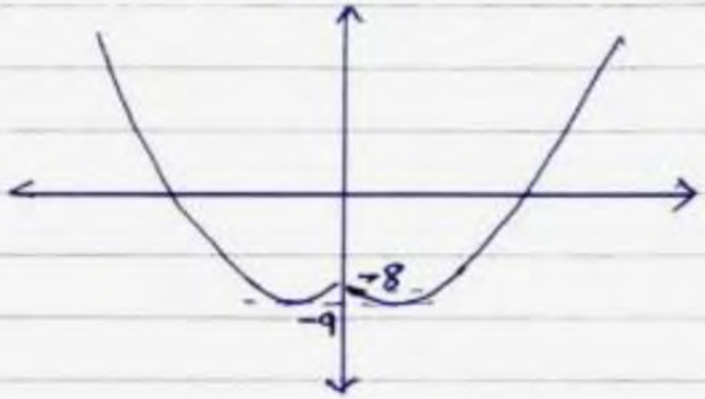
Ans. C

Let $f(x) = x^2 - 2x - 8$ where $x \in R$. Then number of distinct real roots of $|f(|x|)| = 8$ is

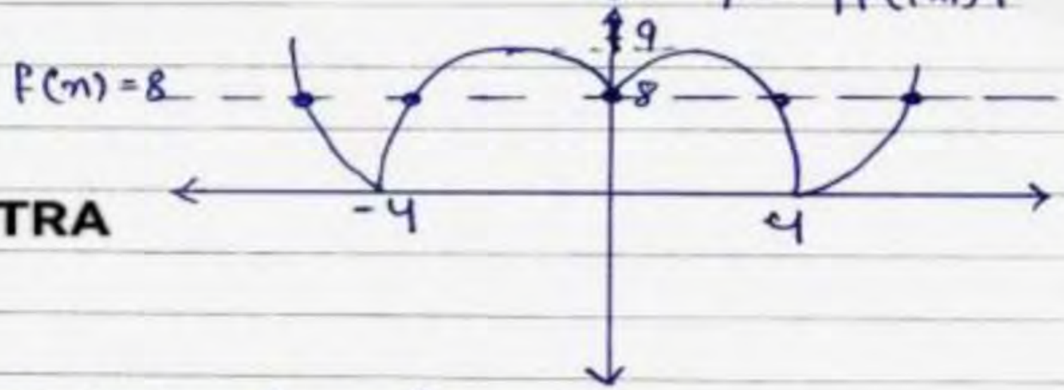
$\Rightarrow$ $f(x) = x^2 - 2x - 8$



$\Longrightarrow$



$\Downarrow$ $y = |f(|x|)|$



RPP 1
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ODISHA

So, 5 distinct real roots

QUESTION

(RPP 2)

Let $f(x) = x^2 - 6x + 8$, then number of real roots of the equation $f(f(x)) = 0$ is

A 4

B 2

C 1

D 0

Ans. A

Let $f(x) = x^2 - 6x + 8$, then no. of real roots of the Eqn $f(f(x)) = 0$ is

RPP 2
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ODISHA

$\Rightarrow f(x) = (x-2)(x-4)$

$f(f(x)) = [(x^2-6x+8)-2][(x^2-6x+8)-4] = 0$

$f(f(x)) = (x^2-6x+6)(x^2-6x+4) = 0$

$x = 3+\sqrt{3}, 3-\sqrt{3}$ $\quad$ $x = 3+\sqrt{5}, 3-\sqrt{5}$

4 Real Roots

Let $f(x) = x^2 - 6x + 8$, then no. of real roots of the Eqn $f(f(x)) = 0$ is

RPP 2
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ODISHA

$\Rightarrow f(x) = (x-2)(x-4)$

$f(f(x)) = [(x^2-6x+8)-2][(x^2-6x+8)-4] = 0$

$f(f(x)) = (x^2-6x+6)(x^2-6x+4) = 0$

$x = 3+\sqrt{3}, 3-\sqrt{3}$ $\quad$ $x = 3+\sqrt{5}, 3-\sqrt{5}$

4 Real Roots

QUESTION

(RPP 3)

Paragraph

Consider, $f(x) = 3(|a^2 - 3a - 2| + 4)x^2 - 12x + 4, a \in R$. Let α, β be the zeros of the polynomial $f(x)$.

The maximum value of $(1 + \alpha)(1 + \beta)$ is

A $\frac{1}{3}$

B $\frac{7}{3}$

C $\frac{7}{12}$

D $\frac{5}{12}$

Ans. B

QUESTION

(RPP 3)

Paragraph

Consider, $f(x) = 3(|a^2 - 3a - 2| + 4)x^2 - 12x + 4, a \in R$. Let α, β be the zeros of the polynomial $f(x)$.

If the maximum value of $f(x)$ is $g(a)$ for $0 \leq x \leq 2$ then minimum value of $g(a)$ is

A 1

B 4

C 28

D 32

Ans. C

(Solution to Home Challenge)

Home Challenge - 09

$$\lim_{x \to \frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x + \sin x)}{\left(2 \sin 2x \sin \frac{3x}{2} + \cos \frac{5x}{2}\right) - \left(\sqrt{2} + \sqrt{2} \cos 2x + \cos \frac{3x}{2}\right)}$$

Homechallange — 09

Q $\lim_{x \to \frac{\pi}{2}} \dfrac{4\sqrt{2}\,(\sin 3x + \sin x)}{(2\sin 2x \sin\frac{3x}{2} + \cos\frac{5x}{2}) - (\sqrt{2} + \sqrt{2}\cos 2x + \cos\frac{3x}{2})}$

Sol: Numerator ⇒ $4\sqrt{2}\,(\sin 3x + \sin x)$

$4\sqrt{2} \cdot 2\sin 2x \cdot \cos x$

$8\sqrt{2}\ \sin 2x \cdot \cos x$

$16\sqrt{2}\ \sin x\,(1 - \sin^2 x)$

$16\sqrt{2}\ \sin x \cdot \cos^2 x$

Denominator ⇒

$(2\sin 2x \cdot \sin\frac{3x}{2} + \cos\frac{5x}{2}) - (\sqrt{2} + \sqrt{2}\cos 2x + \cos\frac{2x}{2})$

$(\cos(2x - \frac{3x}{2}) - \cos(2x + \frac{3x}{2}) + \cos\frac{5x}{2}) - (2\sqrt{2}\cos^2 x + \cos\frac{3x}{2})$

$(\cos\frac{x}{2} + \cos\frac{5x}{2} - \cos(\frac{7x}{2})) - 2\sqrt{2}\cos^2 x - \cos\frac{3x}{2}$

$\cos\frac{x}{2} + 2\sin 2x \cdot \sin\frac{x}{2} - 2\sqrt{2}\cos^2 x - \cos\frac{3x}{2}$

$2\sin x \cdot \sin\frac{x}{2} + 2\sin 3x \cdot \sin\frac{x}{2} - 2\sqrt{2}\cos^2 x$

$2\sin\frac{x}{2}\,[\sin x + \sin 3x] - 2\sqrt{2}\cos^2 x$

$8\sin\frac{x}{2}\ \sin x \cos^2 x - 2\sqrt{2}\cos^2 x$

$\cos^2 x\,(8\sin\frac{x}{2} \cdot \sin x - 2\sqrt{2})$

$\lim_{x \to \frac{\pi}{2}} \dfrac{16\sqrt{2}\ \sin x \cdot \cos^2 x}{\cos^2 x\,(8\sin\frac{x}{2} \cdot \sin x - 2\sqrt{2})}$

$= \dfrac{16\sqrt{2}}{4\sqrt{2}} = \dfrac{16\sqrt{2}}{4\sqrt{2}} =$ **B** Ans

HOME CHALLENGE 09 RAHUL DHAKAD ❤❤
FROM AGRA UP

Mann ki Baat Ashish Sir ke Saath

Overthinking


THANK
YOU

PRAYAS

JEE 2025

Lecture-11 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Problem points on Differentiability
2. Some Important Points

Recap *of previous lecture*

1. A function f is derivable at x = a if LHD (at x=a) = RHD (at x=a)

2. R.H.D $f'(a^+) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

   L.H.D = $f'(a^-) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

3. If f is non derivable at x = a then f is discontinuous at x = a. (True/false)

Recap *of previous lecture*

4. If LHD ≠ RHD but both are finite at $x = a$ then the function is _____ at $x = a$.

5. If f is discontinuous at $x = a$ then f can be differentiable at $x = a$

(True/False)

6. If f is differentiable at $x = a$ then it can be discontinuous at $x = a$.

(True/False)

7. If f is continuous at $x = a$ then it can be non differentiable at $x = a$.

(True/False)

Recap *of previous lecture*

8. If f is differentiable at $x = a$ then it implies that we can draw a tangent of finite slope at $x = a$. (True/False)

9. If we can draw a tangent to $y = f(x)$ at $x = a$ then f is differentiable at $x = a$. (True/False)

10. Number of points of discontinuity of $f(x) = [n + p\sin x]$ where $n \in I, P \in I^+$ on $[\pi, 2\pi]$ is 2P.

$f(x) = n + [p\sin x]$

$p\sin x \in [-p, 0]$

$p\sin x = 0, -1, -2, -3, \ldots -(p-1), -p$

$\pi \quad 2\pi$

Homework Discussion

QUESTION [JEE Mains 2024 (5 April)]

Let, $f:[-1,2] \to R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is

(A) 5

(B) 6

(C) 4

(D) 3

$f(x) = \underbrace{2x^2 + x}_{\text{continuous}} + [x^2] - [x]$

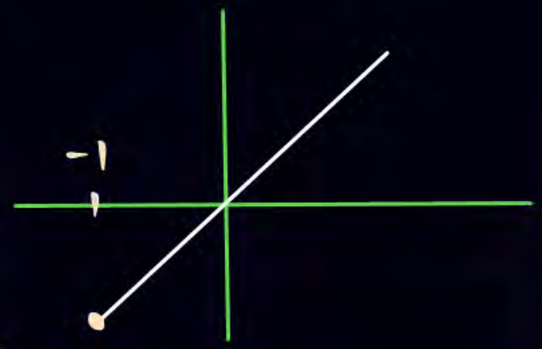
$x^2 \in [0,4]$

$x^2 = 0, 1, 2, 3, 4$

$x = 0, -1, 1, \sqrt{2}, \sqrt{3}, 2.$

$x \in [-1,2]$

$x = -1, 0, 1, 2.$

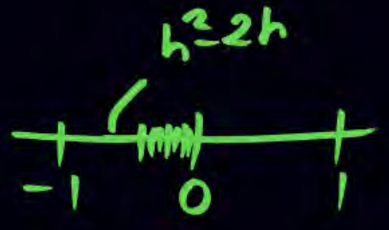


	$[x^2]$	$[x]$
-1^-	A	C
-1^+	A	A
$\sqrt{2}^-$	A	C
$\sqrt{2}^+$	A	C
$\sqrt{3}^-$	A	C
$\sqrt{3}^+$	A	C
2^-	A	C
0^-	A	C
0^+	A	A

$f(x)$ is discontinuous at $x = -1, \sqrt{2}, \sqrt{3}, 0$

Ans. C

We need to check at $x=1$ & $x=2$



$1^2=1$

$$\lim_{x\to 1^+} 2x^2+x+[x^2]-[x] = 3+1-1=3$$

$$\lim_{x\to 1^-} 2x^2+x+[x^2]-[x] = 3+0-0$$

$$\lim_{h\to 0} 2(1-h)^2+(1-h)+[1^2+h^2-2h]-[1-h]$$

$$\lim_{h\to 0} 2(1+h)^2+(1-h)+1+[h^2-2h]-[1-h]$$

(h^2-2h : -ve)

$$=\lim_{h\to 0} 2(1-h)^2+(1-h)+1-1-0 = 3$$

$f(1)=2+1+1-1=3$

f is continuous at $x=1$

$$\lim_{x\to 2^-} 2x^2+x+[x^2]-[x] = 10+3-1=12=f(2)$$

f is continuous at $x=2$

QUESTION [JEE Mains 2023 (11 April)]

Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is :

(A) continuous at $x = 0$, but not continuous at $x = 1$

(B) continuous at $x = 0$ and $x = 1$

(C) continuous at $x = 1$, but not continuous at $x = 0$

(D) not continuous at $x = 0$ and $x = 1$

$\lim_{x \to 0^+} [x^2-x] + |-x+[x]|$

$\lim_{h \to 0} [h^2-h] + |-h+[h]|$

$[h^2-h]$: −ve

$\lim_{h \to 0} (-1 + |-h+0|)$

$= \lim_{h \to 0} (-1+h) = -1$

$\lim_{x \to 0^-} [x^2-x] + |-x+[x]|$

$= \lim_{h \to 0} [h^2+h] + |h+[-h]|$

$[h^2+h]$: +ve

$= \lim_{h \to 0} 0 + |h-1|$

$\lim_{h \to 0} 1-h = 1$

Not continuous at $x = 0$.

AT $x=1$

RHL $\lim_{h\to 0} [(1+h)^2 - (1+h)] + |-(1+h) + [1+h]|$

$\lim_{h\to 0} [h^2+h] + |-1-h+1|$

$\lim_{h\to 0} 0 + |-h| = 0$

Also $f(1) = 0 + |-1+[1]| = 0$

f is continuous at $x=1$

LHL

$\lim_{h\to 0} [(1-h)^2 - (1-h)] + |-(1-h) + [1-h]|$

$\lim_{h\to 0} [h^2-h] + |-1+h+0|$

$\lim_{h\to 0} -1 + (1-h) = 0.$

QUESTION

(KTK 2)

The function $f(x) = \frac{4-x^2}{4x-x^3}$, is

(A) discontinuous at only one point in its domain.

(B) discontinuous at two points in its domain.

(C) discontinuous at three points in its domain.

(D) continuous everywhere in its domain.

$f(x) = \frac{4-x^2}{x(4-x^2)}$

$f(x) = \frac{1}{x}$ Df : $R-\{0,-2,2\}$

fn is continuous in its domain

Ans. D

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION

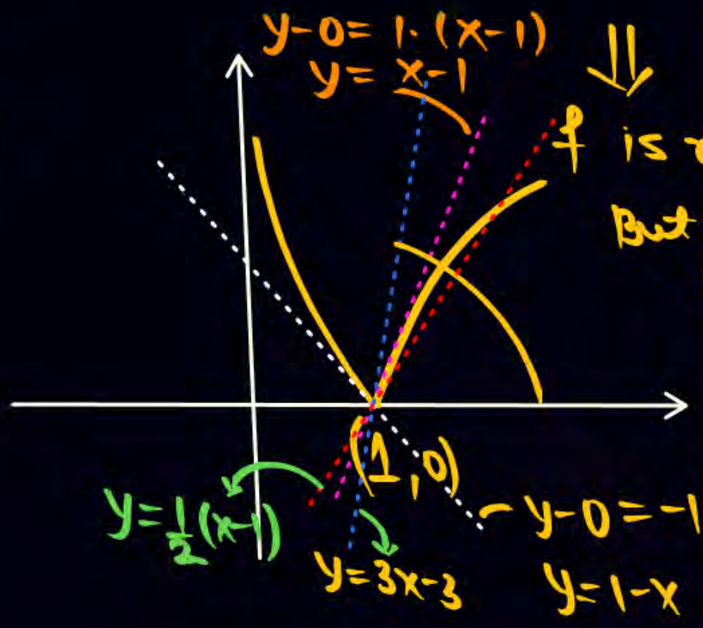
(a) $f(x) = |\ln x|$ at $x = 1$

(b) $f(x) = \ln^2 x$ at $x = 1$

(c) $f(x) = e^{-|x|}$ at $x = 0$

RHD $\lim_{h\to 0} \frac{|\ln(1+h)| - |\ln 1|}{h} = \lim_{h\to 0} \frac{\ln(1+h)}{h} = 1$

LHD $\lim_{h\to 0} \frac{|\ln(1-h)| - |\ln 1|}{-h} = \lim_{h\to 0} -\frac{\ln(1-h)}{-h} = -1$



f is non derivable at x=1
But continuous at x=1
with sharp corner at x=1

Tan 1

Ex: $|\ln x| = 3(x-1)$ → No: of soln = 1

Ex: $|\ln x| = \frac{1}{2}(x-1)$

QUESTION

Tan 2

Check the differentiability at $x = 0$ of $f(x) = \begin{cases} \frac{e^{1/x}}{1+e^{1/x}}; & x \neq 0 \\ 0; & x = 0 \end{cases}$

QUESTION

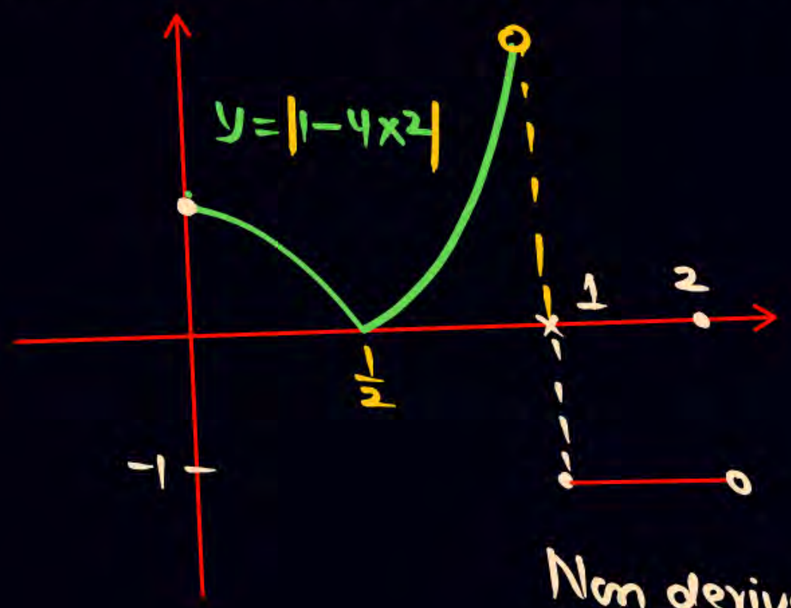
$$f(x) = \begin{cases} |1-4x^2| & 0 \le x < 1 \\ [x^2 - 2x] & 1 \le x \le 2 \end{cases}$$

$2 > x \ge 1$

$0 \le x-1 < 1$

$0 \le (x-1)^2 < 1 \Rightarrow [(x-1)^2] = 0$

Check the differentiability in $(0, 2)$, where [ ] denotes greatest integer function.



$y = [x^2 - 2x + 1 - 1]$

$y = [(x-1)^2] - 1$

$= \begin{cases} -1 & 1 \le x < 2 \\ 0 & x = 2 \end{cases}$

Non derivable at $x = 1/2, 1$

QUESTION

$$f(x)=\begin{cases}|1-4x^2| & 0\le x<1\\ [x^2-2x] & 1\le x\le 2\end{cases}$$

Check the differentiability in $(0,2)$, where [ ] denotes greatest integer function.

$$f(x)=\begin{cases}|4x^2-1| & 0\le x<1\\ [(x-1)^2]-1 & 1\le x<2\end{cases}=\begin{cases}1-4x^2 & 0\le x\le 1/2\\ 4x^2-1 & \frac{1}{2}<x<1\\ -1 & 1\le x<2\\ 0 & x=2\end{cases}$$

at $x=1$

LHL $=3$, RHL $=-1$ $\Rightarrow$ discontinuity $\Rightarrow$ non derivable $x=1$

at $x=\frac{1}{2}$

RHD $=\lim_{x\to 1/2}\frac{d(4x^2-1)}{dx}=\lim_{x\to 1/2}8x=4$

LHD $=\lim_{x\to 1/2}\frac{d}{dx}(1-4x^2)=\lim_{x\to 1/2}-8x=-4$

$\Rightarrow$ Non derivable.

QUESTION

Tan3

Examine for continuity & differentiability the points $x = 1$ & $x = 2$, the function f defined by $f(x) = \begin{bmatrix} x[x] & , & 0 \le x < 2 \\ (x-1)[x] & , & 2 \le x \le 3 \end{bmatrix}$ where $[x] =$ greatest integer less than or equal to x.

$$f(x) = \begin{cases} 0 & 0 \le x < 1 \\ x \cdot 1 & 1 \le x < 2 \\ 2(x-1) & 2 \le x < 3 \\ 6. & x = 3 \end{cases}$$

QUESTION

(ASRQ)

If $f(x) = \begin{cases} |x|^m \cdot \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, then find the set of value of m for which

(i) f(x) is continuous but not differentiable — $m > 0$ & $m \leq 1 \Rightarrow m \in (0, 1]$

(ii) f(x) is continuous and differentiable — $m > 0$ & $m > 1 \Rightarrow m \in (1, \infty)$

(ii) f(x) is discontinuous — $m \leq 0$

(i) for continuity

$\lim_{x \to 0} |x|^m \sin\frac{1}{x} = f(0) = 0$

RHL $\lim_{h \to 0} h^m \sin 1/h$

LHL $\lim_{h \to 0} |-h|^m \sin(-1/h)$

$-\lim_{h \to 0} h^m \sin 1/h$

Both should be 0 $\Longrightarrow m > 0$

(ii) for differentiability

RHD $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$\lim_{h \to 0} \frac{h^m \sin 1/h - 0}{h}$

$\lim_{h \to 0} h^{m-1} \sin 1/h$

LHD

$$\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$\lim_{h\to 0}\frac{|-h|^m \sin\left(\frac{-1}{h}\right)-0}{-h}$$

$$\lim_{h\to 0}\frac{h^m \sin 1/h}{h}$$

$$\lim_{h\to 0} h^{m-1}\sin 1/h$$

for derivability $m-1>0 \Rightarrow m>1$

- continuity ke liyay $m>0$
- Diff. ke liyay $m>1$

Some Golden Points

G1 $$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\ell=f'(a)$$

the common finite value of LHD & RHD at $x=a$ is called the value of derivative at $x=a$ & is represented by $f'(a)$

RHL

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{a+h-a}$$

$$=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$\|$

RHD

LHL

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{a-h-a}$$

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}=\text{LHD}.$$

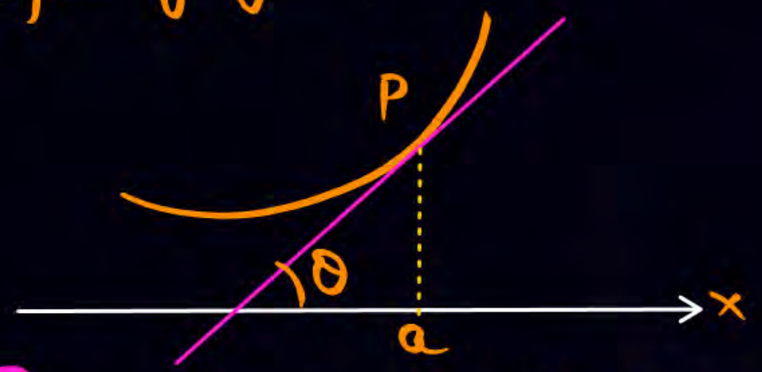
Now $$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\ell=\text{LHD}=\text{RHD}$$

Ex: AT $x=3$

if $f'(3^+)=1=f'(3^-)$

then $f'(3)=1$

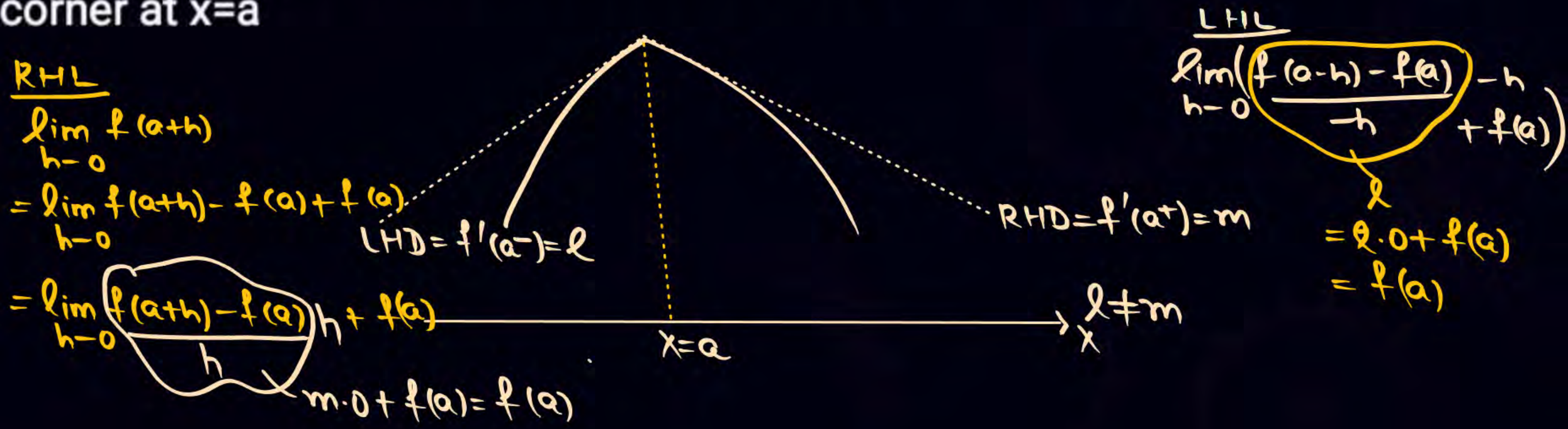
$f'(a)$ represents the slope of tangent drawn at point $(a, f(a))$ on the graph of $y = f(x)$



$\tan\theta = f'(a)$

Some Golden Points

G2 if LHD & RHD at x=a are finite but unequal then the function is nonderivable at x=a but continuous at x=a.It geometrically implies a sharp corner at x=a



QUESTION [JEE 2002 (Scr.)]

Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. The $\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ equals

derivable at $x=1$ ⇒ continuous at $x=1$

$\lim_{x\to 0} f(1+x) = f(1)$

(A) 1

(B) $e^{1/2}$

(C) e^2

(D) e^3

$$\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x} \quad (1^{\infty})$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)}{f(1)}-1\right)}$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left(\frac{f(1+x)-f(1)}{f(1)}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{f(1+x)-f(1)}{x}\cdot\frac{1}{f(1)}\right)}$$

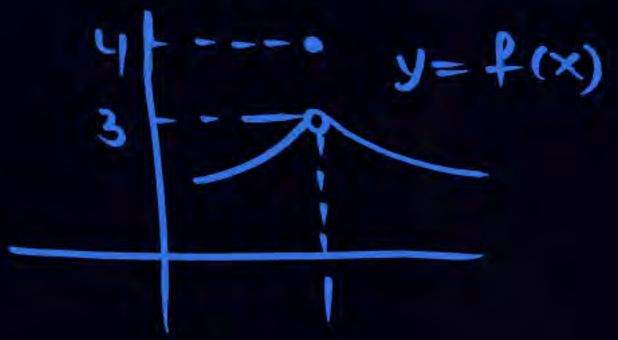
$$= e^{6/3} = e^2$$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a) = \lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$$

$f(1) = 4$

$\lim_{x \to 0} \frac{f(1+x)}{f(1)} = \frac{4}{4} = 1$

(True / ~~false~~)



QUESTION

arbitary = koisaa bhi

Let R be the set of real numbers and $f : R \to R$, be a differentiable function such that $|f(x) - f(y)| \leq |x - y|^3 \ \forall\ x, y \in R$. If $f(10) = 100$, then the value of $f(20)$ is equal to

$$\left|\frac{f(x)-f(y)}{x-y}\right| \leq |x-y|^2$$

(A) 0

(B) 10

(C) 20

(D) 100

Let $a \in R$ be arbitary real no:

$x = a+h$

$y = a$

$$\left|\frac{f(a+h)-f(a)}{h}\right| \leq h^2$$

$$\lim_{h\to 0}\left|\frac{f(a+h)-f(a)}{h}\right| \leq \lim_{h\to 0} h^2$$

$|f'(a)| \leq 0$

$f'(a) = 0 \Rightarrow f'(x) = 0 \ \forall\ x \in R \Rightarrow f$ is constant

$f(10) = 100$

$f(20) = 100$

QUESTION [JEE 2003 (Scr.)]

Tan 4

Given $f'(2) = 6$ and $f'(1) = 4$, $\lim_{h \to 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ is equal to

A $3/2$

B 3

C $5/2$

D -3

Some Golden Points

G3 Short Cut Method

$$h(x)=\begin{cases} f(x) & x>a \\ m & x=a \\ g(x) & x<a \end{cases}$$

check differentiability at $x=a$

This can be applied only if $h(x)$ is continuous at $x=a$ & $f(x)$ & $g(x)$ are smooth fn i.e they have a continuous first derivative.

Then $RHD=\lim_{x\to a} f'(x)$

$LHD=\lim_{x\to a} g'(x)$

Altmash Badmash

$$h(x)=\begin{cases} x\sin\frac{1}{x} & x>0 \\ 0 & x=0 \\ x^2\sin\frac{1}{x} & x<0 \end{cases}$$

clearly continuous at $x=0$

$RHL = LHL = 0 = f(0)$

RHD $\lim_{x\to 0}\frac{d}{dx}(x\sin 1/x)$

$\lim_{x\to 0}\sin 1/x - \frac{1}{x}\cos 1/x = DNE$

$\Downarrow$

f is non derivable at $x=0$

He missped upon smothness

Ghochu Naa Banoo

$$h(x)=\begin{cases} 2x+3 & x>1 \\ x^2+7 & x\le 1 \end{cases}$$

$RHD = \lim_{x\to 1}\frac{d}{dx}(2x+3) = \lim_{x\to 1} 2 = 2$

$\underline{LHD} = \lim_{x\to 1}\frac{d}{dx}(x^2+7) = \lim_{x\to 1} 2x = 2$

$\Downarrow$

$LHD = RHD$

$f(x)$ is differentiable

He missed upon Continuity point

Chimakaari parinam

QUESTION

If $f(x) = \begin{cases} ax + b & \text{for } x \le -1 \\ ax^3 + x + 2b & \text{for } x > -1 \end{cases}$ is differentiable for all $x \in R$, find 'a' & 'b'.

Since f is derivable $\Rightarrow$ f is continuous.

AT x=-1

LHL $= -a+b$

RHL $= -a-1+2b$

$\Rightarrow -a+b = -a-1+2b$.

$b = 1$.

LHD $\lim_{x \to -1} \frac{d}{dx}(ax+b) = a$

RHD $\lim_{x \to -1} \frac{d}{dx}(ax^3+x+2b) = \lim_{x \to -1} 3ax^2+1 = 3a+1$

$a = 3a+1$

$2a = -1 \Rightarrow a = -1/2$.

QUESTION [JEE Mains 2024 (30 Jan)]

Tan 5

If the function, $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ is differentiable on R, then $48(a + b)$ is equal to

$$f(x) = \begin{cases} \frac{1}{x} & x \geq 2 \\ -\frac{1}{x} & x \leq -2 \\ ax^2 + 2b. & -2 < x < 2 \end{cases}$$

Ans. 15

QUESTION [JEE Mains 2024 (1 Feb)]

Tan 6

Let $f: R \to R$ be defined as: $f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2; & 0 \le x \le 1 \\ 2x + 1; & x > 1 \end{cases}$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals:

(A) 1

(B) 4

(C) 3

(D) 2

Ans. D

QUESTION [JEE Mains 2023 (8 April)]

Let k and m be positive real numbers such that the function

$$f(x)=\begin{cases}3x^2+k\sqrt{x+1}, & 0<x<1\\ mx^2+k^2, & x\geq 1\end{cases}$$ is differentiable for all $x>0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to

AT $x=1$

LHL $= 3+k\sqrt{2}$

RHL $= m+k^2$

$\Rightarrow 3+k\sqrt{2}=m+k^2$ — solve.

$f'(8) = 2mx\big|_{x=8} = 16m$

$f'(1/8) = 6x+\frac{k}{2\sqrt{x+1}}\Big|_{x=1/8}$

$= \frac{3}{4}+\frac{k}{2\sqrt{9/8}}$.

$= 3/4+\frac{\sqrt{2}k}{3}$

LHD $\lim_{x\to 1}\frac{d}{dx}(3x^2+k\sqrt{x+1}) = \lim_{x\to 1} 6x+\frac{k}{2\sqrt{x+1}} = 6+\frac{k}{2\sqrt{2}}$

RHD $\lim_{x\to 1}\frac{d}{dx}(mx^2+k^2) = \lim_{x\to 1} 2mx+0 = 2m.$

$\Rightarrow 6+\frac{k}{2\sqrt{2}}=2m.$

Ans. 309

$$3+K\sqrt{2} = 3+\frac{K}{4\sqrt{2}}+K^2$$

$$8K = K + 4\sqrt{2}K^2$$

$$7K = 4\sqrt{2}K^2$$

$$7 = 4\sqrt{2}K$$

$$K = \frac{7}{4\sqrt{2}}$$

$$m = 3+\frac{K}{4\sqrt{2}} = 3+\frac{7}{32} = \frac{103}{32}$$

$$8.\ \frac{f'(8)}{f'(1/8)} = \frac{16m}{3/4+\frac{\sqrt{2}K}{3}} \cdot 8 = \frac{16\cdot\frac{103}{32}\cdot 8}{\frac{3}{4}+\frac{7}{12}} = \frac{4\times103}{9+7}\times 12 = \frac{48\times103}{16} = 309$$ Ans

QUESTION

TAH 7

If the function $f(x) = \begin{cases} ax + b, & -\infty < x \leq 2 \\ x^2 - 5x + 6, & 2 < x < 3 \\ px^2 + qx + 1, & 3 \leq x < \infty \end{cases}$ is differentiable in $(-\infty, \infty)$, then

A $a = -1, p = \frac{-4}{9}$

B $b = 2, q = \frac{5}{3}$

C $a = 1,\ b = 2$

D $a = -1, q = \frac{-5}{3}$

QUESTION

TAH 8

Let $g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \geq 1 \end{cases}$.

If $g(x)$ is continuous and differentiable for all numbers in its domain, then

(A) $a = b = 4$

(B) $a = b = 4$

(C) $a = 4$ and $b = -4$

(D) $a = -4$ and $b = 4$

QUESTION [JEE Mains 2024 (1 Feb)]

Tah 9

Let $f(x) = |2x^2 + 5|x| - 3|, x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to:

(A) 5

(B) 3

(C) 2

(D) 0

Draw graph

$y = 2x^2 + 5x - 3 = g(x) \longrightarrow y = g(|x|) = 2|x|^2 - 5|x| - 3 = h(x)$

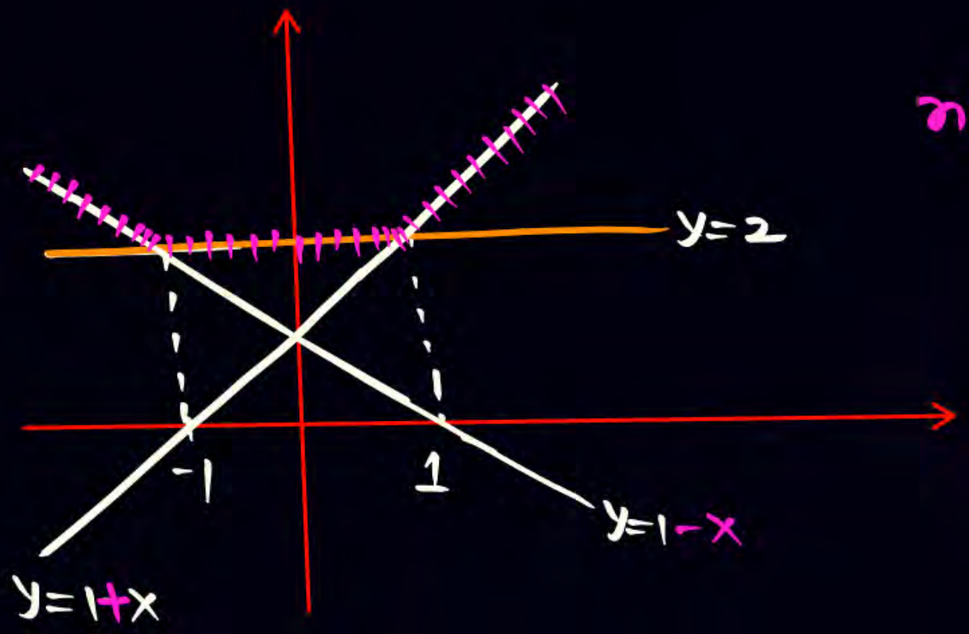
$\downarrow$

$y = |h(x)| = |2|x|^2 - 5|x| - 3|$

Ans. B

QUESTION

$f(x) = \max\{(1-x), (1+x), 2\}$ number of points where f is not differentiable



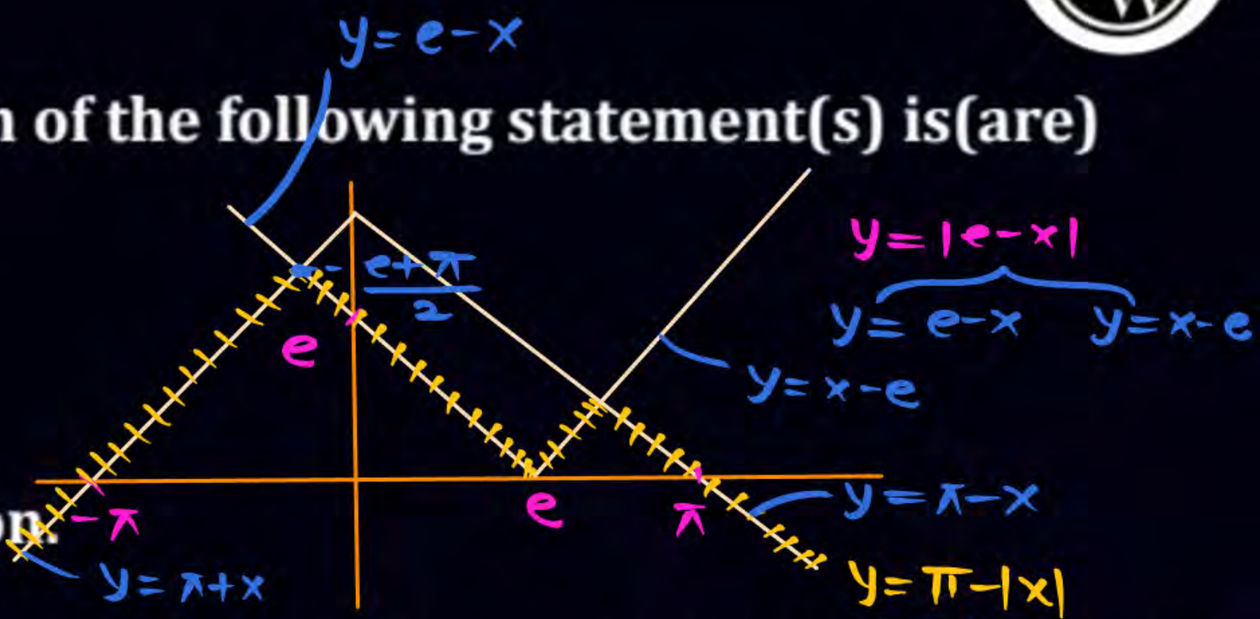
non derivable at $x=1,-1$

2 points

QUESTION

ASRQ

Let f(x) = Min. ($|e - x|, \pi - |x|$). Then which of the following statement(s) is(are) correct?



A Range of f(x) is $\left(-\infty, \frac{\pi-e}{2}\right]$.

B f(x) is many one but not even function.

C f(x) is continuous everywhere but non-derivable at three points.

D f(x) is continuous everywhere but non-derivable at two points.

$y = e - x$, $y = x + \pi$

Solve

$e - x = x + \pi$

$x = \frac{e - \pi}{2}$

$y = e - \frac{e - \pi}{2}$

$y = \frac{2e + \pi - e}{2}$

$y = \frac{e + \pi}{2}$

Range $(-\infty, \frac{e + \pi}{2}]$

QUESTION

(ASRQ)

Number of points at which the function

$$f(x)=\begin{cases}\min(|x|,x^2) & \text{if } -\infty<x<1\\ \min(2x-1,x^2) & \text{if } x\geq 1\end{cases}$$

is not derivable is

observe $y=x^2$ — P$(1,1)$

$\frac{dy}{dx}=2x\big|_{x=1}=2$

T_P: $y-1=2(x-1)$

$y=2x-1$





only one point of non derivability

QUESTION [JEE Mains 2022 (25 June)]

ASRQ

Tan 10

Let [t] denote the greatest integer less than or equal to t. Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$.
Then h is :

(A) continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

(B) not continuous at exactly three points in $[-2, 2]$

(C) continuous in $[-2, 2]$ but not differentiable at exactly three points in $(-2, 2)$

(D) not continuous at exactly four points in $[-2, 2]$

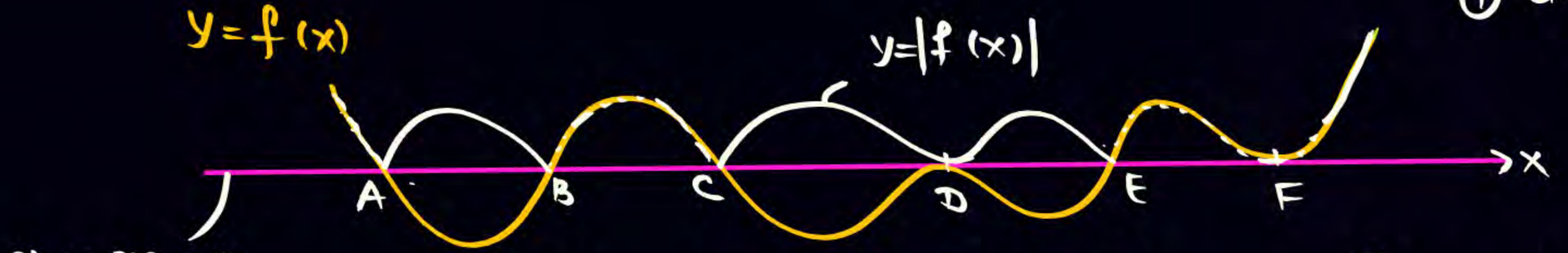
Some Golden Points

G3 Modulus Function Ki Kahaani

If f is a derivable fn & $f(x)=0$ at say $x=\alpha$ then $|f(x)|$ will be

① derivable at $x=\alpha$ ↓ if $\left|f'(x)\right|_{x=\alpha}=0$

② Non derivable at $x=\alpha$ if $\left|f'(x)\right|_{x=\alpha}\neq 0$.

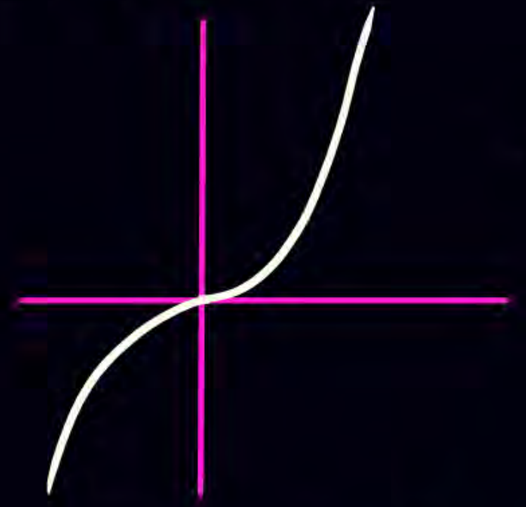


slope $m=0$ of x axis

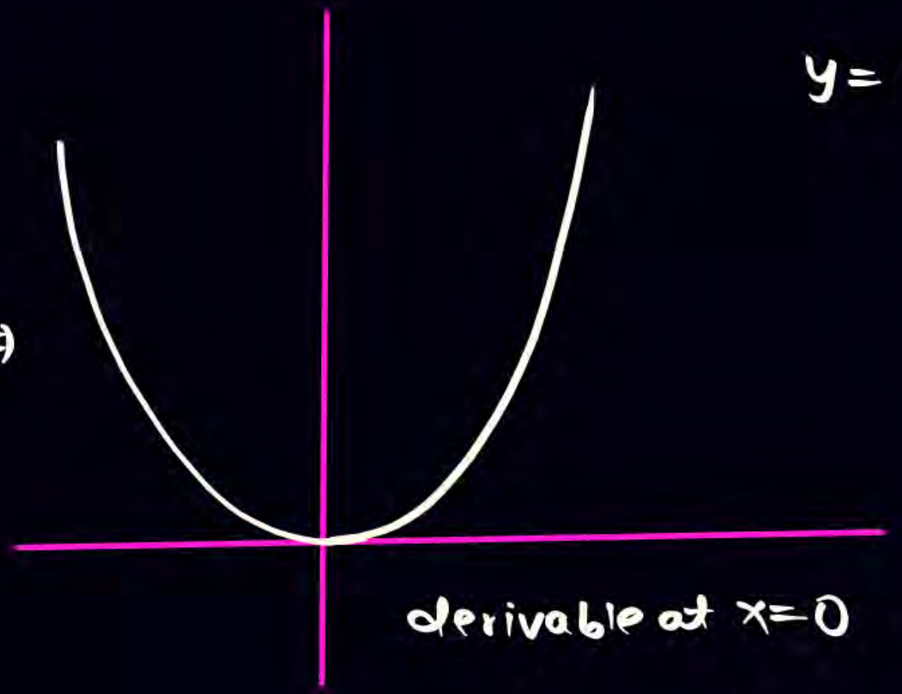
Non derivable at $x=A, B, C, E$

Derivable at F, D.

Ex: $y = x^3 = f(x)$



$\Rightarrow$



derivable at $x=0$

$y = |x^3| = |f(x)|$

$f(x) = 0$ at $x=0$

$f'(x) = 3x^2$

$f'(x)\big|_{x=0} = 0$

$\Downarrow$

x axis is tangent at $x=0$

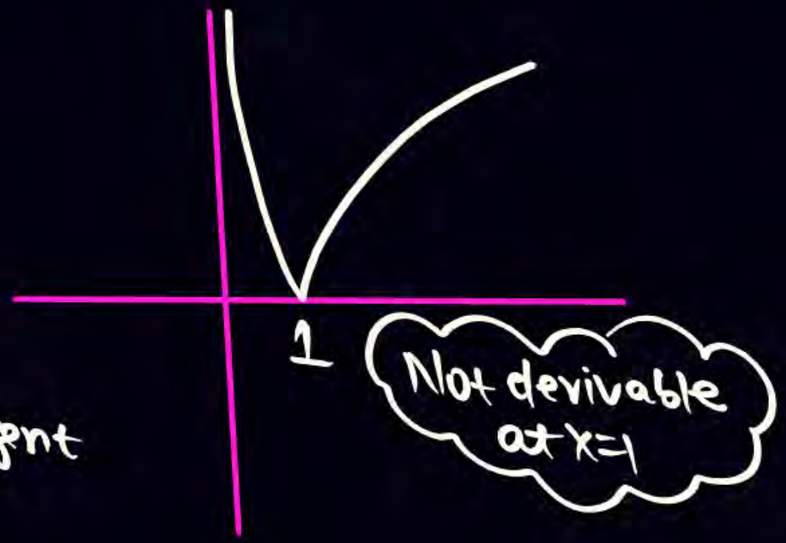
Ex: $y = |\ln x|$

$f(x) = \ln x = 0$

$x = 1$

$f'(x) = 1/x$

$f'(x)\big|_{x=1} = 1 \neq 0$ — x axis is not tangent



QUESTION

Find point of non differentiability of following functions:

(a) $f(x) = |x \sin x|$

(b) $f(x) = |(x^2 - 5x) \cdot \sin \pi x|$

(a) $f(x) = |x \sin x|$

$= |g(x)|$

$g(x) = x \sin x = 0$

$x = 0, \; x = n\pi$

$x = n\pi$

$g'(x) = x\cos x + \sin x$

at $x = 0$ $\quad g'(x) = 0$

at $x = n\pi, x \neq 0$ $\quad g'(x) \neq 0$

$|x \sin x|$ non derivable at $x = n\pi$, $n \in I_0$

QUESTION

Find point of non differentiability of following functions:

(a) $f(x) = |x \sin x|$

(b) $f(x) = |(x^2 - 5x) \cdot \sin \pi x|$

@ $f(x) = |\underbrace{(x^2-5x)\sin\pi x}_{g(x)}|$

$g(x) = (x-5)\cdot x \sin\pi x = 0$

$x = 0, 5,$ or $\pi x = n\pi$

$x = n$

$x = n \in I$

$g'(x) = (2x-5)\sin\pi x + (x^2-5x)\cdot\pi\cos\pi x$

$g'(0) = 0 = g'(5)$ $\quad g'(n) = (n^2-5n)\cos(n\pi)\cdot\pi$

$g'(n) \neq 0 \quad \forall n \in I, n \neq 0, 5.$

$\Downarrow$

$f(x) = |g(x)|$ is non derivable $\forall x \in I$, $x \neq 0, 5$



f is derivable on R & $g(x)=|f(x)|$
then g is non derivable at all
points where $f(x)=0$ (True/False)



Home Challenge-11

Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \dfrac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4}, & x \neq 2p \\ 0, & x = 2p \end{cases}$$

Then $\lim\limits_{x \to 2p^+} [f(x)]$, where [.] denotes greatest integer function, is

A) 2

B) 1

C) 0

D) -1



Sabse Important Baat Yaad Rahe

A) Sab samajh aaya

B) Thodaa nahi aayaa lekin pataa hai doobara dekhugaa aajaygaa

Sabhi Class Illustrations Retry Karnay hai...

C) Kuch samajh nahi aaya

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

If $a = \lim_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}-\sqrt{2}}}{x^4}$ and $b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is:

A 36

B 25

C 32

D 30

Ans. C

QUESTION

(KTK 2)

$y = f(x)$ is a continuous function such that its graph passes through $(a, 0)$. Then $\lim_{x \to a} \frac{\log_e(1+3f(x))}{2f(x)}$ is

A 1

B 0

C $\frac{3}{2}$

D $\frac{2}{3}$

Ans. C

QUESTION

(KTK 3)

Which of the following functions are continuous at $x = 0$?
[Note: sgn x denotes signum function of x.]

(A) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) + \operatorname{sgn}|x|$

(B) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) - \operatorname{sgn}|x|$

(C) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) + \operatorname{sgn}|x|$

(D) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) - \operatorname{sgn}|x|$

Ans. A, D

QUESTION

(KTK 4)

f is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

(A) can not be determined

(B) is $2(1 - \sqrt{3})$

(C) is zero

(D) is $\frac{2(\sqrt{3}-2)}{\sqrt{3}}$

Ans. B

QUESTION

(KTK 5)

Which of the following function(s) will have atleast one real root in $\left[0, \frac{\pi}{2}\right]$?

(A) $f(x) = x - 1 - \cos x$

(B) $g(x) = \frac{1}{2} + \sin 3x$

(C) $h(x) = \frac{1}{1 - x}$

(D) $k(x) = \frac{x - \frac{3}{2}}{x - 1}$

Ans. A, B, D

Homework From Module

LCD

PRARAMBH : Q1 TO Q43,Q44 TO Q61,Q71 TO Q108

PRABAL : Q1 TO Q39

PARIKSHIT : Q1 TO Q8

(Revision Practice Problems)

QUESTION

(RPP 1)

If $|\tan\theta - 1| + |\sqrt{3} + \tan\theta| = 1 + \tan\frac{\pi}{3}$ then number of integral solution(s) in $[0, 2\pi]$ is (are)

A 2

B 3

C 4

D 5

Ans. B

QUESTION

(RPP 2)

If $\log_e \pi + \sqrt{2 + \tan^2 x + \cot^2 x} = 3$, then number of solution(s) satisfying the equation is

A 0

B 2

C 4

D infinite

Ans. A

QUESTION

(RPP 3)

Let $P(x) = 2x^2 - 12x + c \ \forall \ x \in R$ where c is a real constant, then

	Column-I	Column-II
(A)	If greatest value of $p(x)$ for $x \in [1, 2]$ is 1, then c equals	(P) 8
(B)	If smallest value of $P(x)$ for $x \in [1, 5]$ is –1, then c equals	(Q) 11
(C)	If the greatest value of $P(x)$ for $x \in [1, 4]$ is 2, then c equals	(R) 12
		(S) 17

Ans. (A) Q; (B) S; (C) R

QUESTION

(RPP 4)

If $L = \cos^2 84^\circ + \cos^2 36^\circ + \cos 36^\circ \cos 84^\circ$

$M = \cot 73^\circ \cot 47^\circ \cot 13^\circ$

$N = 4 \sin 156^\circ \sin 84^\circ \sin 36^\circ$

then which of the following option is(are) correct?

(A) $L < 1$

(B) $M > \tan 2$

(C) $N > \sin \frac{\pi}{4}$

(D) $LMN > 0$

Ans. A, B, C, D

(Solution to TAH)

QUESTION

TAH 1

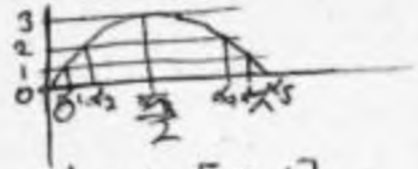
Find number of points of discontinuity of $f(x) = [3 \sin x]$ in

(i) $[0, \pi]$ (ii) $(\pi, 2\pi)$, where [.] denotes G.I.F.

TAH-01

$f(x) = [3\sin x]$

1) in $[0, \pi]$



$\sin x \in [0,1]$

$3\sin x \in [0,3]$

$3\sin x = 0, 1, 2, 3$

$\sin x = 0, \frac{1}{3}, \frac{2}{3}, 1$

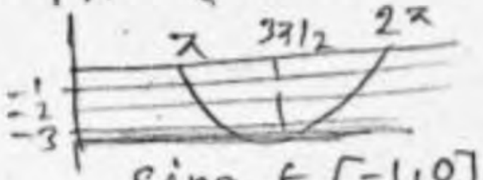
$x = \cancel{0}, \cancel{\pi}, \alpha, \beta, \gamma, \delta, \frac{\pi}{2}$

0, π are rejected

at {0, π} minima occur.

numbe of point of. discont. = 5

2) in $(\pi, 2\pi)$



$\sin x \in [-1, 0]$

$3\sin x \in [-3, 0]$

$3\sin x = -3, -2, -1, 0$

$\sin x = -1, -\frac{2}{3}, -\frac{1}{3}, 0$

$x = \cancel{\frac{3\pi}{2}}, \alpha, \beta, \gamma, \delta, \cancel{\pi, 2\pi}$ (Not in Range)

$\frac{3\pi}{2}$ rejected, @ $\frac{3\pi}{2}$ = min.

point of discontinuity = 4. A

Kripa shankar Maurya Varanasi

August Homework :→

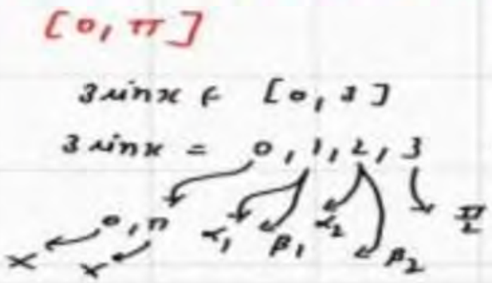
TAH → 1

Q find the nu. of points of discontinuity of $f(x) = [3\sin x]$ in

(i) $[0, \pi]$

$3\sin x \in [0, 3]$

$3\sin x = 0, 1, 2, 3$





↓ point of minima

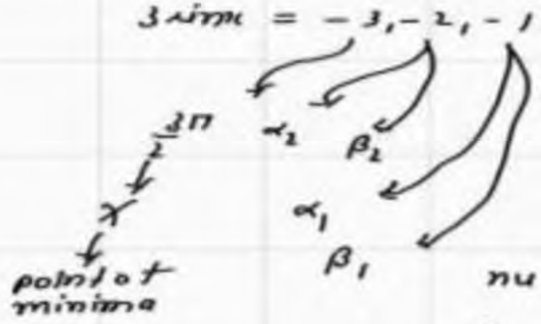
nu. of points of discontinuity = 5

$[\alpha_1, \beta_1, \alpha_2, \beta_2, \frac{\pi}{2}]$

(ii) $(\pi, 2\pi)$

$3\sin x \in [-3, 0)$

$3\sin x = -3, -2, -1$





↓ point of minima

nu. of points of discontinuity = 4

$[\alpha_2, \beta_2, \alpha_1, \beta_1]$

TAH 1 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION

TAH 2

In [1, 3], the function $[x^2 + 1]$, [.] denoting the greatest integer function, is continuous-

A For all x

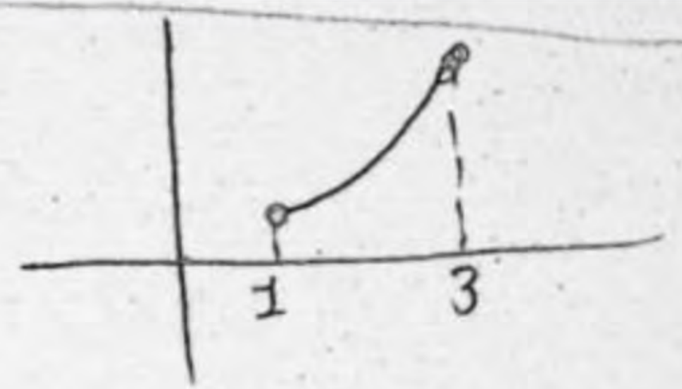
B For all x except at nine points

C For all x except at seven points

D For all x except at eight points

TAH-02

$f(x) = [x^2] + 1$



$x \in [1, 3]$

$x^2 \in [1, 9]$

$x^2 = 1, 2, 3, 4, 5, 6, 7, 8, 9$

become integer at $x = 1, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3$

1 is rejected @ 1 aquire minima.

Hence For all x except 8 point.

Kripa Shankar Maurya Varanasi

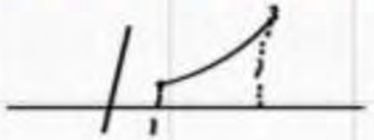
TAH-2

Q In $[1,3]$, the func. $[x^2+1]$. $[\cdot]$ GIF is continuous.

Solu

$f(x) = [x^2+1]$

$f(x) = [x^2] + 1$



$x \in [1,3]$

$x^2 \in [1,9]$

$x^2 = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$x = \pm 1, \pm\sqrt{2}, \pm\sqrt{3}, \pm 2, \pm\sqrt{5}, \pm\sqrt{6}, \pm\sqrt{7}, \pm\sqrt{8}, \pm 3$

$x = 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3$

↳ x point of minima

other are rejected Braus $x \in [1,3]$

∴ nu. of points of discontinuity = 8

nu. of points where $f(x)$ is continuous

↳ for all x except at eight points

Ans (D)

TAH 2 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION

TAH 3

Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$, is equal to (Where $[x]$ denotes greatest integer less than or equal to x)

A 14

B 13

C 10

D 8

TAH 4 RAHUL DHAKAD ❤️❤️ FROM AGRA UP

TAH-4

Q No. of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$ is equal to

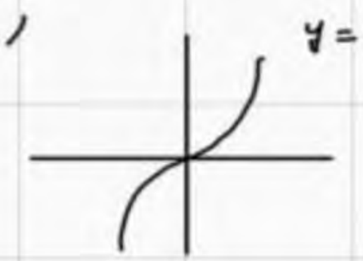
Solution → $f(x) = [2x^3 - 5]$, $x \in [1, 2)$

$f(x) = [2x^3] - 5$

$x \in [1, 2)$

$x^3 \in [1, 8)$

$2x^3 \in [2, 16)$



$2x^3 = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$

$x^3 = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8$

$x = 1, \left(\frac{3}{2}\right)^{1/3}, (2)^{1/3}$ ---------- 2

↓ minima

X

$x = 14$ points of discontinuity

QUESTION [JEE Mains 2024 (5 April)]

TAH 4

Let, $f:[-1,2] \rightarrow R$ be given by $f(x)=2x^2+x+\left[x^2\right]-[x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is

A 5

B 6

C 4

D 3

Ans. C

TAH-3

Q → Let $f: [-1,2] \to R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[\cdot]$ denotes the greatest Integer function that on equal to the no. of points where f is not continuous

Solve → $x \in [-1,2]$, $f(x) = 2x^2 + x + [x^2] - [x]$

$[x^2]$

$x \in [-1,2]$

$x^2 \in [0,4]$

$x^2 = 0, 1, 2, 3, 4$

$x = 0, \pm 1, \pm\sqrt{2}, \pm 2, +\sqrt{3}$

$x = 0, \pm 1, +\sqrt{2}, +2, +\sqrt{3}$

$\times 1$

$[x]$

$x \in [-1,2)$

$x = -1, 0, 1, 2$

$\times$

↓

point of minima

↓

points of minima

point of discontinuity

-1

1

$\sqrt{2}$

2

$+\sqrt{3}$

point of discontinuity

0

1

2

TAH 3 RAHUL DHAKAD

❤❤

FROM AGRA UP

points of discontinuity $0, -1, \sqrt{2}, +\sqrt{3}$

check at $x=1$ and $x=2$

at $x=1$

$\lim_{x \to 1^+} 2x^2 + x + [x^2] - [x]$

$= 2 + 1 + 1 - 1 = 3$, $f(1) = 2 + 1 + 1 - 1 = 3$

$\lim_{x \to 1^-} 2x^2 + x + [x^2] - [x]$

$= 2 + 1 + 0 - 0 = 3$

at $x=1$ $f(x)$ is continuous

at $x=2$

$\lim_{x \to 2^+} 2x^2 + x + [x^2] - [x]$

$= 8 + 2 + 4 - 2 = 12$ $\quad f(2) = 12$

$\lim_{x \to 2^-} = 8 + 2 + 3 - 1 = 12$

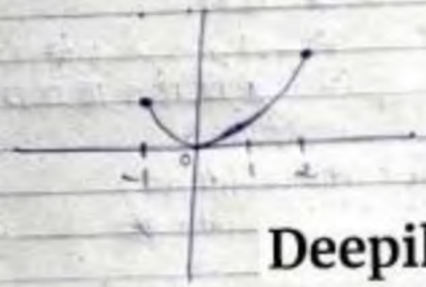
at $x=2$ $f(x)$ is continuous

$\Rightarrow$ total no. of point of discontinuity $= 4$

Toh3

Let $f:[-1,2] \to R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$ where $[t]$ denotes the greatest integer less than or equal to t. Then no. of points of discontinuity, is-

$[x^2]$



$x \in [-1,2]$

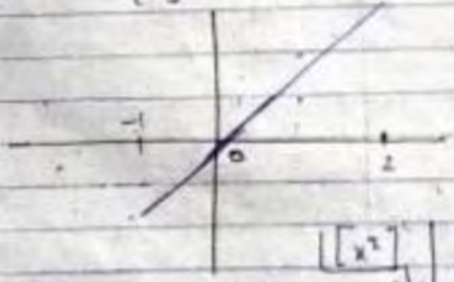
$x^2 \in [0,4]$

$x^2 = 0, 1, 2, 3, 4$

$x = 0, \pm 1, \sqrt{2}, \sqrt{3}, 2$

Deepika Dubey from UP

$[x]$



$x \in [-1,2]$

$x = 0, 1, 2$

$[x^2]$		$[x]$
-1	dis	cont
0	dis	discont
1	dis	dis
2	dis	dis
$\sqrt{2}$	dis	cont
$\sqrt{3}$	dis	cont

$f(x)$ is discontinuous at $x = -1, 0, \sqrt{2}, \sqrt{3}$

We need to check at $x = 1, 2$

TAH 3 continue

at $x = 1$ RHL $\lim_{x \to 1^+} 2x^2 + x + [x^2] - [x]$

$= \lim_{h \to 0} 2(1+h)^2 + (1+h) + [(1+h)^2] - [1+h]$

$\Rightarrow 2 + 1 + 1 - 1$

$\Rightarrow 3$

LHL $\lim_{x \to 1^-} 2x^2 + x + [x^2] - [x]$

$= \lim_{h \to 0} 2(1-h)^2 + (1-h) + [(1-h)^2] - [(1-h)]$

$= 2 + 1 + 0 - 0$

$\Rightarrow 3$

LHL = RHL = $f(1) = 3$

$f(1) = 2 + 1 + 1 - 1$

$= 3$

so at $x = 1$ fn is continuous

$x = 2$ RHL $\lim_{x \to 2^+} 2x^2 + x + [x^2] - [x]$

$= \lim_{h \to 0} 2(2+h)^2 + (2+h) + [(2+h)^2] - [(2+h)]$

$= 8 + 2 + 4 - 2 \Rightarrow 12$

LHL $\lim_{x \to 2^-} 2x^2 + x + [x^2] - [x]$

$\lim_{h \to 0} 2(2-h)^2 + (2-h) + [(2-h)^2] - [2-h]$

$\Rightarrow 8 + 2 + 3 - 1 \Rightarrow 12$

LHL = RHL = $f(2) = 12$

$f(2) = 2(4) + 2 + [4] - [2]$

$\Rightarrow 8 + 4 = 12$

so at $x = 2$ fn is continuous

no. of points of discontinuity $= -1, 0, \sqrt{2}, \sqrt{3}$

$= 4$ Ans

QUESTION [JEE Mains 2023 (11 April)]

TAH 5

Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is :

A continuous at $x = 0$, but not continuous at $x = 1$

B continuous at $x = 0$ and $x = 1$

C continuous at $x = 1$, but not continuous at $x = 0$

D not continuous at $x = 0$ and $x = 1$

Jan-25

Deepika Dubey from UP

Let $f(x) = [x^2 - x] + |-x + [x]|$, where $x \in R$ and $[t]$ t = GIF, then f is :

(A) continuous at $x = 0$ but not at $x = 1$
(B) continuous at $x = 0$ and $x = 1$
(C) continuous at $x = 1$ but not at $x = 0$
(D) not continuous at $x = 0$ and $x = 1$

$f(x) = [x^2 - x] + |-x + [x]|$

at $x = 0$

LHL $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} [(0-h)^2 - (0-h)] + |-(0-h) + [0-h]|$

$= [0 - 0] + |-0 + (-1)|$

$= 1$

RHL $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} [(0+h)^2 - (0+h)] + |-(0+h) + [0+h]|$

$= [0 - 0] + 0$

$= 0$

so $f(x)$ is discontinuous at $x = 0$

$f(0) = 0$, $f(0) \neq$ LHL + RHL

at $x = 1$

LHL $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} [(1-h)^2 - (1-h)] + |(-(1-h)) + [1-h]|$

$= 0 + |-1| = 1$

RHL at $x = 1$

$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} [(1+h)^2 - (1+h)] + |-(1+h) + [1+h]|$

$= [1 - 1] + |-1 + 1|$

$= 0$

$f(1) = 0$

$f(1) \neq$ LHL $\neq$ RHL

so f is not continuous at $x = 1$

(C) Ans

(Solution to KTK)

QUESTION

(KTK 1)

The following functions are not defined at $x = 0$. Find $f(0)$ so that f is continuous at $x = 0$.

(a) $f(x) = \frac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}$

(b) $f(x) = \frac{2-\sqrt[4]{x^2+16}}{\cos 2x-1}$

(c) $f(x) = \frac{1}{x} - \frac{2}{(e^{2x}-1)} (x \neq 0)$

(d) $f(x) = \frac{a-\sqrt{x+a^2}}{\sin bx}$ $(a > 0$ and $b \neq 0)$

Ans. (a) $(\ln 3)^2$; (b) $\frac{1}{64}$; (c) 1; (d) $-\frac{1}{2ab}$

(a) $F(x) = \dfrac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}$

$\Rightarrow \lim\limits_{x\to 0} F(x) = \lim\limits_{x\to 0}\left(\dfrac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}\right)$

$= \lim\limits_{x\to 0}\left(\left(\dfrac{3^x-1}{x}\right)^2 \cdot \dfrac{1}{\left(\dfrac{\sin x}{x}\right)\left(\dfrac{\ln(1+x)}{x}\right)}\right)$

$= (\ln 3)^2$

(b) $F(x) = \dfrac{9-\sqrt[4]{x^2+16}}{\cos 2x - 1}$

$\lim\limits_{x\to 0} F(x) = \lim\limits_{x\to 0}\left(\left(\dfrac{9-\sqrt[4]{x^2+16}}{\cos 2x-1}\right)\cdot\left(\dfrac{9+\sqrt[4]{x^2+16}}{9+\sqrt[4]{x^2+16}}\right)\right)$

$= \lim\limits_{x\to 0}\left(\left(\dfrac{81-\sqrt{x^2+16}}{\cos 2x-1}\right)\left(\dfrac{81+\sqrt{x^2+16}}{81+\sqrt{x^2+16}}\right)\left(\dfrac{1}{9+\sqrt[4]{x^2+16}}\right)\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{-1}{4\left(\dfrac{1-\cos 2x}{4x^2}\right)}\cdot\left(\dfrac{80-x^2}{81+\sqrt{x^2+16}}\right)\left(\dfrac{1}{9+\sqrt{16}}\right)\right)$

$= \dfrac{8}{8\times 4\times 4\times 2} = \dfrac{1}{64}$

(c) $F(x) = \dfrac{1}{x} - \dfrac{2}{(e^{2x}-1)}$

$\lim\limits_{x\to 0} F(x) = \lim\limits_{x\to 0}\left(\dfrac{e^{2x}-1-2x}{(e^{2x}-1)x}\right) = \lim\limits_{x\to 0}\left(\dfrac{e^{2x}-2x-1}{2x^2}\cdot 2\left(\dfrac{x}{e^{2x}-1}\right)\right)$

$= \lim\limits_{x\to 0}\left(\left(\dfrac{e^{2x}-2x-1}{(2x)^2}\right)\left(\dfrac{2}{\dfrac{e^{2x}-1}{2x}}\right)\right)$

$= \dfrac{1}{2}\cdot 2 = 1$ Ans

KTK 1
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(d) $F(x) = \dfrac{9-\sqrt{x+9^2}}{\sin bx}$

$\lim\limits_{x\to 0} F(x) = \lim\limits_{x\to 0}\left(\dfrac{9-\sqrt{x+9^2}}{\sin bx}\cdot\left(\dfrac{9+\sqrt{x+9^2}}{9+\sqrt{x+9^2}}\right)\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{81-x-81}{\sin bx}\left(\dfrac{1}{9+\sqrt{x+9^2}}\right)\right)$

$= \lim\limits_{y\to 0}\left(\dfrac{-1}{b\left(\dfrac{\sin bx}{bx}\right)}\cdot\left(\dfrac{1}{9+\sqrt{x+9^2}}\right)\right)$

$= \dfrac{-1}{9b}$

PART 1

KTK 1 RAHUL DHAKAD ❤❤ FROM AGRA UP

KTK → 1

Q → The following funs. are not defined at $x=0$. find $f(0)$ so that f is continuous at $x=0$

(i) $f(x) = \dfrac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}$

$$\lim_{x\to 0} \frac{(3^x-1)^2}{\sin x \cdot \ln(1+x)}$$

$$\lim_{x\to 0} \frac{\left(\frac{3^x-1}{x^2}\right)^2}{\frac{\sin x}{x}\cdot\frac{\ln(1+x)}{x}} = \frac{(\ln 3)^2}{1\cdot 1} = (\ln 3)^2 \text{ Ans.}$$

(ii) $f(x) = \dfrac{2-(x^4+16)^{1/4}}{\cos 2x - 1}$

$$f(x) = \frac{2-2\left(1+\left(\frac{x}{4}\right)^4\right)^{1/4}}{\cos 2x-1}$$

$$\lim_{x\to 0} \frac{\left[1-\left(1+\left(\frac{x}{4}\right)^4\right)^{1/4}\right]}{-(\sin x)^2}$$

$$\lim_{x\to 0} \frac{\left[\cancel{1}-\left(\cancel{1}+\frac{1}{4}\left(\frac{x}{4}\right)^4+(\,)\left(\frac{x}{4}\right)^8+\cdots\infty\right)\right]}{-\left[x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots\infty\right]^2}$$

$$\lim_{x\to 0} \frac{\left(\frac{1}{4}\frac{x^4}{16}+(\,)\frac{x^8}{4^8}+\cdots\infty\right)}{\left[x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots\infty\right]^2}$$

$$= \frac{\frac{1}{4}\cdot\frac{1}{16}}{1} = \boxed{\frac{1}{64} \text{ Ans}}$$

(iii) $f(x) = \dfrac{1}{x} - \dfrac{2}{(e^{2x}-1)} \quad (x\neq 0)$

$$\lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$\lim_{x\to 0} \frac{e^{2x}-1-2x}{x\cdot(e^{2x}-1)} = \lim_{x\to 0} \frac{\left[\cancel{1}+\frac{2x}{1!}+\frac{4x^2}{2!}+\cdots\infty\right]-\cancel{1}-2x}{x\left[\left(\cancel{1}+\frac{2x}{1!}+\frac{4x^2}{2!}+\cdots\infty\right)-\cancel{1}\right]}$$

$$= \lim_{x\to 0} \frac{\left[2x^2+\frac{8x^3}{3!}+\cdots\infty\right]}{x\left[2x+\frac{4x^2}{2!}+\cdots\infty\right]}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}\left[2+\frac{8x}{3!}+\cdots\infty\right]}{\cancel{x^2}\left[2+\frac{4x}{2!}+\cdots\infty\right]}$$

$$= \frac{2}{2} = \boxed{1 \text{ Ans}}$$

(iv) $f(x) = \dfrac{a-\sqrt{x+a^2}}{\sin bx} \quad (a>0 \text{ and } b\neq 0)$

$$= \lim_{x\to 0} \frac{a-(x+a^2)^{1/2}}{\sin bx}$$

$$= \lim_{x\to 0} \frac{a-a\left(1+\frac{x}{a^2}\right)^{1/2}}{\sin bx} = \lim_{x\to 0} \frac{a\left[1-\left(1+\frac{x}{a^2}\right)^{1/2}\right]}{\sin bx}$$

$$= \lim_{x\to 0} \frac{a\left[\cancel{1}-\left(\cancel{1}+\frac{1}{2}\cdot\frac{x}{a^2}+(\,)\frac{x^2}{a^4}+\cdots\infty\right)\right]}{\left[bx-\frac{b^3x^3}{3!}+\frac{b^5x^5}{5!}-\cdots\infty\right]}$$

$$= \lim_{x\to 0} \frac{-a\cancel{x}\left[\frac{1}{2a^2}+(\,)\frac{x}{a^4}+\cdots\infty\right]}{\cancel{x}\left[b-\frac{b^3x^2}{3!}+\cdots\infty\right]}$$

$$= \boxed{\frac{-1}{2ab} \text{ Ans}}$$

PART 2

KTK 1 RAHUL DHAKAD ❤❤ FROM AGRA UP

QUESTION

(KTK 2)

The function $f(x) = \frac{4-x^2}{4x-x^3}$, is

A discontinuous at only one point in its domain.

B discontinuous at two points in its domain.

C discontinuous at three points in its domain.

D continuous everywhere in its domain.

Ans. D

KTK-2

Q → The func. $f(x) = \frac{4-x^2}{4x-x^3}$ is

Solve $f(x) = \frac{4-x^2}{4x-x^2}$

$4x - x^2 \neq 0$

$x^2 - 4x \neq 0$

$x(x-4) \neq 0$

$x \neq 0, 4$

Domain: $R - \{0, 4\}$

$f(x)$ is polynomial func. ⇒ His continuous everywhere in its domain

KTK 2 RAHUL DHAKAD

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FROM AGRA UP

QUESTION

(KTK 3)

If $f(x) = \begin{cases} \frac{\cos \pi x + \sin\left(\frac{\pi x}{2}\right)}{(x-1)(3x^2-2x-1)} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$

is continuous at $x = 1$, then find the value of k.

Ans. $\frac{3\pi^2}{32}$

$$f(x) = \begin{cases} \dfrac{\cos \pi x + \sin\left(\frac{\pi x}{2}\right)}{(x-1)(3x^2-2x-1)} & , x \neq 1 \\ k & , x = 1 \end{cases}$$

$$\lim_{x \to 1^+} \frac{\cos \pi x + \sin\left(\frac{\pi x}{2}\right)}{(x-1)(x-1)(3x+1)}$$

Let $x \to 1+t$

as $x \to 1$, $t \to 0$

$$\lim_{t \to 0} \frac{-\cos \pi t + \cos \frac{\pi}{2} t + 1 - 1}{t^2(3t+4)}$$

$$\lim_{t \to 0} \frac{\left(\frac{1-\cos \pi t}{(\pi t)^2}\right)(\pi t)^2 - \left(\frac{1-\cos \frac{\pi}{2} t}{\left(\frac{\pi}{2} t\right)^2}\right)\left(\frac{\pi}{2} t\right)^2}{t^2(3t+4)}$$

$$\lim_{t \to 0} \frac{\frac{\pi^2}{2} - \frac{\pi^2}{8}}{4} \Rightarrow \frac{3\pi^2}{32}$$

Ans

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KTK-3 :→

Q → If $f(x) = \begin{cases} \dfrac{\cos \pi x + \sin\left(\frac{\pi x}{2}\right)}{(x-1)(3x^2-2x-1)} & , \text{if } x \neq 1 \\ K & , \text{if } x = 1 \end{cases}$

is continuous at $x = 1$, then find the value of K

Solution →

$$\lim_{x \to 1} \frac{\cos \pi x + \sin \frac{\pi x}{2}}{(x-1)(3x^2-2x-1)}$$

put $x = 1+h$

$x \to 1$

$h \to 0$

$$\lim_{h \to 0} \frac{-\cos \pi h + \cos \frac{\pi}{2} h}{h(3(h+1)^2 - 2(1+h) - 1)}$$

$$\lim_{h \to 0} \frac{\cos \frac{\pi}{2} h - \cos \pi h}{h(3h^2 + 6h + 3 - 2 - 2h - 1)}$$

$$\lim_{h \to 0} \frac{\cos \frac{\pi}{2} h - \cos \pi h}{h^2(3h+4)}$$

$$\lim_{h \to 0} \frac{2 \sin \frac{3\pi}{4} h \cdot \sin \frac{\pi}{4} h}{\frac{3\pi}{4} h \cdot \frac{\pi}{4} h \cdot (3h+4)} \cdot \frac{\frac{3\pi}{4} \cdot \frac{\pi}{4}}{}$$

$$= \frac{2 \cdot (1)(1) \cdot 3\pi^2}{16 \cdot 4}$$

$$= \frac{3}{32} \pi^2$$

$$\lim_{x \to 1} f(x) = f(1)$$

$$\frac{3}{32} \pi^2 = K \quad \text{Ans}$$

KTK 3 RAHUL DHAKAD

❤❤

FROM AGRA UP

QUESTION

(KTK 4)

For the function $f(x) = \frac{1}{x+2^{\frac{1}{(x-2)}}}$, $x \neq 2$ which of the following holds?

A $f(2) = 1/2$ and f is continuous at $x = 2$

B $f(2) \neq 0, 1/2$ and f is continuous at $x = 2$

C f can not be continuous at $x = 2$

D $f(2) = 0$ and f is continuous at $x = 2$.

Ans. C

KTG4

$$f(x) = \frac{1}{x + 2^{1/(x-2)}}, \quad x \neq 2$$

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$$\lim_{x \to 2^+} \frac{1}{x + 2^{\frac{1}{(x-2)}}} \qquad 2^{\infty}$$

$$= \frac{1}{\infty} = 0$$

$$\lim_{x \to 2^-} \frac{1}{x + 2^{\frac{1}{(x-2)}}}$$

$\frac{1}{(x-2)}$ → −ve

$$= \frac{1}{2 + 2^{1/0^-}} \qquad 2^{-\infty}$$

$$= \frac{1}{2}$$

LHL ≠ RHL ∴ f Can not be Cont at x = 2

Ans

QUESTION

(KTK 5)



If $f(x) = \begin{cases} \sin x, & x \neq n\pi, n = 0, \pm1, \pm2, \pm3 \ldots \\ 2, & \text{otherwise} \end{cases}$ and $g(x) = \begin{cases} x^2 + 1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$

then $\lim\limits_{x \to 0} g(f(x))$ is equal to

$\lim\limits_{x \to 0} g(f(x))$

$\lim\limits_{x \to 0} g(\sin x)$ nonzero

$= \lim\limits_{x \to 0} (\sin^2 x + 1) = 1$

A) 1

B) 4

C) 5

D) non existent

Ans. A

QUESTION

(KTK 6)

$$\lim_{n\to\infty}\left(\cos\frac{x}{2}\cdot\cos\frac{x}{4}\ldots\ldots\cos\frac{x}{2^n}\right)$$

Ans. $\frac{\sin x}{x}$

Ex-6

$$\lim_{n\to\infty} \left(\cos\frac{x}{2}\cos\frac{x}{4}\cdots\cos\frac{x}{2^n}\right)$$

$$\lim_{n\to\infty} \frac{\sin 2^n\cdot\frac{x}{2^n}}{2^n \sin\frac{x}{2^n}}$$

$$\lim_{n\to\infty} \frac{\sin x}{x}\cdot\frac{\frac{x}{2^n}}{\sin\frac{x}{2^n}} = \frac{\sin x}{x}$$ Ans

QUESTION

(KTK 7)

$$\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2} - \arctan\frac{x}{x+2}\right)$$

Ans. 1/2

KTK-7

$$\lim_{x\to\infty} x\left(\tan^{-1}\frac{x+1}{x+2} - \tan^{-1}\frac{x}{x+2}\right)$$

$$\lim_{x\to\infty} x \tan^{-1}\frac{1/(x+2)}{1+\frac{(x+1)x}{(x+2)^2}}$$

$$\lim_{x\to\infty} x \tan^{-1}\frac{(x+2)}{(x+2)^2+(x+1)x}$$

$$\frac{(x+2)}{(x+2)^2+(x+1)x} \propto \frac{(x+2)^2+(x+1)x}{(x+2)}$$

$$\lim_{x\to\infty} \frac{x}{\frac{(x+2)^2+(x+1)x}{x+2}} \quad (\quad)$$

$$= \lim_{x\to\infty} \frac{x}{\frac{x^2[(1+1/x)^2+(1+1/x)^2]}{x(1/x+1)}} \quad (\quad)$$

$$= 1/2 \quad \text{Ans}$$

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QUESTION

(KTK 8)

$$\lim_{x\to\infty} x\left(\arctan\frac{x+1}{x+2}-\frac{\pi}{4}\right)$$

Ans. $-1/2$

KTK 8

$$\lim_{x\to\infty} x\left(\tan^{-1}\frac{x+1}{x+2} - \text{F/A}\right)$$

$$\lim_{x\to\infty} x\left(\tan^{-1}\frac{x+1}{x+2} - \tan^{-1}1\right)$$

$$\lim_{x\to\infty} x \tan^{-1}\left(\frac{-1/(x+2)}{1+\frac{(x+1)}{(x+2)}}\right)$$

$$-\lim_{x\to\infty} x \tan^{-1}\frac{\frac{1}{2x+3}}{\frac{1}{(2x+3)}\times(2x+3)}$$

$$-\lim_{x\to\infty}\frac{x}{x(2+3/x)}\left(\frac{1}{1}\right)$$

$-1/2$

Sattick Nayok
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QUESTION

(KTK 9)

$$\lim_{x \to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

Ans. $\frac{\alpha}{\beta}$

Ex-9

$$\lim_{x\to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

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$$\lim_{x\to 0} \frac{\frac{(e^{\alpha x}-1)}{\alpha x}\alpha x + \frac{(1-\cos \alpha x)}{(\alpha x)^2}\alpha^2 x^2}{\frac{(e^{\beta x}-1)}{\beta x}\beta x + \frac{1-\cos \beta x}{(\beta x)^2}\beta^2 x^2}$$

$$\lim_{x\to 0} \frac{\frac{(e^{\alpha x}-1)}{\alpha x}\alpha + \frac{1-\cos \alpha x}{(\alpha^2 x^2)}\alpha^2 x}{\frac{(e^{\beta x}-1)}{\beta x}\beta + \frac{1-\cos \beta x}{(\beta x)^2}\beta^2 x} = \alpha/\beta \quad \text{Ans}$$

KTK-9

$$y = \lim_{x \to 0} \left(\frac{e^{ax} - \cos \alpha x}{e^{\beta x} - \cos \beta x} \right)$$

$$y = \lim_{x \to 0} \left(\frac{e^{ax} - 1 + 1 - \cos \alpha x}{e^{\beta x} - 1 + 1 - \cos \beta x} \right)$$

$$= \lim_{x \to 0} \left(\frac{\left(\frac{e^{ax} - 1}{ax}\right) ax + \left(\frac{1 - \cos \alpha x}{\alpha^2 x^2}\right) \alpha^2 x^2}{\left(\frac{e^{\beta x} - 1}{\beta x}\right) \beta x + \left(\frac{1 - \cos \beta x}{\beta^2 x^2}\right) \beta^2 x^2} \right)$$

$$= \boxed{\frac{a}{\beta}}$$

KTK 9
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KTK-9

KTK 9 RAHUL DHAKAD ❤❤ FROM AGRA UP

$$\text{Q} \rightarrow \lim_{x\to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

METHODE 1

Solve → (m-1)

$$\lim_{x\to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

$$\lim_{x\to 0} \frac{\alpha x \dfrac{e^{\alpha x}-1}{\alpha x} + \dfrac{1-\cos \alpha x}{\alpha^2 x^2}\cdot \alpha^2 x^2}{\beta x \dfrac{e^{\beta x}-1}{\beta x} + \dfrac{1-\cos \beta x}{\beta^2 x^2}\beta^2 x^2}$$

$$\lim_{x\to 0} \frac{x\left[\alpha \dfrac{e^{\alpha x}-1}{\alpha x} + \dfrac{1-\cos \alpha x}{\alpha^2 x^2}\cdot \alpha^2 x\right]}{x\left[\beta \dfrac{e^{\beta x}-1}{\beta x} + \dfrac{1-\cos \beta x}{\beta^2 x^2}\cdot \beta^2 x\right]}$$

$$= \frac{\alpha}{\beta} \quad \text{Ans}$$

(m-2)

$$\lim_{x\to 0} \frac{e^{\alpha x} - \cos \alpha x}{e^{\beta x} - \cos \beta x}$$

$$= \lim_{x\to 0} \frac{\left(1+\frac{\alpha x}{1!}+\frac{\alpha^2 x^2}{2!}+\frac{\alpha^3 x^3}{3!}+\cdots\infty\right)-\left(1-\frac{\alpha^2 x^2}{2}+\frac{\alpha^4 x^4}{4}-\frac{\alpha^6 x^6}{6}+\cdots\infty\right)}{\left(1+\frac{\beta x}{1!}+\frac{\beta^2 x^2}{2!}+\frac{\beta^3 x^3}{3!}+\cdots\infty\right)-\left(1-\frac{\beta^2 x^2}{2}+\frac{\beta^4 x^4}{4}-\frac{\beta^6 x^6}{6}+\cdots\infty\right)}$$

$$= \lim_{x\to 0} \frac{x}{x} \frac{\left(1+\frac{\alpha}{1!}+\frac{\alpha^2 x}{2!}+\frac{\alpha^3 x^2}{3!}+\cdots\infty\right)-\left(1-\frac{\alpha^2 x^2}{2}+\frac{\alpha^4 x^4}{4}-\frac{\alpha^6 x^6}{6}+\cdots\infty\right)}{\left(1+\frac{\beta}{1!}+\frac{\beta^2 x}{2!}+\frac{\beta^3 x^2}{3!}+\cdots\infty\right)-\left(1-\frac{\beta^2 x^2}{2}+\frac{\beta^4 x^4}{4}-\frac{\beta^6 x^6}{6}+\cdots\infty\right)}$$

$$= \frac{\alpha}{\beta} \quad \text{Ans}$$

METHODE 2

QUESTION

(KTK 10)

The value of the limit $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$ is:

A 1

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

Ans. D

KTK-10

Q The value of the limit $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)$

Ans $= \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\cdots 1.$

$= \left(1-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\cdots 1.$

$= \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{3}{4}\cdot\frac{5}{4}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdots 1$

$= \frac{1}{2}\cdot 1 = \frac{1}{2}$ Ans

KTK 10 RAHUL DHAKAD❤️❤️ FROM AGRA UP

(Solution to RPP)

QUESTION [JEE Mains 2024 (6 April)] (RPP 1)

Let α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0, t \in \mathbb{R}$ and $a_n = \alpha^n + \beta^n$. Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

A $-1/2$

B $-1/4$

C $1/4$

D $1/2$

Ans. B

R.P.1. ⓪

$$x^2 - (t^2-5t+6)x + 6_1 = 0$$

$\Rightarrow$. using Newton's law,

$\Rightarrow$ $S_{2025} - (t^2-5t+6)S_{2024} + S_{2023} = 0$

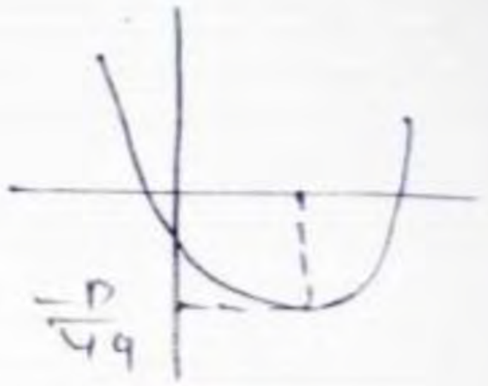
$\Rightarrow$ $\dfrac{S_{2025} + S_{2023}}{S_{2024}} = t^2-5t+6$

$$(t^2-5t+6)_{min} = -\frac{D}{4a}$$

$$= -\left(\frac{25-24}{4}\right)$$

$$= -\frac{1}{4}$$

$$\left(\frac{S_{2025}+S_{2023}}{S_{2024}}\right)_{min} = -\frac{1}{4}$$ Ans



RPP 1
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QUESTION [JEE Advanced 2021 (Paper 1)] **(RPP 2)**

For $x \in R$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is

Ans. 4

R.P.P.② $3x^2 - 4|x^2-1| + x - 1 = 0$,

$\Rightarrow$ Case① $x^2 - 1 \geq 0$

$(x+1)(x-1) \geq 0$

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty)$

$\Rightarrow 3x^2 - 4x^2 + 4 + x - 1 = 0$

$\Rightarrow x^2 - x - 3 = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{13}}{2}$

$x = \frac{1 \pm \sqrt{13}}{2} \in (-\infty, -1] \cup [1, \infty)$ (Accepted)

Case② $x^2 - 1 \leq 0$

$x \in (-1, 1)$

$\Rightarrow 3x^2 + 4x^2 - 4 + x - 1 = 0$

$\Rightarrow 7x^2 + x - 5 = 0$

$x = \frac{-1 \pm \sqrt{141}}{14}$

$x = \frac{-1 \pm \sqrt{141}}{14}$

$x = \frac{-1 \pm \sqrt{141}}{14} \in (-1, 1)$ (Accepted)

No. of real root = 4

RPP 2
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QUESTION [JEE Mains 2024 (27 Jan)] (RPP 3)

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then:

A $2S_{12} = S_{11} + S_{10}$

B $S_{12} = S_{11} + S_{10}$

C $S_{11} = S_{10} + S_{12}$

D $2S_{11} = S_{12} + S_{10}$

Ans. B

R.P.P (III)

$x^2 - x - 1$ → α, β

$$S_n = 2023\alpha^n + 2024\beta^n.$$

$\Rightarrow$.

$$S_{n+2} - S_{n+2} - S_n = 0$$

$$S_{n+2} = S_{n+2} + S_n \quad (n=10)$$

$$\boxed{S_{12} = S_{11} + S_{10}}$$

Ans

RPP 3
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(Solution to Home Challenge)

Home Challenge - 10

Which of the following is the solution set of the equation

$2\cos^{-1}(x) = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$ **?**

A $(0, 1)$

B $(-1, 1) - \{0\}$

C $(-1, 0)$

D $[-1, 1]$

Homechallenge → 10

Q → which of the following is the solution set of the equation

$$2\cos^{-1}(x) = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$$

Solv: put $x = \cos\theta$, $\theta \in (0,\pi) - \{\pi/2\}$, $x \in (-1,1) - \{0\}$

$$2\cos^{-1}(\cos\theta) = \cot^{-1}\left(\frac{2\cos^2\theta - 1}{2\cos\theta\sqrt{1-\cos^2\theta}}\right)$$

$$2\cos^{-1}(\cos\theta) = \cot^{-1}\left(\frac{\cos 2\theta}{2\sin\theta\cdot\cos\theta}\right)$$

$$2\cos^{-1}(\cos\theta) = \cot^{-1}(\cot 2\theta)$$

$$2\theta = 2\theta \Rightarrow \quad 2\theta \in (0,\pi)$$

$$\theta \in \left(0, \frac{\pi}{2}\right)$$

$$\cos^{-1}x \in \left(0, \frac{\pi}{2}\right)$$

$$\boxed{x \in (0,1)}$$ Ans

HOME CHALLENGE 10

RAHUL DHAKAD ❤❤
FROM AGRA UP


THANK
YOU

PRAYAS

JEE 2025

Lecture-12 | Mathematics

Limit, Continuity and Differentiability



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Topics *to be covered*

1. Some Important Points
2. Introduction to MOD

Recap *of previous lecture*

1. If LHD & RHD of a function 'f' are finite but unequal at x = a then f' is non derivable at x = a but continuous at x = a & it geometrically implies a sharp corner at x = a.

2. f is derivable in [a, b] if

(i) f is derivable in (a, b)

(ii) (At x = a) $RHD = f'(a^+) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ = a fq

(iii) (At x = b) $LHD = f'(b^-) = \lim_{h \to 0} \frac{f(b-h) - f(b)}{-h}$ = a fq

Recap of previous lecture

for limit Exist

$\lim_{x\to 2}(f(x)-3)=0$

$\lim_{x\to 2} f(x)=3$

f is derivable ⇓ f is continuous

$f(2)=3$

3. ~~(a)~~ Let $y = f(x)$ be a derivable function s.t. $\lim_{x\to 2}\frac{f(x)-3}{x-2} = 5$ then equation of tangent at $x = 2$ is $y=5x-7$

$\frac{0}{0}$

$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=5$

$\Rightarrow f'(2)=5$

T: $y-3=5(x-2)$

(b) Let $g: R \to R$ be a differentiable function such that $g(2) = -40$ and $g'(2) = -5$. Then $\lim_{x\to 0}\left(\frac{g(2-x^2)}{g(2)}\right)^{\frac{4}{x^2}}$ is equal to

(A) e^{32} (B) $\sqrt{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) e^{-5}

T: $y=5x-7$

Recap *of previous lecture*

3. (a) Let $y = f(x)$ be a derivable function s.t. $\lim_{x\to 2} \frac{f(x)-3}{x-2} = 5$ then equation of tangent at $x = 2$ is $y = 5x - 7$

(b) Let $g: R \to R$ be a differentiable function such that

$g(2) = -40$ and $g'(2) = -5$. Then $\lim_{x\to 0} \left(\frac{g(2-x^2)}{g(2)}\right)^{\frac{4}{x^2}}$ is equal to 1^{∞}

(A) e^{32} (B) $\sqrt{e}$ (C) $\frac{1}{\sqrt{e}}$ (D) e^{-5}

$$e^{\lim_{x\to 0} \frac{4}{x^2}\left(\frac{g(2-x^2)}{g(2)} - 1\right)}$$

$$e^{\lim_{x\to 0} \frac{g(2-x^2)-g(2)}{g(2)} \cdot \frac{4}{x^2}}$$

LHD. at $x=2$

$$e^{4\lim_{x\to 0} \frac{g(2-x^2)-g(2)}{-x^2} \cdot \frac{-x^2}{g(2)} \cdot \frac{1}{x^2}}$$

$$= e^{\frac{-4\cdot g'(2)}{g(2)}}$$

$$= e^{\frac{-4\cdot(-5)}{-40}} = e^{-1/2} = \frac{1}{\sqrt{e}}.$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$$

Recap *of previous lecture*

4. If f(x) is a derivable function |f(x)| is non derivable at x = a if _____ $f(a)=0$ _____ & _____ $f'(a)\neq 0$ _____

5. If f(x) is a derivable function f(a) = 0 then |f(x)| is derivable at x = a if _____ $f'(a)=0$ _____

6. If f is discontinuous at x = a then it is non derivable at x = a. (T/F)

7. If f is non derivable at x = a then f may be continuous at x = a. (T/F)

Recap *of previous lecture*

8. For applying shortcut to check differentiability at x = a for

$$f(x) = \begin{cases} g(x) & x > a \\ P & x = a \\ h(x) & x < a \end{cases}$$

two conditions should be satisfied which are

① f should be continuous at $x=a$

② $g(x), h(x)$ should be smooth fn i.e they should have 1st derivative continuous

Recap *of previous lecture*

9. If $f(x) = \begin{cases} \sin x + \cos x, & 0 \le x < \pi/2 \\ \sin^2 x - \cos x + 7, & \pi/2 \le x < \pi \\ \cos^2 x - \sin x + 6, & \pi \le x < 2\pi \end{cases}$

discontinuous at $x = \pi/2, \pi$

$\Downarrow$

non derivable at $x = \pi/2, \pi$

then $f'(x) = \begin{cases} \cos x - \sin x & 0 \le x < \pi/2 \\ 2\sin x \cos x + \sin x & \pi/2 < x < \pi \\ -2\sin x \cos x - \cos x & \pi < x < 2\pi \end{cases}$

Homework Discussion

QUESTION [JEE Mains 2024 (1 Feb)] TAH 6

Let $f: R \to R$ be defined as: $f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2; & 0 \le x \le 1 \\ 2x + 1; & x > 1 \end{cases}$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals:

A 1

B 4

C 3

D 2

Continuous at $x=0, x=1$

at $x=0$ LHL $\lim_{x\to 0^-} \frac{a-b\cos 2x}{x^2} = \lim_{x\to 0^+} x^2+cx+2 = f(0)=2$. (RHL)

$$\lim_{x\to 0^-} \frac{a-b\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\cdots\infty\right)}{x^2} = 2$$

$$\lim_{x\to 0^-} \frac{(a-b)-b\left(-\frac{4x^2}{2!}+\frac{16x^4}{4!}+\cdots\infty\right)}{x^2} = 2$$

Ans. D

$a-b=0$

$$\lim_{x\to 0^-} \frac{-b\left(-\frac{4x^2}{2!}+\cdots-\infty\right)}{x^2}=2$$

$$\frac{4b}{2}=2$$

$a=1$ $\quad b=1$

AT $x=1$

$$LHL=\lim_{x\to 1^-} x^2+cx+2=\lim_{x\to 1^+} 2x+1=f(1)=1+c+2$$

$1+c+2=3$

$c=0$

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2} & x<0\\ x^2+2 & 0\le x\le 1\\ 2x+1 & x>1\end{cases}$$

AT $x=1$ $\quad LHD=\lim_{x\to 1}\frac{d}{dx}(x^2+2)=2$ $\quad RHD\ \lim_{x\to 1}\frac{d}{dx}(2x+1)=2$

Derivable at $x=1$

At $x=0$

RHD $\lim_{x\to 0} \frac{d}{dx}(x^2+2) = 0$

LHD $\lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

Derivable at $x=0$

$\lim_{h\to 0} \frac{1-\cos(-2h)-2}{(-h)^2}$

$-h.$

$\lim_{h\to 0} \frac{1-\cos 2h-2h^2}{-h^3}.$

$\lim_{h\to 0} \frac{1-\left(1-\frac{(2h)^2}{2!}+\frac{(2h)^4}{4!}-\cdots\right)-2h^2}{h^3} = 0$

$m=0$

$a=1 \quad c=0$

$b=1$

$m+a+b+c=2$

QUESTION [JEE Mains 2022 (25 June)]

ASRQ

TAH 10

Let [t] denote the greatest integer less than or equal to t. Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$.
Then h is :

(A) continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$

(B) not continuous at exactly three points in $[-2, 2]$

(C) continuous in $[-2, 2]$ but not differentiable at exactly three points in $(-2, 2)$

(D) not continuous at exactly four points in $[-2, 2]$

$f(x) = x - [x] = \{x\}$ $g(x) = 1 - (x - [x]) = 1 - \{x\}$

We know

$$\{x\}+\{-x\}=\begin{cases}0 & x\in I\\ 1 & x\notin I\end{cases}$$

If $x\notin I$ $\quad \{x\}+\{-x\}=1$

$$\{-x\}=1-\{x\}$$

& Obsiously $x\in I$, $\{x\}=0$

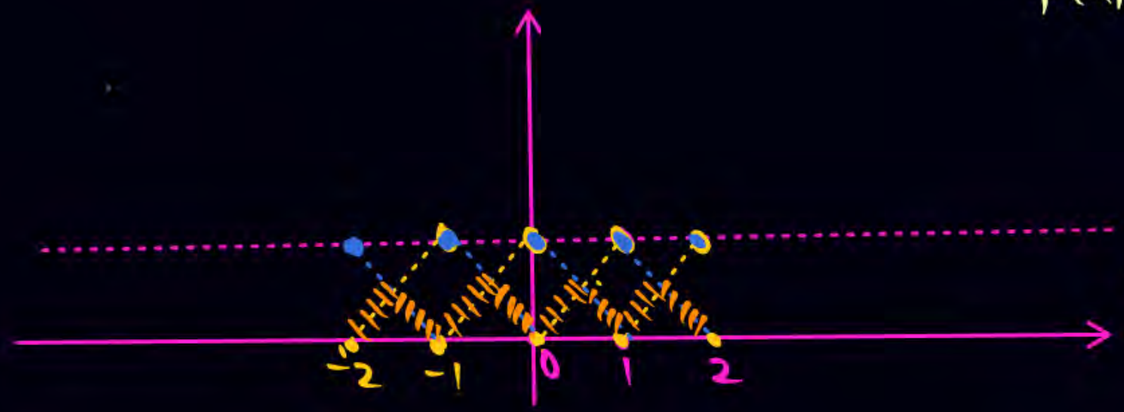
$y=f(x)=\{x\}$

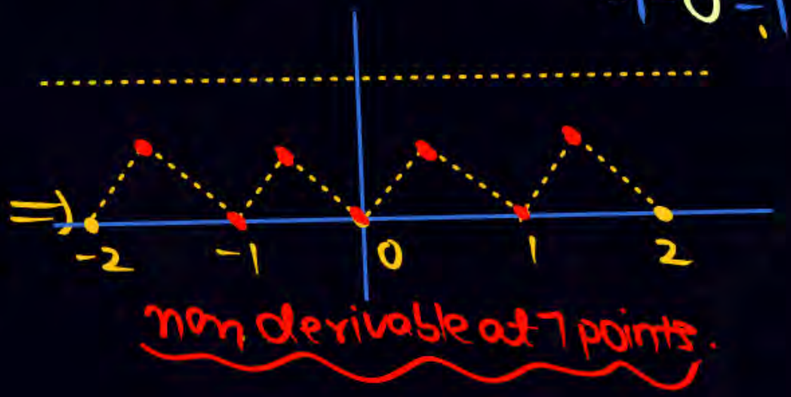
↓ reflect graph about y axis

$y=f(-x)=\{-x\}$

⇓

$$f(x)=\{x\},\ g(x)=1-\{x\}=\begin{cases}\{-x\} & x\notin I\\ 1-0=1 & x\in I\end{cases}$$





non derivable at 7 points.

QUESTION

(KTK 4)

f is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

A can not be determined

B is $2(1 - \sqrt{3})$

C is zero

D is $\frac{2(\sqrt{3}-2)}{\sqrt{3}}$

$x^2 - 2x + 2\sqrt{3} - 3 + (x - \sqrt{3}) f(x) = 0$

$f(x) = -\frac{(x^2 - 2x + 2\sqrt{3} - 3)}{x - \sqrt{3}}$.

$f(x) = -\frac{(x^2 - 3 - 2x + 2\sqrt{3})}{x - \sqrt{3}}$. $= -\frac{((x-\sqrt{3})(x+\sqrt{3}) - 2(x-\sqrt{3}))}{(x-\sqrt{3})}$

$= -\frac{(x-\sqrt{3})(x+\sqrt{3}-2)}{x-\sqrt{3}}$

$f(x) = -(x + \sqrt{3} - 2) \quad x \neq \sqrt{3}$.

Ans. B

Since f is continuous

$$\lim_{x \to \sqrt{3}} f(x) = f(\sqrt{3})$$

$$\lim_{x \to \sqrt{3}} -(x + \sqrt{3} - 2) = -(2\sqrt{3} - 2)$$

$$= \underline{2 - 2\sqrt{3}}$$

$$= \underline{2(1 - \sqrt{3})}$$

Aao Machay Dhmaal Deh Swaal
pe Deh Swaal

QUESTION [JEE Advanced 2016]

Let $a, b \in R$ and $f : R \to R$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is

(A) differentiable at $x = 0$ if $a = 0$ and $b = 1$

(B) differentiable at $x = 1$ if $a = 1$ and $b = 0$

(C) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$

(D) NOT differentiable at $x = 1$ if $a = 1$ and $b = 1$

QUESTION

ASRQ

★★★★★

Gadho/Gadhiyoo aisaa naa karo!! $\lim_{h\to 0}\frac{\sin\frac{1}{h}}{\frac{1}{h}}=1$

Let $f(x)=\begin{cases} x^2\cos\frac{1}{x}, & x<0 \\ 0, & x=0 \\ x^2\sin\frac{1}{x}, & x>0 \end{cases}$, then which of the following is(are) correct?

(A) f(x) is continuous but not differentiable at x = 0.

(B) f(x) is continuous and differentiable at x = 0.

(C) f′(x) is continuous but not differentiable at x = 0.

(D) f′(x) is discontinuous at x = 0.

RHD

$\lim_{h\to 0}\frac{h^2\sin 1/h-0}{h}$

$\lim_{h\to 0} h\sin 1/h=0$

LHD

$\lim_{h\to 0}\frac{h^2\cos(-1/h)-0}{-h}$

$\lim_{h\to 0}\frac{h^2\cos 1/h}{-h}=-\lim_{h\to 0} h\cos\frac{1}{h}$

$=0$

LHD = RHD = 0 ⇒ f′(0) = 0

f is derivable at x = 0

⇓

f is continuous at x = 0

$$f(x)=\begin{cases} x^2\sin\frac{1}{x} & x>0 \\ 0 & x=0 \\ x^2\cos\frac{1}{x} & x<0 \end{cases}$$

$$f'(x)=\begin{cases} 2x\sin\frac{1}{x}+x^2\cdot\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right) & x>0 \\ 0 & x=0 \\ 2x\cos 1/x+x^2(-\sin 1/x)(-1/x^2) & x<0 \end{cases}$$

$$f'(x)=\begin{cases} 2x\sin 1/x-\cos 1/x & x>0 \\ 0 & x=0 \\ 2x\cos 1/x+\sin 1/x & x<0 \end{cases}$$

Not continuous at $x=0$

Ex:

$$Df = [0,2) \qquad f(x) = \begin{cases} x^2+2x+3 & 0 \le x \le 1 \\ 5x+1 & 1 < x < 2 \end{cases}$$

$$Df' = [0,2) - \{1\} \qquad f'(x) = \begin{cases} 2x+2 & 0 \le x < 1 \\ 5 & 1 < x < 2 \end{cases}$$

LHD $2x+2\big|_{x=1} = 4$ AT $x=1$

$5 = \underline{RHD}$

$\Downarrow$

AT $x=1$ nonderivable



Tan!

Ex:

$$f(x)=\begin{cases} x^3+6x^2+5x+1 & x<0 \\ x^2+1 & 0\le x<1 \\ x^2+6x+5 & 1\le x\le 2 \end{cases}$$

find $f'(x)$

Some Important Points

Same in limits + continuity

(1) If $f(x)$ and $g(x)$ both are derivable at $x = a$, then

(i) $f(x) \pm g(x)$ will be differentiable at $x = a$.

(ii) $f(x) \cdot g(x)$ will be differentiable at $x = a$.

(iii) $\frac{f(x)}{g(x)}$ will be differentiable at $x = a$ if $g(a) \neq 0$.

(2) If $f(x)$ is derivable at $x = a$ and $g(x)$ is not derivable at $x = a$ then the $f(x) + g(x)$ or $f(x) - g(x)$ will not be derivable at $x = a$, however nothing definite can said about their product or Quotient functions.

	$f(x)$	$g(x)$	$f(x) \pm g(x)$	$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)}$
					$(\lim_{x \to a} g(x) \neq 0)$
Limit	Exist E D.N.E	Exist D.N.E D.N.E	✓ ✗ can not say	✓ can not say can not say	✓ can not say can not say
Continuity	C C D	C D D	C D can not say	C can not say can not say	C $\longrightarrow (g(a) \neq 0)$ can not say can not say.
Diff.	Diff Diff Not Diff	Diff Not Diff Not Diff	Diff Not Diff can not say	Diff can not say can not say	Diff $\longrightarrow (g(a) \neq 0)$ can not say can not say.

Astonishing in continuity

$h(x) = f(x) \cdot g(x)$

f: continuous at $x=a$ & $f(a)=0$

g: g is discontinuous at $x=a$ but $g(a)$ is defined & $g(x)$ has finite LHL & RHL at $x=a$

$h(x)$ is continuous at $x=a$

An Astonishing Result

If f(x) is derivable at x = a and f(a) = 0 and g(x) is continuous at x = a then the product function F(x) = f(x) · g(x) will be derivable at x = a

$F(x) = f(x) \cdot g(x)$ — g is continuous at $x=a$

↓

derivable at $x=a$ & $f(a)=0$

⇓

$F(x)$ is derivable at $x=a$.

Proof : AT $x=a$

$$R.H.D = \lim_{h\to 0} \frac{F(a+h)-F(a)}{h} = \lim_{h\to 0} \frac{f(a+h)\cdot g(a+h) - f(a)\cdot g(a)}{h} \quad (f(a)=0)$$

$$= \lim_{h\to 0} \frac{f(a+h)\cdot g(a+h)}{h} = \lim_{h\to 0} \left(\frac{f(a+h)-f(a)}{h}\right) g(a+h)$$

$$RHD = f'(a)\cdot g(a)$$

LHD

$$\lim_{h\to 0} \frac{f(a-h)\cdot g(a-h) - \overset{0}{\overset{\|}{f(a)\cdot g(a)}}}{-h}$$

$$\lim_{h\to 0} \frac{f(a-h)\cdot g(a-h)}{-h}$$

$$= \lim_{h\to 0} \left(\frac{f(a-h)-f(a)}{-h}\right) g(a-h)$$

$$= f'(a)\cdot g(a)$$

LHD = RHD

$$f'(a)\cdot g(a) = f'(a) * g(a)$$

⇓

F is derivable at $x=a$.

Some Important Points

(3) If both f(x) and g(x) are non derivable then nothing definite can be said about the sum/difference/product function.

Some Important Points

(4) Derivative of a continuous function need not be a continuous function.

RHD AT $x=0$

$$\lim_{h\to 0}\frac{h^2\sin 1/h-0}{h}$$

$$=\lim_{h\to 0} h\sin 1/h=0$$

LHD

$$\lim_{h\to 0}\frac{h^2\sin(-1/h)-0}{-h}$$

$$=\lim_{h\to 0} h\sin 1/h=0$$

$\Rightarrow f'(0)=0$

$$f(x)=\begin{cases}x^2\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$$

– Continuous at $x=0$

& Also derivable at $x=0$

$$f'(x)=\begin{cases}2x\sin 1/x+x^2\cos 1/x\left(-\frac{1}{x^2}\right), & x\neq 0\\ 0 & x=0\end{cases}$$

$$f'(x)=\begin{cases}2x\sin 1/x-\cos 1/x & x\neq 0\\ 0 & x=0\end{cases}$$

discontinuous at $x=0$

★ $\cos(|x|) = \cos x$

★ $|f(x)| \cdot \sin(|f(x)|) = \begin{cases} f(x) \cdot \sin f(x) & f(x) \geq 0 \\ -f(x) \cdot \sin(-f(x)) & f(x) < 0 \end{cases}$

$= \begin{cases} f(x) \cdot \sin f(x) & f(x) \geq 0 \\ f(x) \cdot \sin f(x) & f(x) < 0 \end{cases}$

$\Downarrow$

$|f(x)| \sin(|f(x)|) = f(x) \cdot \sin(f(x))$

QUESTION

The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at

A. -1

B. 0

C. 1

D. 2

$f(x) = (x^2-1)\,|x^2-3x+2| + \cos x$.

$= (x-1)(x+1)\,|(x-1)(x-2)| + \cos x$.

$(x-1)(x+1)$ → derivable on R → value at $x=1$ is 0

$|(x-1)(x-2)|$ → Non derivable at $x=1,2$ → But continuous at $x=1,2$

(By A·R) derivable at $x=1$

hence only point on non derivability $\Rightarrow x=2$.

QUESTION [JEE Mains 2022 (29 July)]

The number of points, where the function $f : R \to R$, $f(x) = |x-1|\cos|x-2|\sin|x-1| + (x-3)|x^2-5x+4|$, is NOT differentiable, is:

A 1

B 2

C 3

D 4

$$f(x) = |x-1|\cdot\cos|x-2|\cdot\sin|x-1| + (x-3)|x^2-5x+4|$$

$$\Downarrow$$

$$f(x) = (x-1)\cdot\cos(x-2)\sin(x-1) + (x-3)|(x-1)(x-4)|$$

$(x-1)\cdot\cos(x-2)\sin(x-1)$ → derivable on R

$(x-3)|(x-1)(x-4)|$ → Non derivable at $x=1,4$

Non derivable at 2 points

Non derivable at $x=1,4$

Ans. B

QUESTION

Find points of non differentiability of following functions:

(a) $f(x) = \ln x \cdot |x - 1|$

(b) $f(x) = \sin \pi x \cdot |(x - 1)(x - 2)(x - 3)|$

(b)

$f(x) = \sin \pi x \, |(x-1)(x-2)(x-3)|$

$\sin \pi x$ → derivable at $x = 1,2,3$ & it's value at $x = 1,2,3$ is 0.

$|(x-1)(x-2)(x-3)|$ → non derivable at $x = 1,2,3$ but continuous at $x = 1,2,3$

By A·R it is derivable at $x = 1,2,3$

hence f is derivable on R.

(a) $f(x) = \ln x \cdot |x-1|$

$\ln x$ → derivable at $x = 1$, value at $x = 1$ is 0

$|x-1|$ → non derivable at $x = 1$ but continuous at $x = 1$

By A·R $f(x)$ is derivable at $x = 1$

hence f is derivable on $(0, \infty)$

QUESTION [JEE Mains 2021 (31 August)]

The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2-12x+4|}$ is not differentiable at exactly:

A four points

B three points

C two points

D one point

$f(x) = |x^2-2x-3|\, e^{|(3x-2)^2|}$

$= |(x-3)(x+1)|\, e^{(3x-2)^2}$

Non derivable at $x = 3, -1$ only.

Ex: $\phi(x) = e^{(3x-2)^2} \cdot |(x-3)(x-2)|$

$g(3) = 0 = g(2)$

Non derivable at $x = 2, 3$.

& finite LHD, RHD at $x = 2, 3$

But $f(3), f(2) \neq 0$

Ans. C

AsNC

$y = \phi(x) = f(x) \cdot g(x)$

$f(x)$: derivable at $x=a$

$g(x)$: non derivable at $x=a$ but $g(a)=0$ with finite LHD & RHD

RHD

$$\lim_{h\to 0} \frac{\phi(a+h)-\phi(a)}{h} = \lim_{h\to 0} \frac{f(a+h)\,g(a+h) - f(a)\cdot g(a)}{h}$$

LHD

$$\lim_{h\to 0} \frac{\phi(a-h)-\phi(a)}{-h}$$

$$= \lim_{h\to 0} \frac{f(a-h)*g(a-h) - f(a)\cdot g(a)}{-h}$$

$$= \lim_{h\to 0} \frac{f(a-h)\cdot g(a-h)}{-h}$$

$$= \lim_{h\to 0} f(a-h)\cdot \frac{g(a-h)-g(a)}{-h} = f(a)\cdot g'(a^-)$$

$$= \lim_{h\to 0} \frac{f(a+h)\cdot g(a+h)}{h}$$

$$= \lim_{h\to 0} f(a+h)\cdot \frac{g(a+h)-g(a)}{h}$$

$$= f(a)\cdot g'(a^+)$$

For derivability at $x=a$

$$f(a)\cdot g'(a^+) = f(a)\cdot g'(a^-)$$

$$f(a)\cdot\left(g'(a^+) - g'(a^-)\right) = 0$$

$$f(a)=0 \text{ or } g'(a^+) = g'(a^-) \quad (N\cdot P)$$

$f(a)=0$

Hence $\phi(x) = f(x)\cdot g(x)$ where
g is non derivable at $x=a$ & $g(a)=0$
& has finite LHD & RHD also f being derivable/continuous is derivable at $x=a$ only if $f(a)=0$

Super NICHOD!!

Astonishing Result : $\phi(x) = f(x) \cdot g(x)$

- $f(x)$ → derivable at $x=a$ & $f(a)=0$
- $g(x)$ → continuous at $x=a$

$\phi(x)$ is derivable at $x=a$.

ASNC : $\phi(x) = f(x) \cdot g(x)$

- $f(x)$ → derivable | continuous at $x=a$
- $g(a)=0$, g is non derivable with finite LHD & RHD.

$\phi(x)$ is derivable at $x=a$ only if $f(a)=0$.

QUESTION

Tan2

Let $f(x) = (x^2 - 3x + 2)\left|(x^3 - 6x^2 + 11x - 6)\right| + \left|\sin\left(x + \frac{\pi}{4}\right)\right|$.

Number of points at which the function f(x) is non-differentiable in $[0, 2\pi]$, is

QUESTION

Tan3

If number of points of discontinuity of the function $f(x) = [2 + 10 \sin x]$, in $x \in \left[0, \frac{\pi}{2}\right]$ is same as number of points of non-differentiability of the function $g(x) = (x-1)(x-2)|(x-1)(x-2) \ldots (x-2m)|$ $(m \in N)$ in $x \in (-\infty, \infty)$ then find the value of m.

[Note: [k] denotes largest integer less than or equal to k.]

QUESTION [JEE 2001]

Tan 4

Which of the following functions is differentiable at x = 0 ?

A $\cos(|x|) + |x|$

B $\cos(|x|) - |x|$

C $\sin(|x|) + |x|$

D $\sin(|x|) - |x|$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

If $\alpha = \lim\limits_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

A $(1, -3)$

B $(-1, 3)$

C $(-1, -3)$

D $(1, 3)$

Ans. D

QUESTION

(KTK 2)

Let $g(x) = \begin{cases} 2x + \tan^{-1}x + a, & -\infty < x \leq 0 \\ x^3 + x^2 + bx, & 0 < x < \infty \end{cases}$.

If $g(x)$ is differentiable for all $x \in (-\infty, \infty)$ then $(a^2 + b^2)$ is equal to

A 20

B 13

C 9

D 4

Ans. C

QUESTION

(KTK 3)

Which of the following functions defined below are NOT differentiable at the indicated point ?

(A) $f(x) = \begin{cases} x^2 \text{ if } -1 \le x < 0 \\ -x^2 \text{ if } 0 \le x \le 1 \end{cases}$ at $x = 0$

(B) $g(x) = \begin{cases} x \text{ if } -1 \le x < 0 \\ \tan x \text{ if } 0 \le x \le \end{cases}$ at $x = 0$

(C) $h(x) = \begin{cases} \sin 2x \text{ if } x \le 0 \\ 2x \text{ if } x > 0 \end{cases}$ at $x = 0$

(D) $k(x) = \begin{cases} x \text{ if } 0 \le x \le 1 \\ 2 - x \text{ if } 1 < x \le 2 \end{cases}$ at $x = 1$

Ans. D

QUESTION

(KTK 4)

Let $g(x) = (x^5 + 1)|x^2 - 4x - 5| + \sin|x| + \cos(|x - 1|)\ \forall\ x \in R$ then find the number of points of non-differentiability of $g(x)$ on R.

Ans. 2

QUESTION

(KTK 5)

Let $f(x) = x^2 - x + k - 2, k \in R$. If the complete set of values of k for which $y = |f(|x|)|$ is non-derivable at 5 distinct points is (a, b) then find the value of $8(b - a)$.

Ans. 2

QUESTION

(KTK 6)

Which of the following function(s) not defined at $x = 0$ has/have non-removable discontinuity at the point $x = 0$?

A $f(x) = \dfrac{1}{1 + 2^{\frac{1}{x}}}$

B $f(x) = \arctan \dfrac{1}{x}$

C $f(x) = \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$

D $f(x) = \dfrac{1}{\ln|x|}$

QUESTION

(KTK 7)

Let $f(x) = \begin{cases} \frac{e^x - 1 + ax}{x^2}, & x > 0 \\ b, & x = 0 \\ \frac{\sin\frac{x}{2}}{x}, & x < 0 \end{cases}$ then-

A $f(x)$ is continuous at $x = 0$ if $a = -1$, $b = \frac{1}{2}$.

B $f(x)$ is discontinuous at $x = 0$, if $b \neq \frac{1}{2}$.

C $f(x)$ has non-removable discontinuity at $x = 0$ if $a \neq -1$.

D $f(x)$ has removable discontinuity at $x = 0$ if $a = -1$, $b \neq \frac{1}{2}$.

QUESTION

(KTK 8)

If $f(x)=\begin{cases} x+a, & x \leq 0 \\ |x-4|, & x>0 \end{cases}$ and $g(x)=\begin{cases} x+1, & x<0 \\ (x-4)^2+b, & x \geq 0 \end{cases}$ are continuous on R, then $(gof)(2)+(fog)(-2)$ is equal to :

A -10

B 10

C 8

D -8

Ans. D

QUESTION

KTK 9

If $[x]$ denotes the integral part of x, then the number of points where $f(x) = [\sin x] + [\cos x], 0 \leq x \leq 2\pi$ is dis continuous

A 3

B 4

C 5

D none of these

QUESTION [JEE Mains 2020]

KTK 10

Let $f(x) = x.\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to

Homework From Module

LCD

PRARAMBH : Q1 TO Q43,Q44 TO Q61,Q71 TO Q108

PRABAL : Q1 TO Q39

PARIKSHIT : Q1 TO Q8

(Revision Practice Problems)

QUESTION

(RPP 1)

The expression $\frac{\sin 7\theta+6\sin 5\theta+17\sin 3\theta+12\sin\theta}{\sin 6\theta+5\sin 4\theta+12\sin 2\theta}$, when $\theta=\frac{\pi}{5}$ reduces to

A $\cos\frac{\pi}{5}$

B $2\sin\frac{3\pi}{10}$

C $\sin\frac{\pi}{5}$

D $2\sin\frac{\pi}{10}$

Ans. B

QUESTION

(RPP 2)

$A_1A_2A_3$ ..... A_{18} is a regular 18 sided polygon. B is an external point such that A_1A_2B is an equilateral triangle. If $A_{18}A_1$ and A_1B are adjacent sides of a regular n sided polygon, then n =

Ans. 9

QUESTION

(RPP 3)

If λ be the minimum value of

$y = (\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 + (\tan x + \cot x)^2$ where $x \in R$.

Find $\lambda - 6$.

Ans. 7

QUESTION

(RPP 4)

If $\cos 20^\circ + 2\sin^2 55^\circ = 1 + \sqrt{2}\sin K^\circ$, $K \in (0, 90)$, then $K =$

Ans. 65

(Solution to TAH)

QUESTION

(a) $f(x) = |\ln x|$ at $x = 1$

(b) $f(x) = \ln^2 x$ at $x = 1$

(c) $f(x) = e^{-|x|}$ at $x = 0$

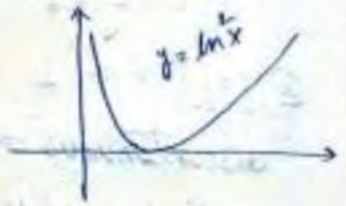
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TAH-1 (a) $f(x) = \ln^2 x$ @ $x=1$

RHD $\lim_{h\to 0} \frac{\ln^2(1+h) - \ln^2 1}{h} = \lim_{h\to 0} \left(\frac{\ln(1+h)}{h}\right)^2 \cdot h = 0$ (the factor $\to 1$, $h \to 0$)

LHD $\lim_{h\to 0} \frac{\ln^2(1-h) - \ln^2 1}{h} = \lim_{h\to 0} \left(\frac{\ln(1-h)}{-h}\right)^2 \cdot h = 0$

$\therefore$ f is derivable @ $x=1$
& also continuous @ $x=1$



(b) $f(x) = e^{-|x|}$ @ $x=0$

RHD $\lim_{h\to 0} \frac{e^{-|h|} - e^0}{h} = \frac{e^{-h} - 1}{h} = -\left(\frac{1-e^{-h}}{-h}\right)\cdot(-1)$

$\lim_{h\to 0} \frac{e^{-h}-1}{-h}(-1) = -1$

LHD $\lim_{h\to 0} \frac{e^{-|h|} - e^0}{h} = \lim_{h\to 0} \frac{e^h - 1}{h} = 1$

$\therefore$ f is non derivable @ $x=0$
But continuous @ $x=0$
with sharp corner @ $x=0$

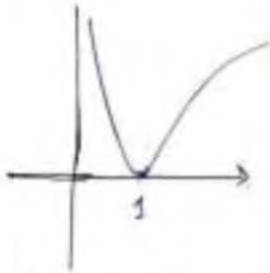
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Tah 1) (b) $f(x) = \ln^2 x$ at $x=1$

RHD $= \lim_{h\to 0} \frac{(\ln(1+h))^2 - \ln^2 1}{1+h-1} = \lim_{h\to 0} \left(\frac{\ln(1+h)}{h}\right)^2 \cdot h = 0$ (the factor $\to 1$, $h \to 0$)

LHD $= \lim_{h\to 0} \frac{(\ln(1-h))^2 - \ln^2 1}{1-h-1} = \left(\frac{\ln(1-h)}{-h}\right)^2 \cdot -h = 0$ ($\to 1$, $h \to 0$)

LHD = RHD = 0 $\Rightarrow$ $f(x) = \ln^2 x$ is derivable at $x=1$



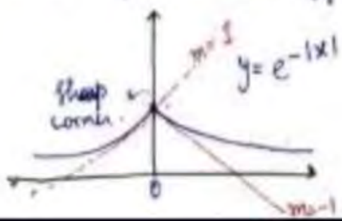
Priyanka
Keshri
WB

(c) $f(x) = e^{-|x|}$ at $x=0$.

LHD $= \lim_{h\to 0} \frac{e^{-|-h|} - e^{-0}}{-h} = \frac{e^{-h} - 1}{-h} = 1$.

RHD $= \lim_{h\to 0} \frac{e^{-h} - e^0}{h} = \frac{e^{-h} - 1}{h} = -1$

RHD $\neq$ LHD $\Rightarrow$ $f(x)$ is non differentiable at $x=0$.



QUESTION

Check the differentiability at $x = 0$ of $f(x) = \begin{cases} \frac{e^{1/x}}{1+e^{1/x}}; & x \neq 0 \\ 0; & x = 0 \end{cases}$

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TAH-2 check the differentiability @ $x=0$

$$f(x)=\begin{cases}\dfrac{e^{1/x}}{1+e^{1/x}} & , x\neq 0\\ 0 & ; x=0\end{cases}$$

∴ discontinuous @ $x=0$
⇒ non derivable @ $x=0$

$$f(x)=\frac{e^{1/x}}{1+e^{1/x}} \quad , x\neq 0$$

$$=$$

RHL $\lim_{x\to 0} \dfrac{e^{\frac{1}{x}}}{1+e^{1/x}} = \lim_{h\to 0} \dfrac{e^{\frac{1}{h}}}{1+e^{\frac{1}{h}}} = \dfrac{1}{1+\frac{1}{e^{\frac{1}{h}}}} = 1$

LHL $\lim_{x\to 0} \dfrac{e^{\frac{1}{x}}}{1+e^{1/x}} = \lim_{h\to 0} \dfrac{e^{-1/h}}{1+e^{-1/h}} = 0.$

QUESTION

Examine for continuity & differentiability the points $x = 1$ & $x = 2$, the function f defined by $f(x) = \begin{bmatrix} x[x] & , & 0 \le x < 2 \\ (x-1)[x] & , & 2 \le x \le 3 \end{bmatrix}$ where $[x]$ = greatest integer less than or equal to x.

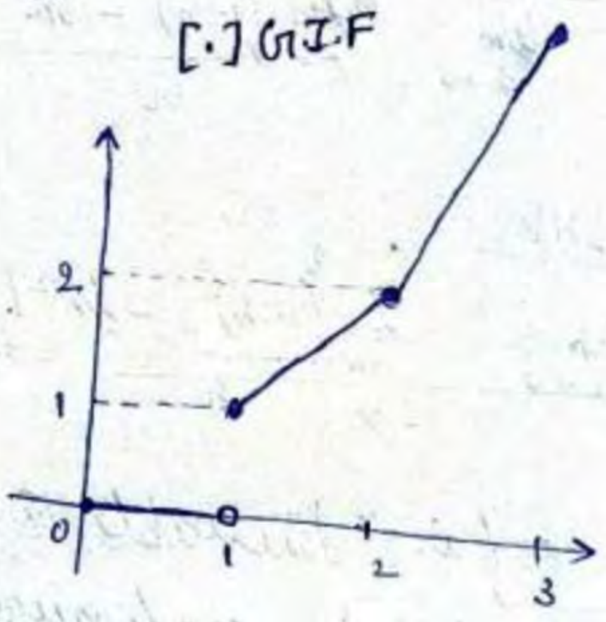
TAH-3

$$f(x) = \begin{cases} x[x] & , 0 \le x < 2 \\ (x-1)[x] & 2 \le x \le 3 \end{cases}$$

Examine for continuity & differentiability @ $x=1$ & $x=2$

$[\cdot]$ G.I.F

$$f(x) = \begin{cases} 0 & 0 \le x < 1 \\ x \cdot 1 & 1 \le x < 2 \\ 2(x-1) & 2 \le x < 3 \\ 6 & x = 3 \end{cases}$$



@ $x=1$

discontinuous b'coz LHL $\neq$ RHL

$\Downarrow$

non derivable.

@ $x=2$

continuous

But non derivable.

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QUESTION [JEE 2003 (Scr.)]

Given $f'(2) = 6$ and $f'(1) = 4$, $\lim_{h \to 0} \frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$ is equal to

A $3/2$

B 3

C $5/2$

D -3

Tah:04

$f'(2) = 6$

$f'(1) = 4$

$$\lim_{h\to 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$$ equals to.

Sol^n

$$\lim_{h\to 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$$

$$\lim_{h\to 0} \frac{f(2h+2+h^2) - f(2)}{2+h^2+2h-2} \times \frac{h(2+h)}{h(1-h)} \times \frac{1}{\frac{f(h-h^2+1) - f(1)}{h-h^2+1-1}}$$

$$\frac{f'(2)}{f'(1)} \times \frac{2}{1} = \frac{6}{4} \times 2 = 3$$

(B)

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TAH-04

Q. Given $f'(2) = 6$ & $f'(1) = 4$, $\lim_{h\to 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)}$ is equal to

$f(x)$ is derivable at 1, 2

$$\lim_{h\to 0} \frac{f(2h+2+h^2)-f(2)}{(2h+2+h^2)-2} \cdot \frac{(h^2-h^2+1)-1}{f(h-h^2+1)-f(1)} \times \frac{(2h+h^2)}{(h-h^2)}$$

$$\frac{h(2+h)}{h(1-h)} \to 0$$

$$\Rightarrow \frac{f'(2)}{f'(1)} \times 2$$

$$\Rightarrow \frac{6^3}{4_2} \times 2$$

$$\Rightarrow 3 \quad \therefore \text{option (B)}$$ Ans.

Srishti Shukla
from (UP)

QUESTION [JEE Mains 2024 (30 Jan)]

If the function, $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ is differentiable on R, then $48(a + b)$ is equal to

Ans. 15

TAH-5 Main -24

$f(x) = \begin{cases} \frac{1}{|x|} & |x| \geq 2 \\ ax^2+2b & |x|<2 \end{cases}$ is differentiable on R, then $48(a+b) = ?$

$f(x) = \begin{cases} \frac{1}{x} & x \geq 2 \\ -\frac{1}{x} & x \leq -2 \\ ax^2+2b & -2<x<2 \end{cases}$

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AT $x=2$ LHL $= 4a+2b$, RHL $= \frac{1}{2}$ $\Rightarrow$ $4a+2b = \frac{1}{2}$, $8a+4b=1$

AT $x=-2$ LHL $= \frac{-1}{(-2)} = \frac{1}{2}$, RHL $= 4a+2b$ $\Rightarrow$ $4a+2b = \frac{1}{2}$, $8a+4b=1$

AT $x=2$

LHD $\lim_{x\to 2} \frac{d}{dx}(ax^2+2b) = \lim_{x\to 2} 2ax = 2\times a\times 2 = 4a$

RHD $\lim_{x\to 2} \frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{x\to 2} -\frac{1}{x^2} = -\frac{1}{4}$

$4a = -\frac{1}{4}$

$a = -\frac{1}{16}$

$8\times\left(-\frac{1}{16}\right)+4b=1$

$4b = 1+\frac{1}{2} = \frac{3}{2}$

$b = \frac{3}{8}$

$\therefore 48(a+b) = 48\left(-\frac{1}{16}+\frac{3}{8}\right) = (-3+18) = 15$

Tah 5 $f(x) = \begin{cases} \frac{1}{|x|} & |x| \geq 2 \\ ax^2+2b & |x|<2 \end{cases}$ $48(a+b) = ?$

f is derivable on R $\Rightarrow$ f is continuous on R.

$f(x) = \begin{cases} -\frac{1}{x} & x \leq -2 \\ \frac{1}{x} & x \geq 2 \\ ax^2+2b & -2<x<2 \end{cases}$

at $x=-2$ LHL $= \lim_{x\to -2^-} -\frac{1}{x} = \frac{1}{2}$

RHL $= \lim_{x\to -2^+} ax^2+2b = 4a+2b$.

$4a+2b = \frac{1}{2}$ (1)

$\Rightarrow 4\cdot\left(-\frac{1}{4}\right)+2b = \frac{1}{2}$

$b = \frac{1}{4}+\frac{1}{8}$

$b = \frac{3}{8}$

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Keshri
W.B

~~$ax^2+2b = 4a+2b$~~

~~$\frac{1}{x} = \frac{1}{2}$~~

differentiability at $x=-2$

RHD $= \lim_{x\to -2} \frac{d}{dx}(ax^2+2b) = \lim_{x\to -2} 2ax = -4a$

LHD $= \lim_{x\to -2} \frac{d}{dx}\left(-\frac{1}{x}\right) = \lim_{x\to -2} +\frac{1}{x^2} = \frac{1}{4}$

$-4a = \frac{1}{4}$

$a = -\frac{1}{16}$

Put in (1)

48 $48(a+b) = 48\left(-\frac{1}{16}+\frac{3}{8}\right)$

$= 3\cancel{48}\times\frac{5}{\cancel{16}} = 15$ Ans

QUESTION [JEE Mains 2024 (1 Feb)]

Let $f: R \to R$ be defined as: $f(x) = \begin{cases} \frac{a-b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2; & 0 \le x \le 1 \\ 2x + 1; & x > 1 \end{cases}$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals:

A 1

B 4

C 3

D 2

Ans. D

Tah : 06

let $f: R \to R$ be defined as $f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}, & x < 0 \\ x^2 + cx + 2, & 0 \le x \le 1 \\ 2x + 1, & x > 1 \end{cases}$

If f is continuous everywhere in R and m is the no of points where f is NOT Differentiable then $a+b+c+m = ?$

Part 1

Sol^n

$$f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2}, & x < 0 \\ x^2 + cx + 2, & 0 \le x \le 1 \\ 2x + 1, & x > 1 \end{cases}$$

at $x = 0$

At $x = 1$

RHL $\lim_{x \to 1^+} 2x + 1 = 2 + 1 = 3$

$c + 3 = 3 \Rightarrow c = 0$

LHL $\lim_{x \to 1^-} x^2 + x.c + 2 = 1 + c + 2 = c + 3$

$\therefore c = 0$

At $x = 0$:

$RHL = \lim_{x \to 0^+} x^2 + cx + 2 = 2$

LHL $\lim_{x \to 0^-} \frac{a - b\cos 2x}{x^2} = \cdot$

$a = +1$, $b = 1$

$LHL = RHL$

$\lim_{x \to 0^-} \frac{a - b\cos 2x}{4x^2} = \frac{2}{4} \Rightarrow \lim_{x \to 0^-} \frac{a - b\cos 2x}{4x^2} = \frac{1}{2}$

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$\therefore f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2}, & x < 0 \\ x^2 + 2, & 0 \le x \le 1 \\ 2x + 1, & x > 1 \end{cases}$

Part :02

checking Differentiability

at $x = 0$ LHL $= \lim_{x \to 0} \frac{1 - \cos 2x}{4x^2} \cdot 4 = 4 \cdot \frac{1}{2} = 2$

RHL $= \lim_{x \to 0^+} x^2 + 2 = 2$ } Continuous

$\therefore$ LHD $= \lim_{x \to 0^-} \frac{d}{dx}\left(\frac{1 - \cos 2x}{4x^2}\right) = \frac{-b}{}$

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$\lim_{x \to 0^-} \frac{d}{dx} \frac{(2\sin^2 x)}{4x^2} = \lim_{x \to 0^-} \frac{1}{2}\left(-2\sin^2 x \cdot x^{-3} + \frac{2\sin x \cos x}{x^2}\right)$

$= \frac{1}{2}(0 + 0) = 0.$

RHD $\lim_{x \to 0^+} \frac{d}{dx}(x^2 + 2) = \lim_{x \to 0^+} 2x = 0$

Derivable at $x = 0$.

At $x = 1$ $LHL = 1^2 + 2 = 3$, $RHL = 2 \times 1 + 1 = 3$ } Continuous

LHD $\lim_{x \to 1^-} \frac{d}{dx}[x^2 + 2] = 2 \times 1 = 2$

RHD $\lim_{x \to 1^+} \frac{d}{dx}[2x + 1] = 2$

Derivable at $x = 1$

$\therefore$ no of Non Derivable Point $= m = 0$

$\therefore a + b + c + m = 1 + 1 + 0 + 0 = 2$ (D)

Ans

QUESTION

TAH

If the function $f(x)=\begin{cases} ax+b, & -\infty < x \le 2 \\ x^2-5x+6, & 2 < x < 3 \\ px^2+qx+1, & 3 \le x < \infty \end{cases}$ is differentiable in $(-\infty, \infty)$, then

(A) $a=-1, p=\frac{-4}{9}$

(B) $b=2, q=\frac{5}{3}$

(C) $a=1,\ b=2$

(D) $a=-1, q=\frac{-5}{3}$

Tqh:07

Part :01

$$f(x)=\begin{cases} ax+b, & -\infty<x\le 2 \\ x^2-5x+b, & 2<x<3 \\ px^2+qx+1, & 3\le x<\infty \end{cases}$$

is differentiable in $(-\infty,\infty)$ then,

Sol^n =

At $x=2$ LHL $= 2a+b$ $\Rightarrow$ $\boxed{2a+b=0}$

RHL $= 0$

$$LHD=\lim_{x\to 2^-}\frac{d}{dx}(ax+b)=a$$

$$RHD=\lim_{x\to 2^+}\frac{d}{dx}(x^2-5x+b)=\lim_{x\to 2}2x-5$$

$$=-1$$

LHD = RHD

$\therefore \boxed{a=-1}$ $\therefore \boxed{b=2}$

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Part :2

At $x=3$

LHL $= 0$

RHL $= 9p+3q+1$ $\Rightarrow \boxed{9p+3q+1=0}$ —— (i)

$$LHD=\lim_{x\to 3^-}\frac{d}{dx}(x^2-5x+6)=1$$

$$RHD=\lim_{x\to 3^+}\frac{d}{dx}(px^2+qx+1)=6p+q$$

LHD = RHD

$6p+q=1$ —— (ii)

Eq^n (ii) $\times 3$ − Eq^n (i)

$18p+3q-(9p+3q+1)=3$

$9p=4$

$\boxed{p=\frac{4}{9}}$

Put in Eq^n (ii)

$\therefore \frac{24}{9}+q=1$

$q=1-\frac{24}{9}=\frac{-15}{9}=\frac{-5}{3}$

$\boxed{q=\frac{-5}{3}}$ Ans//

(D) ✓ $a=-1$, $q=\frac{-5}{3}$

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QUESTION

TAH

Let $g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \geq 1 \end{cases}$.

If $g(x)$ is continuous and differentiable for all numbers in its domain, then

(A) $a = b = 4$

(B) $a = b = 4$

(C) $a = 4$ and $b = -4$

(D) $a = -4$ and $b = 4$

Tab:08

$$g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & , x < 1 \\ ax + b & , x \geq 1 \end{cases}$$

$g(x)$ is continuous and differentiable for all numbers in its domain

Soln

At $x = 1$

LHL $= 3 - 4 + 1 = 0$

RHL $= a + b$

$a + b = 0$

LHD $\lim_{x \to 1^-} \frac{d}{dx}(3x^2 - 4\sqrt{x} + 1)$

$= 6 - 2 = 4$

$b = -4$

RHD $\lim_{x \to 1^+} \frac{d}{dx}(ax + b) = a$

$a = 4$

(c)

✓ Ans

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TAH-08

Q). Let $g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \geq 1 \end{cases}$

If $g(x)$ is continuous and differentiable for all no. in its domain, then —

$x = 1$

LHL $\lim_{x \to 1^-} 3x^2 - 4\sqrt{x} + 1$

$\Rightarrow 0$

RHD $\lim_{x \to 1^+} ax + b$

$a + b$

$\boxed{a+b = 0}$ —(i)

Diff — (i)

LHD $\lim_{x \to 1^-} 6x - \frac{2}{\sqrt{x}}$

$\Rightarrow 4$

RHD $\lim_{x \to 1^+} a$

$\Rightarrow a$

$\boxed{a = 4}$ $\boxed{b = -4}$

So, $a = 4$ & $b = -4$ .. option (c) Ans.

Srishti Shukla
from (UP)

QUESTION [JEE Mains 2024 (1 Feb)]

Let $f(x) = |2x^2 + 5|x| - 3|, x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then $m + n$ is equal to:

(A) 5

(B) 3

(C) 2

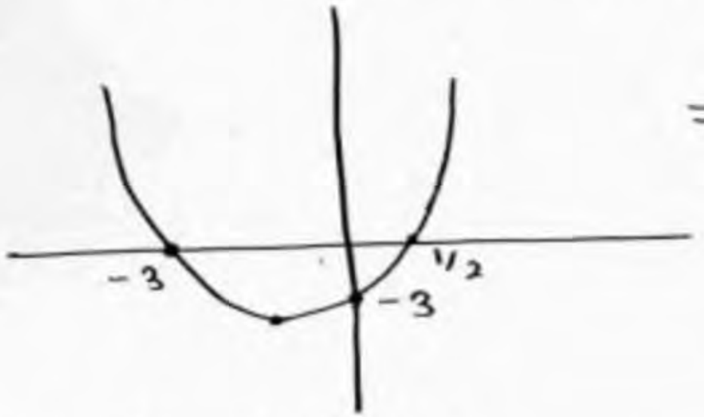
(D) 0

Ans. B

Tah: 09

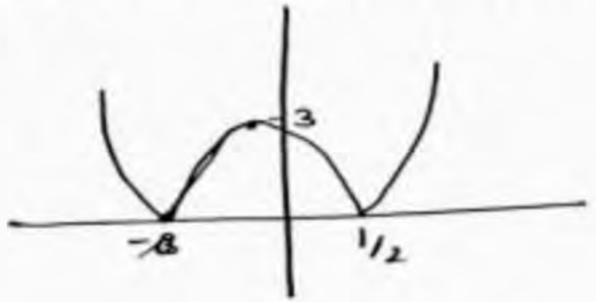
$f(x) = |2x^2 + 5|x| - 3|$, $x \in R$ If m and n denote the no of points where f is not continuous and not differentiability respectively, then $m+n$ is .

Sol^n $g(x) = y = 2x^2 + 5x - 3$



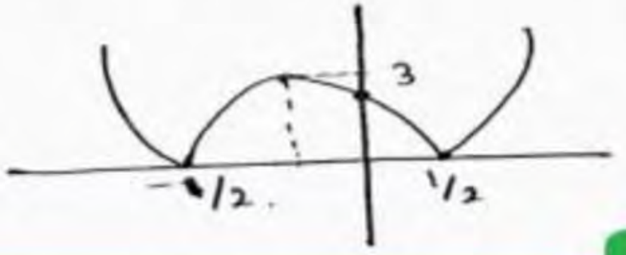
$\Longrightarrow$

$y = g(|x|) = 2|x|^2 + 5|x| - 3 = h(x)$



$\Downarrow$

$y = |h(x)| = |2|x|^2 + 5|x| - 3|$



No of points :-

① f is non deriable
$= n = 3$

② f is not Continuous
$= m = 0$

$\therefore m+n = 3+0 = 3$ (B)

Ritesh Singh, from Devbhoomi Uttarakhand

(Solution to KTK)

QUESTION

(KTK 1)

If $a = \lim_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is:

A 36

B 25

C 32

D 30

Ans. C

KTK-1

$$a = \lim_{x\to 0}\left(\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}\right), \quad b = \lim_{x\to 0}\left(\frac{\sin^2 x}{\sqrt{2}+\sqrt{1+\cos x}}\right), \quad ab^3 = 2$$

$$a = \lim_{x\to 0}\frac{\sqrt{1+x^4}-1}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)}$$

$$a = \lim_{x\to 0}\frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

$$a = \frac{1}{4\sqrt{2}}$$

$$b = \lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{1-\cos x}$$

$$b = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \frac{x^2\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{1-\cos x}$$

$$b = 1\cdot 2\cdot\left(\sqrt{2}+\sqrt{2}\right) = 4\sqrt{2}$$

$$\therefore \quad ab^3 = \frac{1}{4\sqrt{2}}\cdot(4\sqrt{2})^3 \cdot 2 = 16\times 2 = 32$$

Sourik Mondal
Form WestBengal
Murshidabad.

QUESTION

(KTK 2)

$y = f(x)$ is a continuous function such that its graph passes through $(a, 0)$. Then $\lim_{x \to a} \frac{\log_e(1+3f(x))}{2f(x)}$ is

A 1

B 0

C $\frac{3}{2}$

D $\frac{2}{3}$

Ans. C

Topic-2

Que $y = f(x)$ is a continuous function st. its graph passes through $(a, 0)$. then, $\lim_{x \to a} \log_e \frac{(1+3f(x))}{2f(x)}$

$\Rightarrow f(a) = 0$

$\lim_{x \to a} \frac{\log_e (1+3f(x))}{3f(x)} \times \frac{3}{2} = \frac{3}{2}$ Ans

Shivanshi Mishra
Mr. anoop Mishra (UP)
Mrs. Smita Mishra

Topic-3

QUESTION

(KTK 3)

Which of the following functions are continuous at $x = 0$?
[Note: sgn x denotes signum function of x.]

(A) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) + \operatorname{sgn}|x|$

(B) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) - \operatorname{sgn}|x|$

(C) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) + \operatorname{sgn}|x|$

(D) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right) - \operatorname{sgn}|x|$

Ans. A, D

Q Which of foll fn are continuous at $x=0$?

KTK 3

Sol (A) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right)+\operatorname{sgn}|x|$

$LHL=\cos\left(\frac{\pi}{2}\cdot 1\right)+1=1=RHL$

$f(0)=\cos 0+0=1$ } f is continuous at $x=0$

(B) $\cos\left(\frac{\pi}{2}\operatorname{sgn}|x|\right)-\operatorname{sgn}|x|$

$LHL=\cos\left(\frac{\pi}{2}\right)-1=-1=RHL$

$f(0)=1$ — f is not continuous at $x=0$

(C) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right)+\operatorname{sgn}|x|$

$LHL=\sin\left(\frac{\pi}{2}\cdot 1\right)+1=2=RHL$

$f(0)=0+0=0$ → f is not continuous at $x=0$

(D) $\sin\left(\frac{\pi}{2}\operatorname{sgn}|x|\right)-\operatorname{sgn}|x|$

$LHL=\sin\left(\frac{\pi}{2}\cdot 1\right)-1=0$ (RHL)

$f(0)=0$ → f is continuous at $x=0$.

Deepak from Batala punjab

QUESTION

(KTK 4)

f is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3} \cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

(A) can not be determined

(B) is $2(1 - \sqrt{3})$

(C) is zero

(D) is $\frac{2(\sqrt{3}-2)}{\sqrt{3}}$

Ans. B

KTK-4

f is continuous fn on real line.

Given

$$x^2 + (f(x)-2)x - \sqrt{3}f(x) + 2\sqrt{3} - 3 = 0$$

Value of $f(\sqrt{3})$

Soln:- $x^2 + x f(x) - 2x + \sqrt{3}f(x) + 2\sqrt{3} - 3 = 0$

$$(\sqrt{3}-x)f(x) = x^2 - 2x + 2\sqrt{3} - 3$$

$$f(x) = \frac{x^2 - 3 + 2(\sqrt{3}-x)}{\sqrt{3}-x}$$

$$f(x) = \frac{(\sqrt{3}-x)[-x-\sqrt{3}] + 2(\sqrt{3}-x)}{\sqrt{3}-x}$$

Since f is continuous,

$$f(\sqrt{3}) = \lim_{x\to\sqrt{3}} f(x) = \lim_{x\to\sqrt{3}} \frac{\sqrt{3}-x[-x-\sqrt{3}+2]}{(\sqrt{3}-x)}$$

$$f(\sqrt{3}) = \lim_{x\to\sqrt{3}} (-x-\sqrt{3}+2)$$

$$= 2 - 2\sqrt{3}$$

$$\boxed{f(\sqrt{3}) = 2(1-\sqrt{3})}$$

(B)

QUESTION

(KTK 5)

Which of the following function(s) will have atleast one real root in $\left[0, \frac{\pi}{2}\right]$?

(A) $f(x) = x - 1 - \cos x$

(B) $g(x) = \frac{1}{2} + \sin 3x$

(C) $h(x) = \frac{1}{1 - x}$

(D) $k(x) = \frac{x - \frac{3}{2}}{x - 1}$

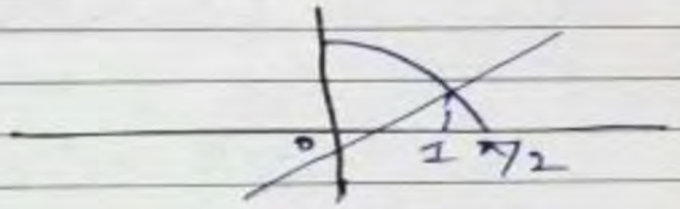
Ans. A, B, D

8. Which of following fn will have atleast one real root in $[0, \frac{\pi}{2}]$?

Soln

(A) $f(x) = x - 1 - \cos x = 0$

$\cos x = x - 1$



(B) $g(x) = \frac{1}{2} + \sin 3x = 0$

$\sin 3x = -\frac{1}{2}$

$3x = \frac{5\pi}{6}$ $\boxed{x = \frac{5\pi}{18}}$

(C) $h(x) = \frac{1}{x-1} = 0$ Not soln

(D) $k(x) = \frac{x - 3/2}{x-1} = 0 \Rightarrow \boxed{x = 3/2}$, $x \neq 1$

Deepak from Batala punjab

(Solution to RPP)

QUESTION

(RPP 1)

If $|\tan\theta - 1| + |\sqrt{3} + \tan\theta| = 1 + \tan\frac{\pi}{3}$ then number of integral solution(s) in $[0, 2\pi]$ is (are)

A 2

B 3

C 4

D 5

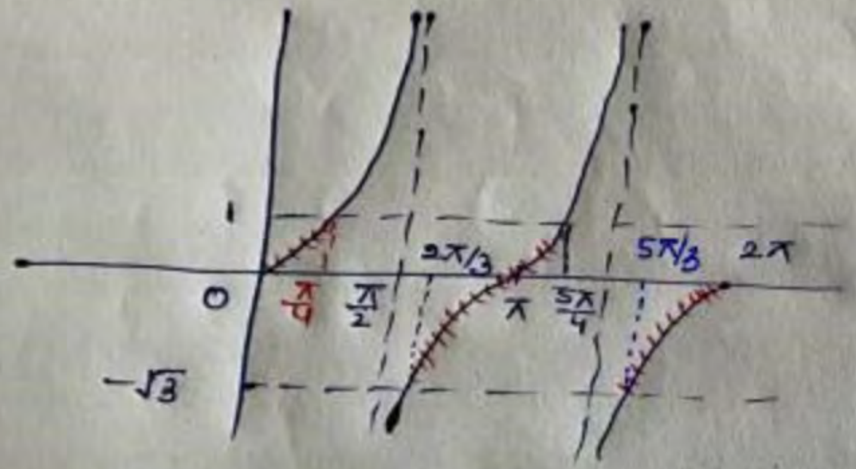
Ans. B

$|\tan\theta - 1| + |\sqrt{3} + \tan\theta| = 1 + \tan\pi/3$

$|\tan\theta - 1| + |\sqrt{3} + \tan\theta| = |\sqrt{3} + \tan\theta - (\tan\theta - 1)|$

$\Rightarrow (\tan\theta - 1)(\sqrt{3} + \tan\theta) \leq 0$

$\tan\theta \in [-\sqrt{3}, 1]$



$\theta \in [2\pi/3, 5\pi/4] = [2.094, 3.926] \Rightarrow$ Integral $\theta = 3$

or $\theta \in [0, \pi/4] \Rightarrow$ Integral $\theta = 0$

or $\theta \in [5\pi/3, 2\pi] = [5.235, 6.28] \Rightarrow$ Integral $\theta = 6$.

⇓

Integral values of $\theta = 3, 0, 6$

Hence three integral values.

QUESTION

(RPP 2)

If $\log_e \pi + \sqrt{2 + \tan^2 x + \cot^2 x} = 3$, then number of solution(s) satisfying the equation is

A 0

B 2

C 4

D infinite

Ans. A

RPP - 2

Sourik Mondal
Form WestBengal
Murshidabad.

$$\log_e \pi + \sqrt{2 + \tan^2 x + \cot^2 x} = 3$$

$$|\tan x + \cot x| = 3 - \ln \pi \approx 3 - 1.6\ldots \approx 1.4\ldots$$

$$\left|\tan x + \frac{1}{\tan x}\right| \approx 1.4$$

$$\geq 2$$

$\therefore$ No. of Soln $= 0$

QUESTION

(RPP 3)

Let $P(x) = 2x^2 - 12x + c \ \forall \ x \in R$ where c is a real constant, then

	Column-I	Column-II
(A)	If greatest value of $p(x)$ for $x \in [1, 2]$ is 1, then c equals	(P) 8
(B)	If smallest value of $P(x)$ for $x \in [1, 5]$ is –1, then c equals	(Q) 11
(C)	If the greatest value of $P(x)$ for $x \in [1, 4]$ is 2, then c equals	(R) 12
		(S) 17

Ans. (A) Q; (B) S; (C) R

Q Let $p(x) = 2x^2 - 12x + c \ \forall\ x \in R$ where c is a real no. then

(A) if greatest value of $f(x)$ for $x \in [1,2]$ is 1, then c equals

Soln $p(x) = 2(x-3)^2 + c - 18$

$x \in [1,2]$

$(x-3) \in [-2,-1]$

$(x-3)^2 \in [1,4]$

$2(x-3)^2 \in [2,8]$

$2(x-3)^2 + c - 18 \in [2+c-18, 8+c-18]$

$8 + c - 18 = 1$

$\boxed{c = 11}$

(B) if smallest value of $p(x)$ for $x \in [1,5]$ is -1, then c equals

Soln $x \in [1,5]$

$x-3 \in [-2,2]$

$(x-3)^2 \in [0,4]$

$2(x-3)^2 \in [0,8]$

$2(x-3)^2 + c - 18 \in [c-18, 8+c-18]$

$c - 18 = 1 \Rightarrow \boxed{c = 17}$

(C) if the greatest value of $p(x)$ for $x \in [1,4]$ is 2, $c = ?$

Soln $x \in [1,4]$

$x-3 \in [-2,1]$

$(x-3)^2 \in [0,4]$

$2(x-3)^2 \in [0,8]$

$2(x-3)^2 + c - 18 \in [c-18, 8+c-18]$

$8 + c - 18 = 2$

$\boxed{c = 12}$

Deepak from Batala punjab

QUESTION

(RPP 4)

If $L = \cos^2 84^\circ + \cos^2 36^\circ + \cos 36^\circ \cos 84^\circ$

$M = \cot 73^\circ \cot 47^\circ \cot 13^\circ$

$N = 4 \sin 156^\circ \sin 84^\circ \sin 36^\circ$

then which of the following option is(are) correct?

(A) $L < 1$

(B) $M > \tan 2$

(C) $N > \sin \frac{\pi}{4}$

(D) $LMN > 0$

Ans. A, B, C, D

RPP 4

Deepak from Batala punjab

Q if $L=\cos^2 84+\cos^2 36+\cos 36\cos 84$
$M=\cot 73^\circ \cot 47^\circ \cot 13^\circ$
$N=4\sin 156^\circ \sin 84 \sin 36^\circ$
which of foll option is (are) correct?

Soln $L=\cos^2 84+\cos^2 36+\cos 36\,84$

$=\frac{1+\cos 168}{2}+\frac{1+\cos 72}{2}+\cos 36^\circ\cos 84^\circ$

$=\frac{1+\cos 168^\circ+1+\cos 72+2\cos 36^\circ\cos 84^\circ}{2}$

$=\frac{2+\cos 168+\cos 72+\cos 120+\cos 48}{2}$

$=\frac{\left(2-\frac{1}{2}\right)+\cos 48+\cos(120+48)+\cos(240+48)}{2}$

$L=\frac{3}{4}<1$ $\boxed{L<1}$

$\cos 72^\circ=\cos(360-72)$

$M=\cot 13^\circ \cot(60-13)\cot(60+13)$
$=\cot 3(13)=\cot 39^\circ > \tan 2$
$\boxed{M>\tan 2.}$

it is radian
$\tan 2<0$
bcoz $2\in(\pi/2,\pi)$

RPP 4

$N = 4\sin 36^\circ \sin 24^\circ \sin 84^\circ$

$= 4\sin 24^\circ \sin(60-24)\sin(60+24)$

$= 4\cdot\frac{1}{4}\sin(3\cdot 24) = \sin 72^\circ > \sin 45^\circ$

$$N > \sin\frac{\pi}{4}$$

$L = \frac{3}{4}$, $m = \cot 39^\circ$, $N = \sin 72^\circ$

$$LMN > 0$$

Deepak from Batala punjab

(Solution to Home Challenge)

Home Challenge-11

Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} \dfrac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4}, & x \neq 2p \\ 0, & x = 2p \end{cases}$$

Then $\lim\limits_{x \to 2p^+} [f(x)]$, where [.] denotes greatest integer function, is

A) 2

B) 1

C) 0

D) -1

HOME CHALLENGE

Date. / Page. /

Q Let $x=2$ be a root of eqn $x^2+px+q=0$ &

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , x\neq 2p \\ 0 & x=2p\end{cases}$$

then $\lim\limits_{x\to 2p^+}[f(x)]$; $[\cdot]$ GIF

Sol $4+2p+q=0$

$$\lim_{x\to 2p}\frac{1-\cos\left(x^2-4px+4p^2-q^2+8q+16-4p^2\right)}{(x-2p)^4}$$

$$\lim_{x\to 2p}\frac{1-\cos\left((x-2p)^2+(q+4)^2-4p^2\right)}{(x-2p)^4}$$

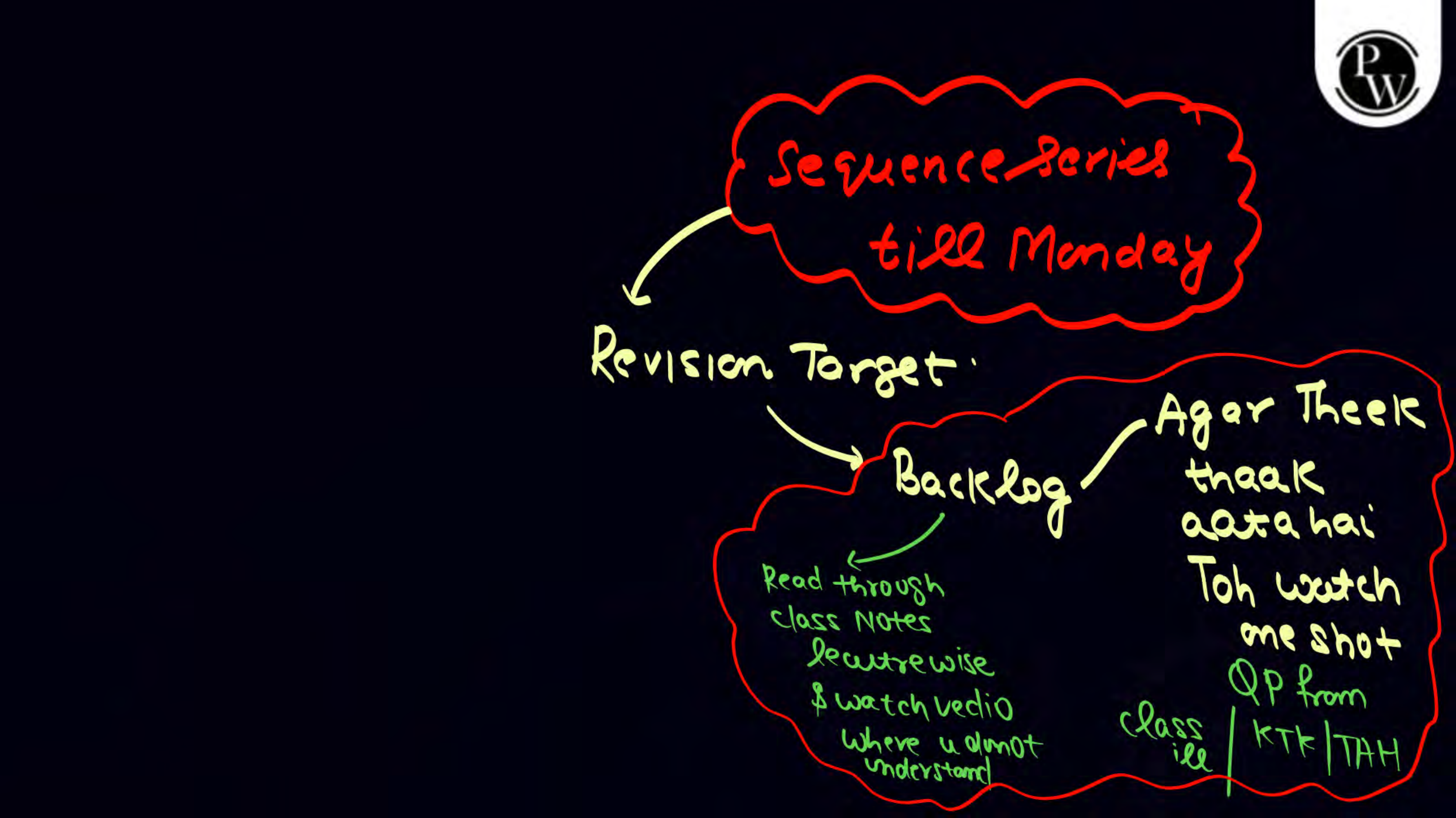
$$\lim_{x\to 2p}\frac{1-\cos\left((x-2p)^2+(q+4+2p)(q+4-2p)\right)}{(x-2p)^4}$$

(0 b'coz $x=2$ is root of $x^2+px+q=0$ ⇓ $4+2p+q=0$)

$$\lim_{x\to 2p}\frac{1-\cos(x-2p)^2}{(x-2p)^4}=\frac{1}{2}$$

$$\lim_{x\to 2p^+}[f(x)]=\lim_{x\to 2p^+}\left[\frac{1}{2}\right]=0 \text{ Ans}$$

Deepak from Batala punjab


Sequence Series till Monday
Revision Target:
Backlog
Agar Theek thaak aata hai Toh watch one shot
Qp from KTK/TAH
class ill
Read through class Notes lecture wise
& watch vedio where u almost understand


THANK
YOU

PRAYAS

JEE 2025

Lecture-13 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Introduction to MOD
2. Practice Problems

Recap *of previous lecture*

1. $\phi(x) = f(x) \cdot g(x)$ if f is Derivable at $x = a$ & value of $f(x)$ is zero at $x = a$ while $g(x)$ is continuous at $x = a$ then $\phi(x)$ is derivable at $x = a$.

2. If f is derivable at $x = a$ & g is not derivable at $x = a$ then $f(x) \pm g(x)$ is non derivable at $x = a$.

3. If $f(x)$ & $g(x)$ are both non derivable at $x = a$ then nothing can be said about $f(x) \pm g(x)$ & $f(x) \cdot g(x)$ @ $x = a$. we need to check.

Recap *of previous lecture*

4. Derivative of a continuous function is also continuous. (T/F)

5. A continuous function always maps a closed interval onto a closed interval. (T/F) $f : [a, b] \longrightarrow [m, M]$

6. A continuous function maps an open interval onto on open interval. (T/F)

7. If a function is derivable at $x = a$ then it is continuous at $x = a$. (T/F)

8. If function is discontinues at $x = a$ then it is non derivable at $x = a$. (T/F)

Recap *of previous lecture*

9. $\phi(x) = f(x) \cdot g(x)$ if 'f' is continuous at $x = a$ & 'g' has is non derivable at $x = a$ with finite LHD & RHD & $g(a) = 0$ then $\phi(x)$ can be derivable only if _____ $f(a)=0$ _____.

10. If a function is increasing on R then as $x \to \infty$ value of function also tends to ∞. (False)

$f(x) = \tan^{-1}x$ — inc on R

but $x \to \infty$, $\tan^{-1}x \to \pi/2$

11. $\lim_{t\to 0} f(1+t) = f(1)$ (T/F)

12. $\lim_{t\to 0} f(1-t) = f(1)$ (T/F)

Homework Discussion

QUESTION

(KTK 5)

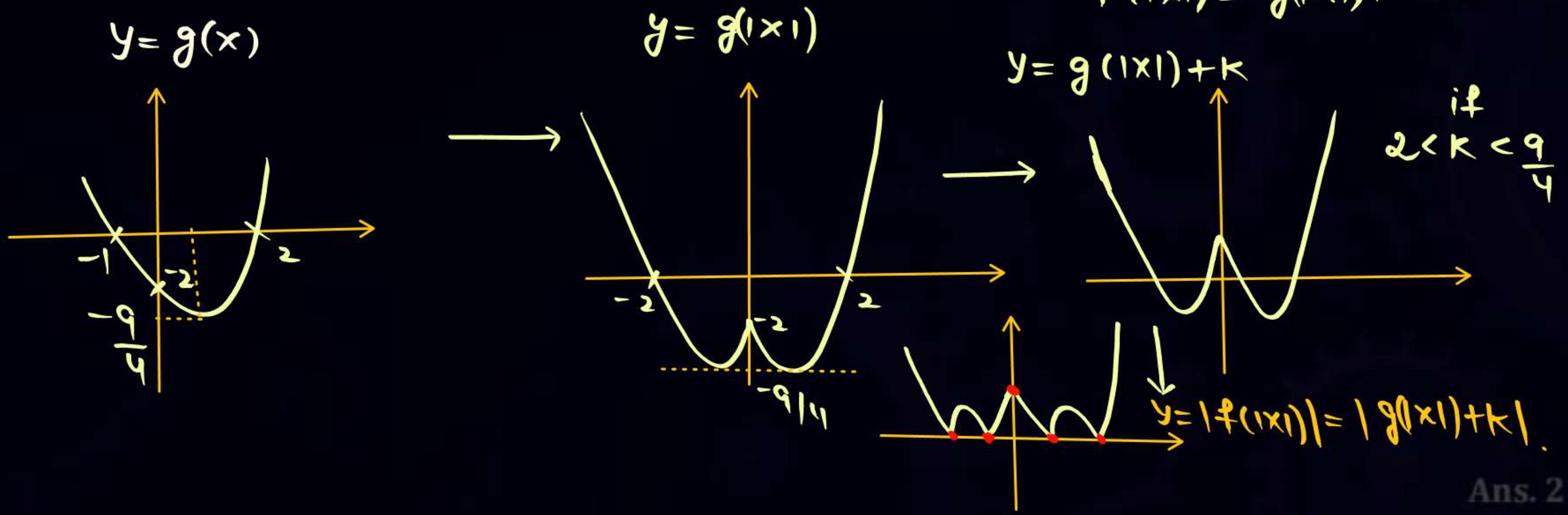
$g(x) = x^2 - x - 2 = (x-2)(x+1)$

$g\left(\frac{1}{2}\right) = -\frac{3}{2} \cdot \frac{3}{2} = -\frac{9}{4}$

Let $f(x) = x^2 - x + k - 2, k \in R$. If the complete set of values of k for which $y = |f(|x|)|$ is non-derivable at 5 distinct points is (a, b) then find the value of $8(b - a)$.

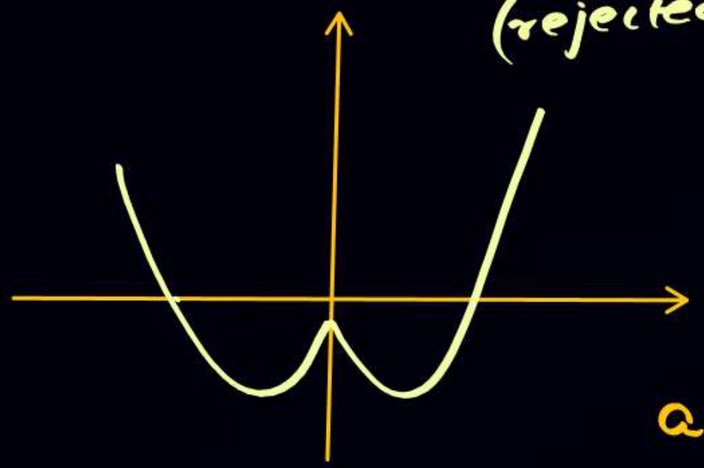
$f(x) = x^2 - x - 2 + k = g(x) + k$ where $g(x) = x^2 - x - 2$

$f(|x|) = g(|x|) + k$



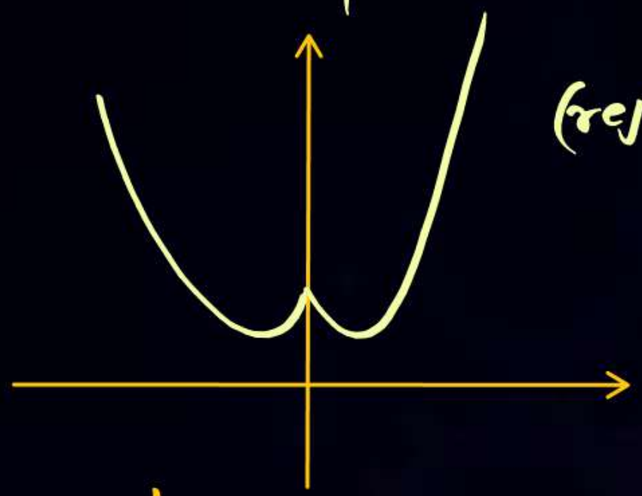
Ans. 2

if $K \leq 2$



(rejected)

$K \geq \frac{9}{4}$



(rejected)

$K \in (2, \frac{9}{4})$ (a, b)

$8(b-a) = 8(\frac{9}{4} - 2) = \underline{2 \text{ Ans}}$

QUESTION [JEE Mains 2020]

KTK 10

Let $f(x) = x.\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to

$f(x) = x \cdot \left[\frac{x}{2}\right]$

$\frac{x}{2} \in (-5, 5)$

$\frac{x}{2} \in I \Rightarrow \frac{x}{2} = -4, -3, -2, -1, 0, 1, 2, 3, 4$

$x = -8, -6, -4, -2, 0, 2, 4, 6, 8$

$\left[\frac{x}{2}\right]$ is discontinuous at above 9 points

Continuous at $x=0$, value $=0$ at $x=0$

discont. & finite LHL & RHL at $x=0$ & defined at $x=0$

$f(x)$ is continuous at $x=0$

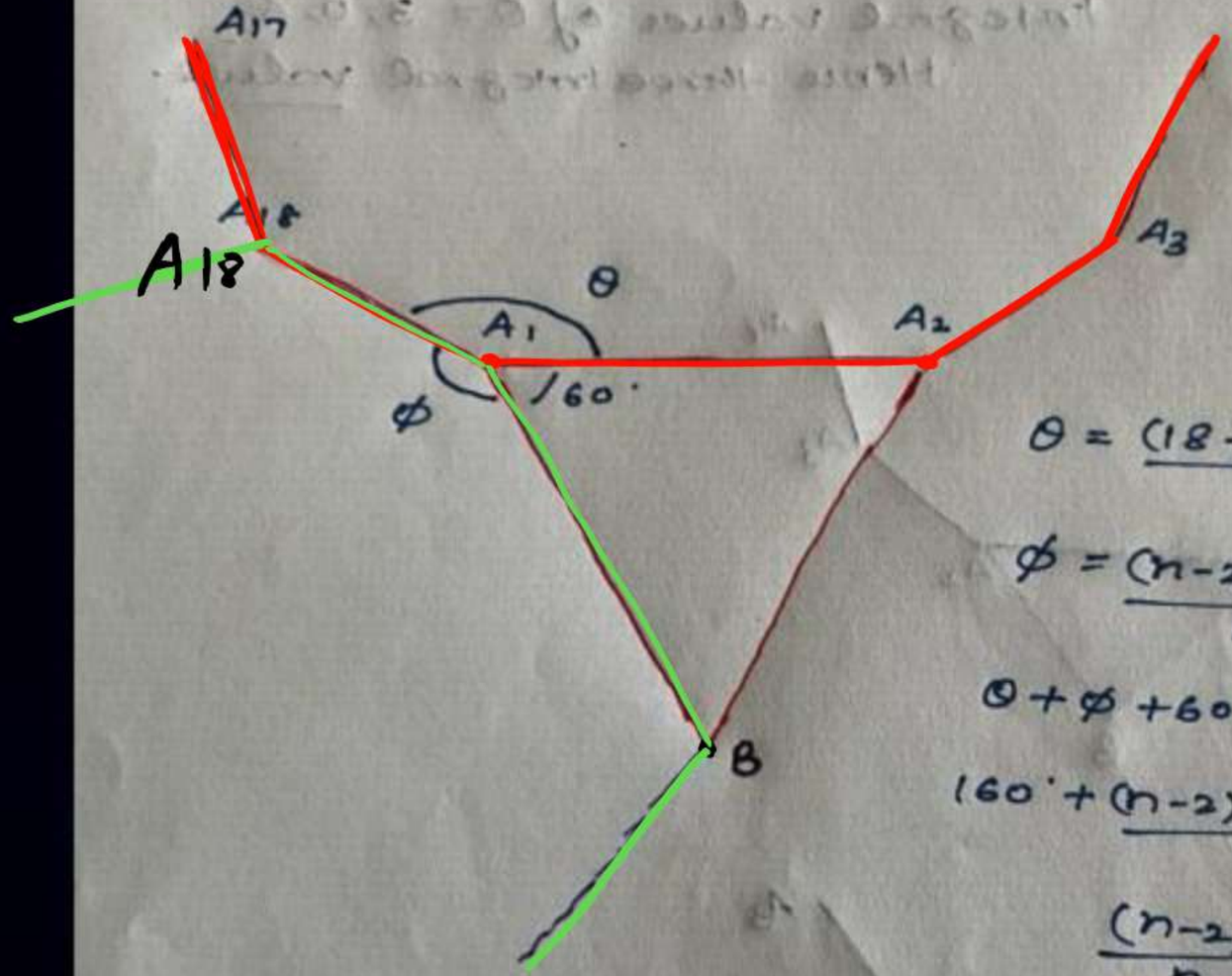
$f(x)$ is discontinuous at 8 points.

QUESTION

(RPP 2)

$A_1A_2A_3$ ..... A_{18} is a regular 18 sided polygon. B is an external point such that A_1A_2B is an equilateral triangle. If $A_{18}A_1$ and A_1B are adjacent sides of a regular n sided polygon, then n =

Ans. 9



$$\theta = \frac{(18-2)\times 180^\circ}{18}$$

$$\phi = \frac{(n-2)\times 180^\circ}{n}$$

$$\theta + \phi + 60^\circ = 360^\circ$$

$$160^\circ + \frac{(n-2)\times 180}{n} = 300$$

$$\frac{(n-2)}{n} \times 180 = 140$$

$$18n - 36 = 14n$$

$$4n = 36$$

$$n = 9$$

QUESTION

TAH 3 DISPLAYED IN Lecture 11

Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$, is equal to (Where $[x]$ denotes greatest integer less than or equal to x)

A 14

B 13

C 10

D 8

TAH 4 RAHUL DHAKAD ❤️❤️ FROM AGRA UP

TAH-4

Q No. of points of discontinuity of $f(x) = [2x^3 - 5]$ in $[1, 2)$ is equal to

Solution → $f(x) = [2x^3 - 5]$, $x \in [1, 2)$

$f(x) = [2x^3] - 5$

$x \in [1, 2)$

$x^3 \in [1, 8)$

$2x^3 \in [2, 16)$



$2x^3 = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$

$x^3 = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8$

$x = 1, \left(\frac{3}{2}\right)^{1/3}, (2)^{1/3}$ ---------- 2

↓ minima

X

$x = 13$ points of discontinuity

Aao Machay Dhmaal Deh Swaal
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QUESTION

KCLS

$\cos(n\pi+\theta) = (-1)^n \cos\theta$

1. $\lim\limits_{n\to\infty} \cos^2\left(\pi\left(\sqrt[3]{n^3+n^2+2n}\right)\right)$ where n is an integer, equals

$$\lim_{n\to\infty} \cos^2\left(\pi n\left(1+\frac{n+2}{n^2}\right)^{1/3}\right)$$

$$\lim_{n\to\infty} \cos^2\left(\pi n\left(1+\frac{1}{3}\left(\frac{n+2}{n^2}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{n+2}{n^2}\right)^2+\text{- - }\infty\right)\right)$$

$$\lim_{n\to\infty} \cos^2\left(n\pi\left(1+\left(\frac{1}{n}+\frac{2}{n^2}\right)\frac{1}{3}+\frac{1/3(1/3-1)}{2!}\cdot\left(\frac{1}{n}+\frac{2}{n^2}\right)^2+\text{— }\infty\right)\right)$$

$= 0$

$$\lim_{n\to\infty} \cos^2(n\pi(1+\theta)) = \lim_{n\to\infty} \cos^2(n\pi+n\pi\theta) = \lim_{n\to\infty} \left((-1)^n \cos n\theta\right)^2 = \lim_{n\to\infty} (\cos^2 n\theta)$$

$$\lim_{n\to\infty} \cos^2\left(n\pi\left(\left(\frac{1}{n}+\frac{2}{n^2}\right)\frac{1}{3} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{n}+\frac{2}{n^2}\right)^2 + \cdots\infty\right)\right)$$

$$\lim_{n\to\infty} \cos^2\left(\pi\left(\left(1+\frac{2}{n}\right)\frac{1}{3} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\cdot n\cdot\frac{1}{n^2}\left(1+\frac{2}{n}\right)^2 + \cdots\infty\right)\right)$$

$\frac{1}{n^2}\left(1+\frac{2}{n}\right)^2 + \cdots\infty \to 0$

$= \cos^2 \pi/3 = (1/2)^2 = 1/4$

QUESTION

KCLS

2. $\lim_{n\to\infty} n \sin\left(2\pi\sqrt{1+n^2}\right)\,(n \in N)$ equals

Tan 1

Determination of f_n

f_n satisfying a given functional rule is presented in the problem & we have to determine the f_n.

Determination of function which are differentiable and satisfying the given functional rule

Basic Steps

1. Write down the expression for $f'(x)$ as $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
2. Manipulate $f(x+h)-f(x)$ in such a way that the given functional rule is applicable. Now apply the functional rule and simplify the RHS to get $f'(x)$ as a function of x along with constants if any.
3. Integrate $f'(x)$ to get $f(x)$ as a function of x and a constant of integration. In some cases a Differential Equation in formed which can be solved to get $f(x)$.
4. Apply the boundary value conditions to determine the value of this constant.

QUESTION

Suppose f is a derivable function that satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y.

Suppose that $\lim_{x\to 0} \frac{f(x)}{x} = 1$, find f(x).

① $f(x+y) = f(x) + f(y) + x^2y + xy^2 \quad \forall\, x, y \in R$ — put $y = h$

$f(x+h) = f(x) + f(h) + x^2h + xh^2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h-0} \frac{f(x) + f(h) + x^2h + xh^2 - f(x)}{h}$

$= \lim_{h\to 0} \left(\frac{f(h)}{h} + x^2 + xh\right) = 1 + x^2 \quad \left(\because \lim_{x\to 0} \frac{f(x)}{x} = 1 \text{ (given)}\right)$

$f'(x) = 1 + x^2$

Integrating both sides

$$f(x) = x + \frac{x^3}{3} + C \quad \cdots (1)$$

put $x = 0, y = 0$ in (1)

$$f(0) = f(0) + f(0) + 0 + 0$$

$$f(0) = 2f(0)$$

$$f(0) = 0$$

$$f(0) = 0 + 0 + C$$

$$0 = f(0) = C$$

$$\Downarrow$$

$$C = 0$$

$$\therefore f(x) = \frac{x^3}{3} + x \quad \text{Ans}$$

QUESTION

Suppose f is a derivable function that satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y.

Suppose that $\lim_{x\to 0} \frac{f(x)}{x} = 1$, find f(x).

M②

$f(x+y) = f(x) + f(y) + x^2y + xy^2 \quad \forall\, x, y \in R$ → put $x=0=y$, $f(0)=0$

Diff. partially w.r.t y

$f'(x+y)\,(0+1) = 0 + f'(y) + x^2\cdot 1 + x\cdot 2y$

$f'(x+y) = f'(y) + x^2 + 2xy$

Put $y=0$ $\quad f'(x) = f'(0) + x^2$

$f'(x) = 1 + x^2$

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$f'(0) = \lim_{h\to 0} \frac{f(h)}{h} = 1$ (given)

QUESTION

A differentiable function satisfies the relation
$f(x+y) = f(x) + f(y) + 2xy - 1 \; \forall \; x, y \in R$.
If $f'(0) = \sqrt{3 + a - a^2}$ **find** $f(x)$ **and prove that** $f(x) > 0 \; \forall \; x \in R$

$f(x+y) = f(x) + f(y) + 2xy - 1 \quad \forall\, x, y \in R$

$x = y = 0 \Rightarrow f(0) = f(0) + f(0) + 0 - 1$

$f(0) = 1$

Diff. partially w.r.t y

$f'(x+y)(1+0) = 0 + f'(y) + 2x$

Put $y = 0$ $\quad f'(x) = f'(0) + 2x$

$f'(x) = \sqrt{3+a-a^2} + 2x$.

$f(x) = x\sqrt{3+a-a^2} + x^2 + C$.

Put $x = 0$

$f(0) = C \Rightarrow C = 1$

$f(x) = x^2 + \sqrt{3+a-a^2}\,x + 1$

$A_1 = 1 > 0$, $D_1 = (3+a-a^2) - 4 \cdot 1 \cdot 1$

$= a - a^2 - 1$

$= -a^2 + a - 1$

$D_2 = 1 - 4(-1)(-1) < 0$

$A = -1 < 0$

$\downarrow$

$-a^2 + a - 1$ always -ve

$\Downarrow$

$D_1 < 0$ hence $f(x) > 0 \ \forall \ x \in R$

QUESTION [JEE Mains 2023 (29 Jan)]



Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1, \forall\, x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to

Ans. 3

QUESTION [JEE Mains 2023 (30 Jan)]

Suppose $f: \mathbb{R} \to (0, \infty)$ be a differentiable function such that $5f(x+y) = f(x) \cdot f(y), \forall x, y \in \mathbb{R}$. If $f(3) = 320$, then $\sum_{n=0}^{5} f(n)$ is equal to :

Range does not have -ve or 0

(A) 6875

(B) 6525

(C) 6575

(D) 6825

$5f(x+y) = f(x) \cdot f(y)$ — $x = y = 0 \Rightarrow 5f(0) = f^2(0)$

$f(0) = 5$

Diff partially w.r.t y

$5 f'(x+y) \cdot (0+1) = f(x) \cdot f'(y)$

$5f'(x+y) = f(x) \cdot f'(y)$

put $y = 0$ $\quad 5f'(x) = f(x) \cdot f'(0)$

$5f'(x) = \alpha f(x)$ where $f'(0) = \alpha$.

$\frac{5f'(x)}{f(x)} = \alpha \Rightarrow 5\int \frac{f'(x)}{f(x)} dx = \int \alpha dx$

Ans. D

$$5\ln|f(x)| = \alpha x + c$$

$$\ln(f(x)) = \frac{\alpha x + c}{5}$$

$$f(x) = e^{\frac{\alpha x + c}{5}}$$

$$f(x) = e^{\frac{\alpha x}{5}} \cdot e^{c/5}$$

put $x = 0$

$$f(0) = e^{c/5} = 5$$

$$f(x) = 5 \cdot e^{\frac{\alpha x}{5}}$$

put $x = 3$

$$f(3) = 5e^{\frac{\alpha \cdot 3}{5}}$$

$$320 = 5 \cdot e^{3\alpha/5}$$

$$64 = (e^{\alpha/5})^3$$

$$e^{\alpha/5} = 4$$

$$f(x) = 5 \cdot 4^x$$

$$f(n) = 5 \cdot 4^n \Rightarrow \sum_{n=0}^{5} 5 \cdot 4^n = 5(1 + 4 + 4^2 + \cdots + 4^5)$$

$$= 5 \cdot \frac{(4^6 - 1)}{4 - 1}$$

QUESTION [JEE Mains 2024 (30 Jan)]

Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all $x, y, f(y) \neq 0$. If $f'(1) = 2024$, then

(A) $x f'(x) + 2024 f(x) = 0$

(B) $x f'(x) - 2023 f(x) = 0$

(C) $x f'(x) - 2024 f(x) = 0$

(D) $x f'(x) + f(x) = 2024$

Ans. C

$$\sum_{n=0}^{5} 5 \cdot 4^n = 6825$$

Method of Differentiation

MOD — 2 Sat
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MOD — Sunday

$y = f(x)$ ① be a derivable fn.

let there be a small change in x say Δx
& corresponding change in y be Δy.

$$y + \Delta y = f(x + \Delta x) \quad \text{②}$$

② − ①

$$\Delta y = f(x+\Delta x) - f(x)$$

Divide both sides by Δx.

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$$

$$\frac{dy}{dx} = f'(x) = \frac{\text{change in } y}{\text{change in } x} \quad \text{such that change in } x \to 0.$$

Ex: $y = x^n$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^n - x^n}{\Delta x} = n \cdot x^{n-1}$$

$f(x) = x^n$

$f'(x) = n \cdot x^{n-1}$

Ex: $y = \sin^{-1} x \quad y \in [-\pi/2, \pi/2]$

$x = \sin y$

$x + \Delta x = \sin(y + \Delta y)$

$$\frac{\Delta x}{\Delta y} = \frac{\sin(y+\Delta y) - \sin y}{\Delta y}$$

$$\frac{\Delta x}{\Delta y} = \frac{2 \sin \frac{\Delta y}{2} \cos\left(\frac{2y + \Delta y}{2}\right)}{\Delta y}.$$

as $\Delta x \to 0$ obviously $\Delta y \to 0$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta y \to 0} \frac{\Delta y}{2 \sin\frac{\Delta y}{2} \cdot \cos\left(\frac{2y+\Delta y}{2}\right)}$$

$$= \lim_{\Delta y \to 0} \frac{\Delta y/2}{\sin\frac{\Delta y}{2} \cdot \cos\left(\frac{2y+\Delta y}{2}\right)}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad x \neq 1, -1$$

Formulae

(1) $\frac{d}{dx}(\text{constant}) = 0$

(2) $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

(3) $\frac{d}{dx}(e^x) = e^x$

(4) $\frac{d}{dx}(a^x) = a^x \cdot \ln a$

(5) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

(6) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$

(7) $\frac{d}{dx}(\sin x) = \cos x$

(8) $\frac{d}{dx}(\cos x) = -\sin x$

(9) $\frac{d}{dx}(\tan x) = \sec^2 x$

(10) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(11) $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

(12) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

Formulae

(13) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$; $|x| \le 1$

(14) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$; $|x| \le 1$

(15) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$; $x \in R$

(16) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$; $x \in R$

(17) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$; $|x| \ge 1$

(18) $\frac{d}{dx}(\text{cosec}^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$; $|x|$ 1

$y = \sec^{-1}x \quad y \in [0,\pi]-\{\frac{\pi}{2}\}$

$x = \sec y$

$x+\Delta x = \sec(y+\Delta y)$

$$\frac{\Delta x}{\Delta y} = \frac{\sec(y+\Delta y)-\sec y}{\Delta y}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta y\to 0}\frac{\Delta y\cdot\cos y\cdot\cos(y+\Delta y)}{\cos y-\cos(y+\Delta y)}$$

$$\frac{dy}{dx} = \lim_{\Delta y\to 0}\frac{\Delta y\cdot\cos y\cos(y+\Delta y)}{2\sin\frac{2y+\Delta y}{2}\sin\frac{\Delta y}{2}}$$

$$\frac{dy}{dx} = \frac{\cos^2 y}{\sin y} = \frac{1}{\frac{\sin y}{\cos y}\cdot\frac{1}{\cos y}} = \frac{1}{\tan y\cdot\sec y}$$

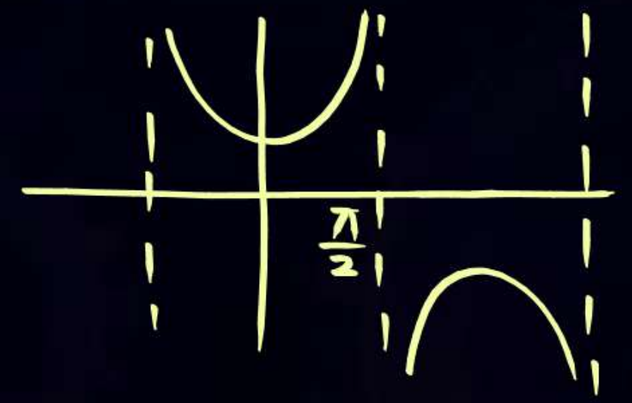
$$\frac{dy}{dx} = \frac{1}{x\cdot\tan y}$$

Case ① If $y \in (0, \pi/2)$

$\Rightarrow \sec y \in (1, \infty)$

$x \in (1, \infty)$

$$\frac{dy}{dx} = \frac{1}{x \cdot \sqrt{\sec^2 y - 1}} = \frac{1}{x\sqrt{x^2-1}}$$



Case ⑪ If $y \in (\pi/2, \pi)$

$\sec y \in (-\infty, -1)$

$x \in (-\infty, -1)$

$\sec^2\theta - 1 = \tan^2\theta$

$\tan\theta = -\sqrt{\sec^2\theta - 1}$

$\theta \in (\pi/2, \pi)$

$$\frac{dy}{dx} = \frac{1}{x(-\sqrt{\sec^2 y - 1})}$$

$$\frac{dy}{dx} = \frac{1}{-x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{dy}{dx} = \begin{cases} \frac{1}{x\sqrt{x^2-1}} & x > 1 \\ \frac{1}{-x\sqrt{x^2-1}} & x < -1 \end{cases} = \frac{1}{|x|\sqrt{x^2-1}} \quad |x| > 1$$

QUESTION

$\frac{dy}{dx} = Dy = y' = y_1$ same.

(i) If $D(\sin x) = \cos x$ then $D\left(\sin\frac{\pi}{3}\right)$ is 0

(ii) $D(\cos x \operatorname{cosec} x) =$ $-\operatorname{cosec}^2 x$

(iii) $D\left(\frac{1}{\log_x e}\right)$ when $x = \frac{1}{e} =$ e $\quad \frac{d}{dx}(\log_e x) = \frac{1}{x}$ put $x = 1/e$

(iv) $D\left(e^{\ln \cot^{-1} x}\right) =$ $D(\cot^{-1}x) = \frac{-1}{1+x^2}$

(v) $D\left(\frac{1-\cos 2x}{\sin 2x}\right) =$ $D(\tan x) = \sec^2 x$ ($1-\cos 2x = 2\sin^2 x$, $\sin 2x = 2\sin x \cos x$)

(vi) $D\left(\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)\right) =$ 0 (where $x \neq 0$)

if $x>0$: $D(\pi/2) = 0$

if $x<0$: $D(-\pi/2) = 0$

$\tan^{-1}x + \tan^{-1}\frac{1}{x}$, $x>0$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\tan^{-1}x + \tan^{-1}1/x$, $x<0$

$\tan^{-1}x + \cot^{-1}x - \pi$

$\pi/2 - \pi = -\pi/2$

QUESTION

(i) $D\left(\frac{1}{\sin x}\right)$

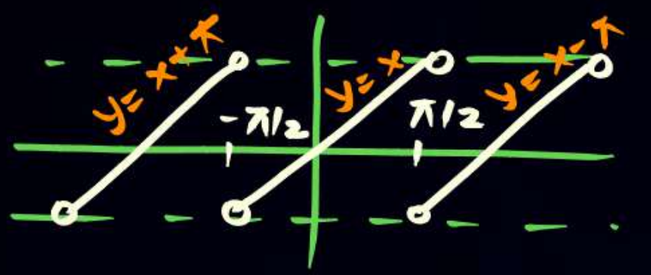
(ii) If $De^x = e^x$ then $D\, e^{\pi}$ is $\underline{0}$

(iii) $D\left(\tan\left(\tan^{-1} x\right)\right) = \underline{D(x)=1}$

(iv) $D\left(\cos^4\frac{x}{2} - \sin^4\frac{x}{2}\right) = \underline{D(\cos x) = -\sin x}$

(v) $D\left(\cos^{-1} x + \sin^{-1} x\right)^n = \underline{D(\pi/2)^n = 0}$.

$y = \tan^{-1}(\tan x)$



$D(\tan^{-1}(\tan x)) = 1$

$D(x+n\pi) = 1$

Product Rule

- $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

- $\frac{d}{dx}(f(x) \cdot g(x) \cdot h(x)) = f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$

OR

$$\frac{d}{dx}(f(x) \cdot g(x) \cdot h(x)) = \frac{(f \cdot g)' \cdot h + (g \cdot h)' \cdot f + (h \cdot f)' \cdot g}{2}$$

$$= \frac{(f' \cdot g + g' \cdot f) \cdot h + (g' \cdot h + h' \cdot g) \cdot f + (h' \cdot f + h \cdot f') g}{2}$$

$$= \frac{2(f' gh + fg'h + fgh')}{2} = f' \cdot gh + f \cdot g' \cdot h + f \cdot g \cdot h'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g^2(x)}$$

Nr. Den

$$\frac{d}{dx}\left(\frac{Nr}{Den}\right) = \frac{Den\,\frac{d}{dx}Nr - Nr\cdot\frac{d}{dx}Den}{(Den)^2}$$



Think twice before applying Quotient Rule, see if the given fn can be simplified or not



QUESTION

If $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$ and $\frac{dy}{dx} = ax + b$ then the value of $a + b$ is equal to

A $\cot\frac{5\pi}{8}$

B $\cot\frac{5\pi}{12}$

C $\tan\frac{5\pi}{12}$

D $\tan\frac{5\pi}{8}$

$$y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1} = \frac{x^4 + 2x^2 + 1 - 3x^2}{x^2 + \sqrt{3}x + 1} = \frac{(x^2+1)^2 - (\sqrt{3}x)^2}{x^2 + \sqrt{3}x + 1}$$

$$y = \frac{(x^2 + 1 + \sqrt{3}x)(x^2 + 1 - \sqrt{3}x)}{x^2 + 1 + \sqrt{3}x}$$

$$y = x^2 - \sqrt{3}x + 1$$

$$\frac{dy}{dx} = 2x - \sqrt{3}$$

$$a = 2, \; b = -\sqrt{3}$$

$$a + b = 2 - \sqrt{3} = \tan\pi/12 \equiv \cot 5\pi/12$$

QUESTION

TAH 1a

If $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$, find $\frac{dy}{dx}\Big]_{x=\pi/4}$

QUESTION

If $f(x) = |\cos x|$, then $f'\left(\frac{3\pi}{4}\right)$ is equal to

A $-\frac{1}{\sqrt{2}}$

B $\frac{1}{\sqrt{2}}$

C 1

D -1

$f(x) = -\cos x$

$f'(x) = \sin x$

$f'(3\pi/4) = \frac{1}{\sqrt{2}}$.

b'coz in nbd of $x = \frac{3\pi}{4} \Rightarrow \cos x < 0$

QUESTION

TAH 1b

If $f(x) = \sin\left(\frac{5\pi}{3}[x] - x\right)$, then $f'(\pi)$ is equal to

[Note : $[\alpha]$ denotes the greatest integer less than or equal to α.]

A 0

B 1

C -1

D $1/2$

QUESTION [JEE Mains 2024 (29 Jan)]

Suppose $f(x)=\frac{(2^x+2^{-x})\tan x\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^3}$. Then the value of $f'(0)$ is equal to

(A) π

(B) $\sqrt{\pi}$

(C) 0

(D) $\frac{\pi}{2}$

$f(x)=\underbrace{\frac{(2^x+2^{-x})\sqrt{\tan^{-1}(x^2-x+1)}}{(7x^2+3x+1)^3}}_{g(x)}\cdot\tan x = g(x)\cdot\tan x.$

$f'(x)=g(x)\cdot\sec^2x+g'(x)\cdot\tan x$

$f'(0)=g(0)\cdot 1+0$

$=\frac{(2^0+2^{-0})\sqrt{\tan^{-1}1}}{1^3}=2\cdot\frac{\sqrt{\pi}}{2}=\sqrt{\pi}$

Ans. B

QUESTION

If $f(x) = (1 + x) \cdot (3 + x^2)^{1/2} \cdot (9 + x^3)^{1/3}$. Find $f'(-1)$.

$g(x)$

$f(x) = g(x) \cdot (1+x)$

$f'(x) = g(x) + (1+x)\, g'(x)$

$f'(-1) = g(-1) = 4^{1/2} \cdot 8^{1/3} = 6.$

QUESTION

Let f, g, h are differentiable function. If $f(0) = 1$, $g(0) = 2$, $h(0) = 3$ and the derivative of their pairwise product at $x = 0$ are
$(f \cdot g)'(0) = 6$, $(g \cdot h)'(0) = 4$; $(h \cdot f)'(0) = 5$. Compute the value of $(f \cdot g \cdot h)'(0)$.

We know $(f \cdot g \cdot h)'(x) = \dfrac{(f \cdot g)' h + (gh)' f + (fh)' g}{2}$

$= \dfrac{6 \cdot 3 + 4 \cdot 1 + 5 \cdot 2}{2}$

$= 16$ Ans

QUESTION

TAH 1c

Let $F(x) = f(x) \cdot g(x) \cdot g(x) \cdot h(x)$. If for some $x = x_0$,

$F'(x_0) = 21\, F(x_0)$; $f'(x_0) = 4f(x_0)$, $g'(x_0) = -7 \cdot g(x_0)$ and $h'(x_0) = k \cdot h(x_0)$. Then find the value of k.

QUESTION

Let $f(x) = \frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1} \cdot x$ then

A $f'(10) = 1$

B $f'(3/2) = -1$

C domain of $f(x)$ is $x \geq 1$

D none

$$f(x) = \frac{\sqrt{x-1+1-2\sqrt{x-1}}}{\sqrt{x-1}-1} x = \frac{\sqrt{(\sqrt{x-1}-1)^2}}{\sqrt{x-1}-1} \cdot x$$

$$f(x) = \frac{|\sqrt{x-1}-1|}{\sqrt{x-1}-1} \cdot x$$

$\sqrt{x-1}-1 \neq 0$, $x-1 \geq 0$

$\sqrt{x-1} \neq 1$, $x \geq 1$

$x \neq 2$

$x \in [1,\infty) - \{2\}$

if $x > 2$ $\quad x-1 \geq 1$

$\sqrt{x-1} \geq 1$

$\sqrt{x-1}-1 \geq 0$

$$f(x) = \frac{\sqrt{x-1}-1}{\sqrt{x-1}-1} \cdot x = x \quad f'(x) = 1$$

if $1 \leq x < 2$

$$f(x) = \frac{-(\sqrt{x-1}-1)}{\sqrt{x-1}-1} \cdot x = -x \quad f'(x) = -1$$

QUESTION

Paragraph

People living at Mars define the derivative of $f(x)$ in an unusual manner as $D * f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$ where $D * f(x)$ denotes the derivative of $f(x)$ at Mars.

Now answer the following.

If $u = f(x), v = g(x)$, then the value of $D * (uv)$ is

A $(D * u)v + (D * v)u$

B $u^2 D * v + v^2 D * u$

C $D * u + D * v$

D $uvd * (u + v)$

$$D * f(x) = \lim_{h \to 0} \frac{(f(x+h) - f(x))(f(x+h) + f(x))}{h}$$

$$= f'(x) \cdot 2f(x)$$

$$D^* f(x) = 2f(x) \cdot f'(x)$$

$$D^*(uv) = 2uv\,(uv)'$$

$$= 2uv(u'v + v'u) = 2(v^2 uu' + u^2 vv')$$

$$= (2uu')v^2 + (2vv')u^2 = v^2 D^* u + u^2 D^* v$$

QUESTION

TAH 2a

Paragraph

People living at Mars define the derivative of $f(x)$ in an unusual manner as $D * f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$ where $D * f(x)$ denotes the derivative of $f(x)$ at Mars.

Now answer the following.

If $u = f(x), v = g(x)$ then the value of $D * \left(\frac{u}{v}\right)$ is

(A) $\sec^2 x$

(B) $2 \sec^2 x$

(C) $\tan x \sec^2 x$

(D) $2 \tan x \sec^2 x$

QUESTION

TAH 2b

For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds?

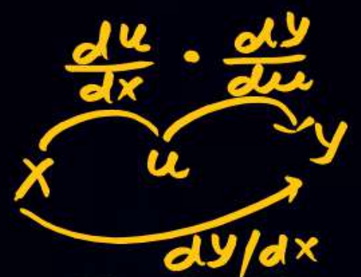
A if $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

B if $f(x) > 0$ for all real $x \Rightarrow f'(x) < 0$

C if $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

D if $f'(x) > 0$ for all real $x \Rightarrow f(x) < 0$

Chain Rule of Differentiation



If $y = f(u)$, $u = g(x)$ are derivable functions of u and x respectively then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

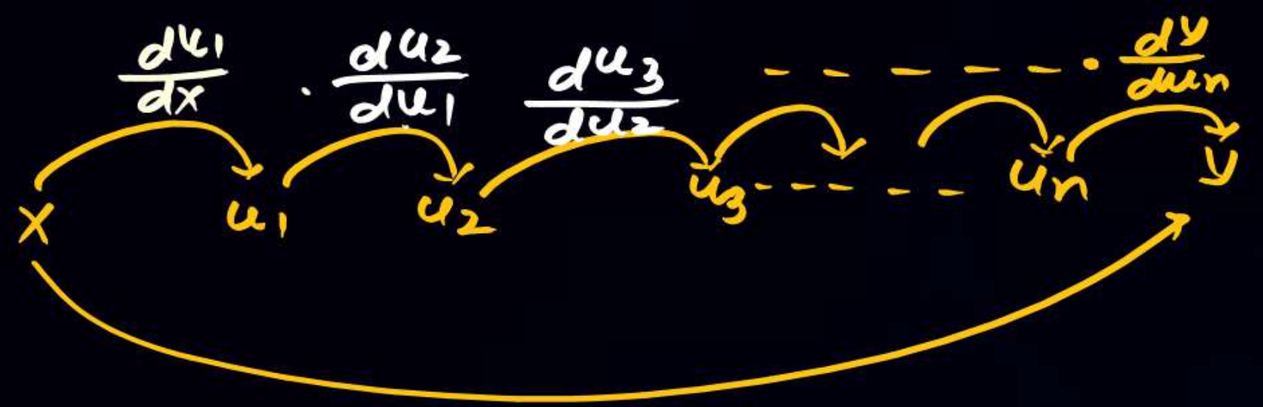
$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \Rightarrow \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x}$$

as $\Delta x \to 0 \Downarrow \Delta u \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$$

$$= \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$



$u_1 = f(x)$

$u_2 = f(u_1)$

$u_3 = f(u_2)$

$u_n = f(u_{n-1})$

$y = f(u_n)$

$$\frac{dy}{dx} = \frac{du_1}{dx} \cdot \frac{du_2}{du_1} \cdot \frac{du_3}{du_2} \cdots \frac{dy}{du_n}.$$

Tameez se

$$y = \sqrt{\sin x}$$

$y = \sqrt{u}$, $u = \sin x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{u}} \cdot \cos x$$

$$= \frac{1}{2\sqrt{\sin x}} \cdot \cos x$$

Shortcut

$y = \sqrt{\sin x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x}} \cdot \cos x.$$

QUESTION

If $y = \ln\left\{\frac{x+\sqrt{(a^2+x^2)}}{a}\right\}$, then the value of $\frac{dy}{dx}$ is

A $\sqrt{a^2-x^2}$

B $a\sqrt{a^2+x^2}$

C $\frac{1}{\sqrt{a^2+x^2}}$

D $x\sqrt{a^2+x^2}$

$$y = \ln(x+\sqrt{a^2+x^2}) - \ln a$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{a^2+x^2}}\left(1 + \frac{1}{2\sqrt{a^2+x^2}}(0+2x)\right) - 0$$

$$= \frac{1}{x+\sqrt{a^2+x^2}}\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)$$

$$= \frac{1}{x+\sqrt{a^2+x^2}} \cdot \frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}}$$

QUESTION [JEE Mains 2024 (6 April)]

Suppose for a differentiable function h, $h(0) = 0, h(1) = 1$ and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to:

(A) 4

(B) 5

(C) 3

(D) 8

$g(x) = h(e^x) \cdot e^{h(x)}$

$g'(x) = h(e^x) \cdot e^{h(x)} \cdot h'(x) + e^{h(x)} \cdot h'(e^x) \cdot e^x$

$g'(0) = h(1) \cdot e^{h(0)} \cdot h'(0) + e^{h(0)} \cdot h'(1) \cdot e^0$

$= 1 \cdot e^0 \cdot 2 + e^0 \cdot 2 \cdot 1$

$= 4.$

Ans. A

QUESTION [JEE Mains 2019 (8 April)]

If $f(1) = 1, f'(1) = 3$, then the derivative of $f\left(f(f(x))\right) + (f(x))^2$ at $x = 1$ is:

A. 33

B. 12

C. 9

D. 15

$$y = f(f(f(x))) + (f(x))^2$$

$$\frac{dy}{dx} = f'(f(f(x)))\, f'(f(x))\, f'(x) + 2(f(x))\, f'(x)$$

Put $x = 1$

$$\left.\frac{dy}{dx}\right|_{x=1} = f'(f(f(1)))\, f'(f(1))\, f'(1) + 2f(1)\cdot f'(1)$$

$$= f'(1)\, f'(1)\cdot f'(1) + 2\cdot 1\cdot f'(1)$$

$$= 3\cdot 3\cdot 3 + 6$$

$$= 33$$

Ans. A

QUESTION

Suppose that f is a differentiable function such that $f(2) = 1$ and $f'(2) = 3$ and let $g(x) = f(x\,f(x))$. Find $g'(2)$.

$g(x) = f(x \cdot f(x))$

$g'(x) = f'(x \cdot f(x)) \cdot (x \cdot f'(x) + f(x))$

$g'(2) = f'(2 \cdot f(2))\,(2 f'(2) + f(2))$

$= f'(2) \cdot (2 \cdot f'(2) + f(2))$

$= 3(6 + 1) = 21.$

QUESTION

TAH 3a

Let $h(x) = f(g(\sin 4x))$, find $h'\left(\frac{\pi}{24}\right)$. If $g\left(\frac{1}{2}\right) = 2; f'(2) = 4, g'\left(\frac{1}{2}\right) = \sqrt{3}$.

QUESTION

TAH 3b

Suppose f and g are derivable functions such that $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$. find $g'(x)$.

QUESTION

TAH 3c

If $u = f(\tan x), v = g(\sec x)$ and $f'(x) = \tan^{-1} x, g'(x) = \operatorname{cosec}^{-1} x$ then $\frac{du}{dv}$ at $x = \frac{\pi}{4}$ is equal to

A $1/\sqrt{2}$

B 1

C $\sqrt{2}$

D 2

QUESTION

TAH 3d

Let $g(x) = x^3 \ln(x^2 f(x))$, where $f(x)$ is a differentiable positive function on $(0, \infty)$ satisfying $f(2) = \frac{1}{4}$ and $f'(2) = -3$, then $g'(2)$ equals

A 77

B -88

C 88

D -77

Logarithmic Differentiation

To find the derivative of :

(i) A function which is the product or quotient of a number of functions.

(ii) A function of the form $[f(x)]^{g(x)}$ where f and g are both derivable functions, it will be found convenient to take the logarithm of the function first and then differentiate

OR

Express $y = (f(x))^{g(x)} = e^{g(x) \ln (f(x))}$ and then differentiate.

M① $y = (f(x))^{g(x)}$

$\ln y = g(x) \cdot \ln f(x)$

M② $y = (f(x))^{g(x)}$

$y = e^{\ln (f(x))^{g(x)}}$

$y = e^{g(x) \cdot \ln f(x)}$

$y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \ln x$

$\frac{dy}{dx} = y(1 + \ln x)$

$\frac{dy}{dx} = x^x(1 + \ln x)$

M② $y = x^x$

$y = e^{\ln x^x}$

$y = e^{x \ln x}$

$\frac{dy}{dx} = e^{x \ln x} \cdot \left(x \cdot \frac{1}{x} + 1 \cdot \ln x\right)$

$\frac{dy}{dx} = e^{\ln x^x}(1 + \ln x)$

$= x^x(1 + \ln x)$

$\frac{d}{dx}(x^x) = x^x(1 + \ln x)$

QUESTION

Find $\frac{dy}{dx}$ if

(i) $y = (\sin x)^{\ln x} = e^{\ln x \cdot \ln \sin x}$

$$\frac{dy}{dx} = e^{\ln x \cdot \ln \sin x}\left(\frac{1}{x} \cdot \ln \sin x + \ln x \cdot \frac{1}{\sin x} \cdot \cos x\right)$$

$$\frac{dy}{dx} = (\sin x)^{\ln x}\left(\frac{\ln(\sin x)}{x} + \cot x \cdot \ln x\right)$$

(ii) $y = x^{\tan x} + (\sin x)^{\cos x}$

Taking log on both sides

$\ln y = \tan x \cdot \ln x + \cos x \cdot \ln \sin x$

Gadho/Gadhiyoo aisaa naa karo!!

Agar aapne yehi sochaa

toh aap —— hai!!

$x = e^{\ln x}$

$$y = x^{\tan x} + (\sin x)^{\cos x}$$

$$y = e^{\tan x \cdot \ln x} + e^{\cos x \cdot \ln \sin x}$$

$$\frac{dy}{dx} = e^{\tan x \cdot \ln x}\left(\sec^2 x \cdot \ln x + \frac{\tan x}{x}\right) + e^{\cos x \cdot \ln \sin x}\left(-\sin x \cdot \ln \sin x + \cos x \cdot \frac{1}{\sin x} \cdot \cos x\right)$$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection —TRISHUL / Apnao IIT Jao→ **Selection with good Rank**

Class illustrations

Module,DPP



KTK,TAH
CHALLENGER

QUESTION

(KTK 1)

Let f(x) be a differentiable function on R such that

$2f(x + y) + f(x - y) = 3f(x) + 3f(y) + 2xy$

for all $x, y \in R$. If $f'(0) = 0$ then find the value of $f(3)$.

Ans. 9

QUESTION

(KTK 2)

Let f be a differentiable function satisfying the functional rule $f(xy) = f(x) + f(y) + \frac{x+y-1}{xy} \forall x, y > 0$ and $f'(1) = 2$. Find the value of $[f(e^{100})]$.

Note: [k] denotes the greatest integer less than or equal to k.

Ans. 99

QUESTION

(KTK 3)

Let a_1 and a_2 be two values of a for which

$$f(x)=\begin{cases} x\cdot\dfrac{\ln(1+x)+\ln(1-x)}{\sec x-\cos x}, & x\in(-1,0) \\ (a^2-3a+1)x+x^2, & x\in[0,\infty) \end{cases}$$

is differentiable at $x=0$, then find the value of $(a_1^2+a_2^2)$.

Ans. 5

QUESTION

(KTK 4)

If the function $f(x)$ defined as $f(x) = \begin{cases} -\frac{x^2}{2} & \text{for } x \leq 0 \\ x^n \sin\frac{1}{x} & \text{for } x > 0 \end{cases}$ is continuous but not derivable at $x = 0$, then find the range of n.

Ans. $0 < n < 1$

QUESTION

KTK 5

If $\lim_{x\to 0^+} f(x) = p$ and $\lim_{x\to 0^-} f(x) = q$ then the correct statement(s) is/are true?

A $\lim_{x\to 0^+} f(x^3 - x) = q$

B $\lim_{x\to 0^-} f(x^3 - x) = p$

C $\lim_{x\to 0^+} f(x^2 - x^4) = q$

D $\lim_{x\to 0^-} f(x^2 - x^4) = p$

Ans. A, B, D

Homework From Module

LCD

PRARAMBH : Complete

PRABAL : Complete

PARIKSHIT : Complete

(Revision Practice Problems)

QUESTION

(RPP 1)

The value of $\frac{\cos 24^\circ}{2 \tan 33^\circ \sin^2 57^\circ} + \frac{\sin 162^\circ}{\sin 18^\circ - \cos 18^\circ \tan 9^\circ} + \cos 162^\circ$ is equal to:

Ans. 2

QUESTION

(RPP 2)

If $\cos 20^\circ + 2\sin^2 55^\circ = 1 + \sqrt{2}\sin K^\circ, K \in (0, 90)$, then $K =$

Ans. 65

QUESTION

(RPP 3)

Find the value of the expression $\frac{\sin 20^\circ (4 \cos 20^\circ + 1)}{\cos 20^\circ \cos 30^\circ}$.

Ans. 2

QUESTION

(RPP 4)

Find the value of $\tan\theta(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)$, when $\theta = \frac{\pi}{32}$.

Ans. 1

(Solution to TAH)

TAH-1

$$f(x)=\begin{cases} x^3+6x^2+5x+1 & x<0 \\ x^2+1 & 0\le x<1 \\ x^2+6x+5 & 1\le x\le 2 \end{cases}$$

find $f'(x)$

$$f'(x)=\begin{cases} 3x^2+12x+5 & ,x<0 \\ 2x & 0<x<1 \\ 2x+6 & 1<x\le 2 \end{cases}$$

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at $x=0$

RHD $\lim_{x\to 0} 2x = 0$

LHD $\lim_{x\to 0} 3x^2+12x+5 = 5$

> non derivable at $x=0$

at $x=1$

RHD $\lim_{x\to 1} 2x+6 = 8$

LHD $\lim_{x\to 1} 2x = 2$

> non derivable at $x=1$

QUESTION

Let $f(x) = (x^2 - 3x + 2)\left|(x^3 - 6x^2 + 11x - 6)\right| + \left|\sin\left(x + \frac{\pi}{4}\right)\right|$.

Number of points at which the function f(x) is non-differentiable in $[0, 2\pi]$, is

TAH-2

Let $f(x) = (x^2-3x+2)\left|(x^3-6x^2+11x-6)\right| + \left|\sin\left(x+\frac{\pi}{4}\right)\right|$

No. of points at which the fn f(x) is non-differen. in $[0, 2\pi]$, is

$f(x) = (x-2)(x-1)\underbrace{\left|(x-1)(x-2)(x-3)\right|}_{\text{continuous at } x=1,2,3} + \left|\sin\left(x+\frac{\pi}{4}\right)\right|$

$\downarrow$ derivable at $x=1,2$

A value $=0$ at both $x=1,2$

By A stonishing Result derivable at $x=1,2$

$\swarrow$

Hence only point of non Diff. is $x=3$

$\sin\left(x+\frac{\pi}{4}\right)=0$

$x+\frac{\pi}{4}=n\pi$

$x=n\pi-\frac{\pi}{4}$

$x=\pi-\frac{\pi}{4}, 2\pi-\frac{\pi}{4}$

$x=\frac{3\pi}{4}, \frac{7\pi}{4}$

$\downarrow\downarrow$

It is non derivable at $x=\frac{3\pi}{4}, \frac{7\pi}{4}$

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Avichita Dwivedi
From (UP)
Radhey ~ Radhey Sir

JEE-2

Q. Let $f(x) = |x^2-3x+2|\,|x^3-6x^2+11x-6| + \sin\left|x+\frac{\pi}{4}\right|$
Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$ is-

Soln $f(x) = |x^2-3x+2|\,|x^3-6x^2+11x-6| + \sin\left|x+\frac{\pi}{4}\right|$

$f(x) = |(x-2)(x-1)|\,|(x-1)(x-2)(x-3)| + \left|\sin\left|x+\frac{\pi}{4}\right|\right|$

↓ derivable at $x = 1, 2$ & also $f(1) = f(2) = 0$

↓ Not derivable at $x = 1, 2, 3$

↓ Not derivable at $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

By R.R $f(x)$ is derivable at $x = 1, 2$
$f(x)$ is not derivable at $x = 3$

No. of points of differentiability = 3

at $x = 3, \frac{3\pi}{4}, \frac{7\pi}{4}$

Ans: 3

Tah-2 $f(x)=(x^2-3x+2)\left|(x^3-6x^2+11x-6)\right|+\left|\sin\left(x+\frac{\pi}{4}\right)\right|$.

no of point at which $f_n \cdot f(x)$ is non differentiable in $[0, 2\pi]$, is.

$$f(x)=(x^2-3x+2)\left|(x^3-6x^2+11x-6)\right|+\left|\sin\left(x+\frac{\pi}{4}\right)\right|$$

$$f(x)=(x-1)(x-2)\left|x^2(x-2)-4x(x-2)+3(x-2)\right|+\left|\sin x\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}\cos x\right|.$$

$$f(x)=(x-1)(x-2)\left|(x-2)(x^2-4x+3)\right|+\left|\frac{1}{\sqrt{2}}[\sin x+\cos x]\right|$$

$$f(x)=(x-1)(x-2)\left|(x-1)|x-2)(x-3)\right|+\left|\frac{1}{\sqrt{2}}(\sin x+\cos x)\right|.$$

differential at $x=1,2$

non derivable at $x=1,2,3$

derivable at $x=1,2$ & non derivable at $x=3$.

non derivable at $x=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$

$x=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$

non derivable at $x=3, \frac{3\pi}{4}, \frac{7\pi}{4}$.

$\frac{1}{\sqrt{2}}(\sin x+\cos x)=0$

$\sin x=-\cos x$

in $[0,2\pi]$

$x=\pi-\frac{\pi}{4}$,

$2\pi-\frac{\pi}{4}$.

Kshitij From (U.P)

Ans: 3 points.

QUESTION

If number of points of discontinuity of the function $f(x) = [2 + 10 \sin x]$, in $x \in \left[0, \frac{\pi}{2}\right]$ is same as number of points of non-differentiability of the function $g(x) = (x-1)(x-2)(x-1)(x-2) \ldots (x-2m), (m \in N)$ in $x \in (-\infty, \infty)$ then find the value of m.

[Note: [k] denotes largest integer less than or equal to k.]

TAH-3

$f(x) = [2 + 10\sin x] \quad x \in [0, \pi/2]$

$= 2 + [10\sin x]$

$\therefore 10\sin x \in [0, 10]$

$10\sin x = 0, 1, 2, 3 \ldots, 10$

$\therefore$ No. of points of non continuity $= 10$

$g(x) = (x-1)(x-2)|(x-1)(x-2)\ldots\ldots(x-2m)|$

$\Downarrow$

non derivable at

$x = 3, 4, \ldots, 2m$

$\Downarrow$

No. of points of non diff. $= 2m - 2$

$\therefore 2m - 2 = 10$

$m = 6$

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QUESTION [JEE 2001]

Which of the following functions is differentiable at $x = 0$?

A $\cos(|x|) + |x|$

B $\cos(|x|) - |x|$

C $\sin(|x|) + |x|$

D $\sin(|x|) - |x|$

TAH-4

Which of the following fn's is differentiable at $x=0$?

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A) $\cos(|x|)+|x|$

↓ Differentiable at $x=0$ — ↓ Non Differen. at $x=0$

B'coz $\cos|x| = \cos x$

$\therefore \cos(|x|)+|x|$ is Non. Differ. at $x=0$

B) $\cos(|x|)-|x|$

↓ Differ. at $x=0$ — ↘ Differ. at $x=0$

$\therefore \cos(|x|)-|x|$ is Non. Differ. at $x=0$

C) $\sin(|x|)+|x|$

RHD $\lim_{h\to 0} \frac{\sin h+h}{h} = 2$

LHD $\lim_{h\to 0} \frac{\sin h+h}{-h} = -2$

LHD $\neq$ RHD

⇓

Non Differ. at $x=0$

D) $\sin(|x|)-|x|$

RHD $\lim_{h\to 0} \frac{\sin h-h}{h} = 0$

LHD $\lim_{h\to 0} \frac{\sin h-h}{-h} = 0$

RHD = LHD

⇓

Hence, Differentiable at $x=0$

(Solution to KTK)

QUESTION

(KTK 1)

If $\alpha = \lim\limits_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

A $(1, -3)$

B $(-1, 3)$

C $(-1, -3)$

D $(1, 3)$

Ans. D

KTK-1

$\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$\lim_{x \to \frac{\pi}{4}} \frac{\tan x (\tan^2 x - 1)}{\cos x \cos\frac{\pi}{4} - \sin x \cos\frac{\pi}{4}}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\tan x \left(\frac{\sin^2 x - \cos^2 x}{\cos^2 x}\right)}{\frac{1}{\sqrt{2}}(\cos x - \sin x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\sin x}{\cos x} \frac{(\sin x + \cos x)(\sin x - \cos x)}{\cos^2 x} \Big/ \frac{1}{\sqrt{2}}(\cos x - \sin x)$

$= \lim_{x \to \frac{\pi}{4}} \sqrt{2} \frac{\sin x}{\cos^3 x} (\sin x + \cos x)(-1)$

$= \sqrt{2} \times \frac{1}{\sqrt{2}} \Big/ \left(\frac{1}{\sqrt{2}}\right)^3 \left(\frac{2}{\sqrt{2}}\right)(-1)$

$= -2\sqrt{2} \times \frac{2}{\sqrt{2}} = -4$

$\alpha = -4$

$\beta = \lim_{x \to 0} (\cos x)^{\cot x}$ $\quad \to 1^\infty$

$= e^{\lim_{x \to 0} \frac{\cos x}{\sin x}(\cos x - 1)}$

$= e^{\lim_{x \to 0} (-1) \frac{\cos x}{\sin x} \frac{(1 - \cos x)}{x} x}$

$= e^{\lim_{x \to 0} (-1) \cos x}$

$= e^0 = 1$

$\therefore \beta = 1$

Now, $ax^2 + bx - 4 = 0$ $\quad \to -4, \to 1$

$-\frac{b}{a} = -3$, $-\frac{4}{a} = -4$

$\boxed{b = 3}$ $\quad \boxed{a = 1}$

$\therefore (a, b) = (1, 3)$ Ans

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KTK-1

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Form (UP)
Radney - Radney Sir

Q. $\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ & $\beta = \lim_{x \to 0} (\cos x)^{\cot x}$ are the roots of the eqn $ax^2 + bx - 9 = 0$, then ordered pair of (a, b) is:

Sol'n $\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2} \tan x (\tan x + 1)(\tan x - 1)}{\cos x - \sin x}$

$\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2} \tan x (\tan x + 1)(\sin x - \cos x)}{\cos x (\cos x - \sin x)}$

$\alpha = -\sqrt{2} \cdot 1 \cdot 9 \Rightarrow \boxed{\alpha = 9}$

$\beta = \lim_{x \to 0} (\cos x)^{\cot x}$

$\beta = \lim_{x \to 0} (\cos x)^{\cot x} = e^{\lim_{x \to 0} (\cos x - 1) \cot x}$

$= e^{\lim_{x \to 0} -2\sin^2 \frac{x}{2} \cdot \frac{\cos x}{2 \sin \frac{x}{2} \cos \frac{x}{2}}}$

$\beta = e^0 = 1$

$\boxed{\beta = 1}$

$ax^2 + bx - 9 = 0$

POR $= \alpha + \beta = 1 - 9$

$-\frac{b}{a} = -3$

$\boxed{b = 3a}$

SOR $= \alpha + \beta = 9$

$\therefore -\frac{9}{a} = 9$

$\boxed{a = 1}$ $\quad \boxed{b = 3}$ $\Rightarrow$ Answer.

QUESTION

(KTK 2)

Let $g(x) = \begin{cases} 2x + \tan^{-1}x + a, & -\infty < x \leq 0 \\ x^3 + x^2 + bx, & 0 < x < \infty \end{cases}$.

If $g(x)$ is differentiable for all $x \in (-\infty, \infty)$ then $(a^2 + b^2)$ is equal to

A 20

B 13

C 9

D 4

Ans. C

KTK-2

Let $g(x) = \begin{cases} 2x + \tan^{-1}x + a, & -\infty < x \le 0 \\ x^3 + x^2 + bx, & 0 < x < \infty \end{cases}$

If $g(x)$ is differentiable for all $x \in (-\infty, \infty)$ then (a^2+b^2) is equal to,

Since $g(x)$ is differentiable it is continuous at $x = 0$

$\therefore$ LHL = RHL

$\boxed{a = 0}$

LHD = RHD at $x = 0$

$$2 + \frac{1}{1+x^2} + a\Big|_{x=0} = 3x^2 + 2x + b\Big|_{x=0}$$

$$2 + 1 + a = b$$

$$b = a + 3$$

$\boxed{b = 3}$

$\therefore \quad a^2 + b^2 = 9$ Ans

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Form (UP)
Radhey ~ Radhey Sir

KTK-2

Q. Let $g(x) = \begin{cases} 2x + \tan^{-1} x + a & -\infty < x \le 0 \\ x^3 + x^2 + bx & 0 < x < \infty \end{cases}$

If $g(x)$ is differentiable at $x \in (-\infty, \infty)$ then (a^2+b^2) is equal to -

Soln: LHL

$\lim_{x \to 0^-} 2x + \tan^{-1} x + a = a.$

RHL

$\lim_{x \to 0^+} x^3 + x^2 + bx = 0$ ([a = 0])

LHD

$\lim_{h \to 0} \frac{2(x-h) + \tan^{-1}(x-h) + a - 2x - \tan^{-1} x - a}{-h}$

$\lim_{h \to 0} \frac{-2h + \tan^{-1}(x-h) + \tan^{-1}(x)}{-h}$

$\lim_{h \to 0} \frac{2h + \tan^{-1} h}{h} = 2 + 1 = 3.$

RHD

$\lim_{h \to 0} \frac{(x+h)^3 + (x+h)^2 + b(x+h) - x^3 - x^2 - bx}{h}$

$\lim_{h \to 0} \frac{h^3 + h^2 + bh}{h}$

$\lim_{h \to 0} h^3 + h^2 + bh = b$

[b = 3]

$[a^2 + b^2 = 9]$ Answer.

So Ordered pair of $(a, b) = (3, 3)$

QUESTION

(KTK 3)

Which of the following functions defined below are NOT differentiable at the indicated point ?

(A) $f(x) = \begin{cases} x^2 \text{ if } -1 \le x < 0 \\ -x^2 \text{ if } 0 \le x \le 1 \end{cases}$ at $x = 0$

(B) $g(x) = \begin{cases} x \text{ if } -1 \le x < 0 \\ \tan x \text{ if } 0 \le x \le \end{cases}$ at $x = 0$

(C) $h(x) = \begin{cases} \sin 2x \text{ if } x \le 0 \\ 2x \text{ if } x > 0 \end{cases}$ at $x = 0$

(D) $k(x) = \begin{cases} x \text{ if } 0 \le x \le 1 \\ 2 - x \text{ if } 1 < x \le 2 \end{cases}$ at $x = 1$

Ans. D

KTK-3

A) $P(x) = \begin{cases} x^2 & , -1 \le x < 0 \\ -x^2 & , 0 \le x \le 1 \end{cases}$ at $x = 0$

$f(x)$ is continuous $\Rightarrow$ LHL = RHL

LHD : $2x|_{x=0} = 0$

RHD : $-2x|_{x=0} = 0$ $\Rightarrow$ differentiable

B) $g(x) = \begin{cases} x & , -1 \le x < 0 \\ \tan x & , 0 \le x 1 \end{cases}$ at $x = 0$

LHL = 0
RHL = 0 $\Rightarrow$ continuous at $x = 0$

LHD $\lim_{h \to 0} \frac{h+0-0}{h} = 1$

RHD $\lim_{h \to 0} \frac{\tan(0-h)-0}{-h} = \lim_{h \to 0} -\frac{\tan h}{-h} = 1$

$g(x)$ is differentiable at $x = 0$

C) $h(x) = \begin{cases} \sin 2x & , x \le 0 \\ 2x & , x > 0 \end{cases}$ at $x = 0$

LHD $2\cos 2x|_{x=0} = 2$

RHD $2|_{x=0} = 2$ $\Rightarrow$ Differentiable.

D) $K(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 < x \le 2 \end{cases}$ at $x = 1$

LHD : 1

RHD : $-x|_{x=1} = -1$ $\Rightarrow$ LHD $\neq$ RHD

$\Rightarrow$ $K(x)$ is not differentiable

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QUESTION

(KTK 4)

Let $g(x) = (x^5 + 1)|x^2 - 4x - 5| + \sin|x| + \cos(|x - 1|)\ \forall\ x \in R$ then find the number of points of non-differentiability of $g(x)$ on R.

Ans. 2

KTK-4

$g(x) = (x^5+1)|x^2-4x-5| + \sin|x| + \cos(|x-1|) \quad \forall x \in R$

(x^5+1): derivable at $x=-1$ & value at $x=-1$ is 0

$|x^2-4x-5|$: continuous at $x=-1,5$

$\sin|x|$: Non derivable at $x=0$

$\cos(|x-1|)$: derivable on R

by Astonishing Result
derivable at $x=-1$
hence only point of non differentiability is $x=5$

2 points of non-differentiability

Hng

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QUESTION

(KTK 5)

Let $f(x) = x^2 - x + k - 2, k \in R$. If the complete set of values of k for which $y = |f(|x|)|$ is non-derivable at 5 distinct points is (a, b) then find the value of $8(b - a)$.

Ans. 2

QUESTION

(KTK 6)

Which of the following function(s) not defined at $x = 0$ has/have non-removable discontinuity at the point $x = 0$?

A $f(x) = \dfrac{1}{1 + 2^{\frac{1}{x}}}$

B $f(x) = \arctan \dfrac{1}{x}$

C $f(x) = \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$

D $f(x) = \dfrac{1}{\ln|x|}$

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Form (UP)
Radhey ~ Radhey Sir

KTG-6

Q. Which of the following function(s) not defined at $x=0$ has/have non-removable discontinuity at $x=0$?

(A) $f(x) = \frac{1}{1+2^{\frac{1}{x}}}$

Soln: LHL $\lim_{h\to 0} \frac{1}{1+2^{\frac{1}{h}}} = 1$

RHL $\lim_{h\to 0} \frac{1}{1+2^{-\frac{1}{h}}} = 0$

} It has non-removable dis.

(B) $f(x) = \tan^{-1}\left|\frac{1}{x}\right|$

Soln: LHL: $\lim_{h\to 0} \tan^{-1}\left|\frac{1}{h}\right| = -\infty$

RHL: $\lim_{h\to 0} \tan^{-1}\left|\frac{1}{h}\right| = \infty$

} It has non removable discontinuity

(C) $f(x) = \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$

Soln It has non-removable discontinuity

(D) $f(x) = \frac{1}{\ln|x|}$

Soln. LHL: $\lim_{h\to 0} \frac{1}{\ln|-h|} = 0$

RHL: $\lim_{h\to 0} \frac{1}{\ln|h|} = 0$

it has removable discontinuity.

QUESTION

(KTK 7)

Let $f(x) = \begin{cases} \frac{e^x - 1 + ax}{x^2}, & x > 0 \\ b, & x = 0 \\ \frac{\sin\frac{x}{2}}{x}, & x < 0 \end{cases}$ then-

(A) $f(x)$ is continuous at $x = 0$ if $a = -1$, $b = \frac{1}{2}$.

(B) $f(x)$ is discontinuous at $x = 0$, if $b \neq \frac{1}{2}$.

(C) $f(x)$ has non-removable discontinuity at $x = 0$ if $a \neq -1$.

(D) $f(x)$ has removable discontinuity at $x = 0$ if $a = -1$, $b \neq \frac{1}{2}$.

KTK: 07

$$f(x) = \begin{cases} \dfrac{e^x - 1 + ax}{x^2}, & x > 0 \\ b, & x = 0 \\ \dfrac{\sin x/2}{x}, & x < 0 \end{cases}$$

Sol^n At $x = 0$

LHL $\lim\limits_{x \to 0^-} \dfrac{\sin x/2}{\frac{x}{2} \cdot 2} = \dfrac{1}{2}$

RHL :- $\lim\limits_{x \to 0^+} \dfrac{e^x - 1 + ax}{x^2}$

$$\lim_{x \to 0^+} \frac{\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right) - 1 + ax}{x^2}$$

$$\lim_{x \to 0^+} \frac{x(1+a) + \frac{x^2}{2} + \ldots \infty}{x^2}$$

for limit to be Exist if $1 + a = 0$

$a = -1$

$$\lim_{x \to 0} \frac{\frac{x^2}{2!} + \frac{x^3}{3!} + \ldots}{x^2}$$

$$= \frac{1}{2}$$

$\therefore$ LHL = RHL = $f(0)$

$\frac{1}{2} = \frac{1}{2} = b$

$b = 1/2$

Ans is ABCD

(A) $\therefore$ $f(x)$ is continuous at $x = 0$ if $a = -1, b = \frac{1}{2}$

(B) $f(x)$ is discontinuous at $x = 0$ if $b \neq \frac{1}{2}$

(C) $f(x)$ has non-removable discont. at $x = 0$ if $a \neq -1$

(D) " " " " " at $x = 0$ if $a = -1$ $b \neq 1/2$

Ritesh Singh, from Devbhoomi Uttarakhand 🙏

QUESTION

(KTK 8)

If $f(x)=\begin{cases} x+a, & x \leq 0 \\ |x-4|, & x>0 \end{cases}$ and $g(x)=\begin{cases} x+1, & x<0 \\ (x-4)^2+b, & x \geq 0 \end{cases}$ are continuous on R, then $(gof)(2)+(fog)(-2)$ is equal to :

A -10

B 10

C 8

D -8

Ans. D

KTK:08

$$f(x) = \begin{cases} x+a, & x \le 0 \\ |x-4|, & x > 0 \end{cases}$$ and

$$g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \ge 0 \end{cases}$$ are Continuous

on R then $(gof)(2) + fog(-2) = ?$

Sol:- $$f(x) = \begin{cases} x+a, & x \le 0 \\ |x-4|, & x > 0 \end{cases}$$

At $x = 0$

LHL, $0 + a = a$

RHL, $|-4| = 4$ $\Rightarrow$ $a = 4$

$$g(x) = \begin{cases} x+1, & x < 0 \\ (x-4)^2 + b, & x \ge 0 \end{cases}$$

Ritesh Singh, from Devbhoomi Uttarakhand

LHL, 1

RHL, $16 + b$ $\Rightarrow$ $b = -15$

$\therefore gof(2) = g(f(2)) = g(2)$

$= (2)^2 + (-15) = -11$

$fog(-2) = f(g(-2)) = f(-1)$

$= -1 + 4 = 3$

$\therefore gof(2) + fog(-2) = -11 + 3 = -8$

(D) Ans

QUESTION

KTK 9

If $[x]$ denotes the integral part of x, then the number of points where $f(x) = [\sin x] + [\cos x], 0 \leq x \leq 2\pi$ is dis continuous

A 3

B 4

C 5

D none of these

Esha
From: Karnataka

PAGE NO. :
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KTK-9

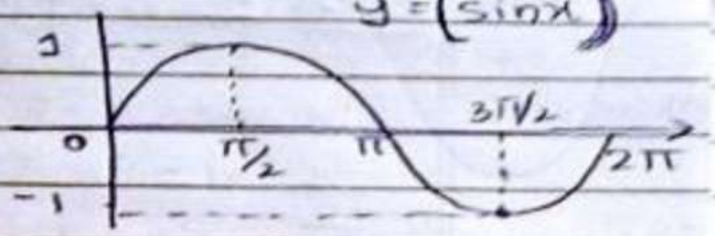
$[\cdot] \rightarrow$ GIF no. of pts where $f(x) = [\sin x] + [\cos x]$, $0 \le x \le 2\pi$ is discontinuous. ~~abs~~ 5 pts.

$[\sin x] + [\cos x] = f(x)$

$[\sin x]$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

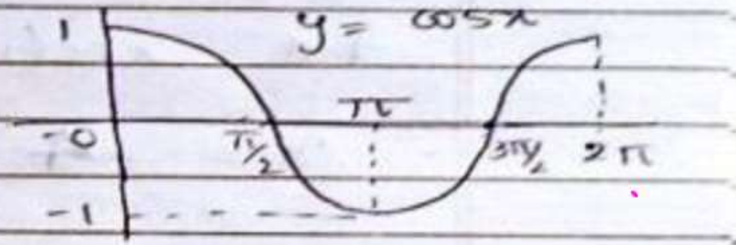
↑ min



$[\cos x]$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

↑ min



∴

x	$[\sin x]$	$[\cos x]$
0	discontinuous	discont.
$\pi/2$	discont.	discont.
π	discont.	cont.
$3\pi/2$	cont.	discont.
2π	discont.	discont.

$f(x)$ is discontinuous at $x = \pi, 3\pi/2$

Ans:- f is discontinuous at $x = 0, \pi/2, \pi, 3\pi/2, 2\pi$
5 pts

We need to check for $x = 0, \frac{\pi}{2}, 2\pi$

at $x=0$

RHL $\lim_{x\to 0^+} [\sin x] + [\cos x] = \lim_{h\to 0} [\sin(+h)] + [\cos(+h)] = 0 + 0 - 1 = -1$

$f(0) = [\sin 0] + [\cos 0] = 0 + 1 = 1$

RHL $\neq f(0)$ $\therefore$ f is discontinuous at $x=0$

at $x = \pi/2$, $f(\pi/2) = 1$

LHL $\lim_{x\to \frac{\pi}{2}^-} [\sin x] + [\cos x] = 0 + 0 = 0$

RHL $\lim_{x\to \frac{\pi}{2}^+} [\sin x] + [\cos x] = 0 - 1 = -1$

$\Big\}$ LHL $\neq$ RHL $\neq f(\pi/2)$ $\Rightarrow$ f is discontinuous at $x=\pi/2$

at $x = 2\pi$ $f(2\pi) = 0+1 = 1$

LHL $\lim_{x\to 2\pi^-} [\sin x] + [\cos x] = -1 + 0 = -1$

$\Big\}$ at $x = 2\pi$, LHL $\neq f(2\pi)$ $\Rightarrow$ Not continuous at $x = 2\pi$

QUESTION [JEE Mains 2020]

KTK 10

Let $f(x) = x.\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to

KTK-10 [JM-2020]

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Q. Let $f(x) = x\left[\frac{x}{2}\right]$ $-10 < x < 10$. Then no. of points of discontinuity of f:

Soln $f(x) = x\left[\frac{x}{2}\right]$ → Non-Derivable at nine points. $x \in (-10, 10)$

$\frac{x}{2} \in (-5, 5)$

derivable at $x=0$
(Its value at $x=0$ is 0)

$\frac{x}{2} = -4, -3, -2, -1, 0, 1, 2, 3, 4$

By A.R
f is derivable at $x=0$

Hence f is non-derivable at
$\frac{x}{2} = -4, -3, -2, -1, 1, 2, 3, 4$
Eight Points of derival discontinuity.

(Solution to RPP)

QUESTION

(RPP 1)

The expression $\frac{\sin 7\theta+6\sin 5\theta+17\sin 3\theta+12\sin\theta}{\sin 6\theta+5\sin 4\theta+12\sin 2\theta}$, when $\theta=\frac{\pi}{5}$ reduces to

A $\cos\frac{\pi}{5}$

B $2\sin\frac{3\pi}{10}$

C $\sin\frac{\pi}{5}$

D $2\sin\frac{\pi}{10}$

Ans. B

Rpp1

$$\frac{\sin 7\theta + 6\sin 5\theta + 17\sin 3\theta + 12\sin\theta}{\sin 6\theta + 5\sin 4\theta + 10\sin 2\theta}$$

$$= \frac{(\sin 7\theta + \sin 5\theta) + 5(\sin 5\theta + \sin 3\theta) + 12(\sin 3\theta + \sin\theta)}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$= \frac{2\sin 6\theta\cos\theta + 5(2\sin 4\theta\cos\theta) + 12(2\sin 2\theta\cos\theta)}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$= \frac{2\cos\theta(\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta)}{(\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta)}$$

$$= 2\cos\frac{\pi}{5} = 2\sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$$

$$= 2\sin\frac{3\pi}{10}$$ (B) Ans

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RDP-1

Q. The expression $\frac{\sin 7\theta + 6\sin 5\theta + 17\sin 3\theta + 12\sin\theta}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$

where $\theta = \frac{\pi}{5}$ reduced to -

Soln

$$\frac{\sin 7\theta + 6\sin 5\theta + 5\sin 5\theta + 5\sin 3\theta + 12\sin 3\theta + 12\sin\theta}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$\frac{2\sin 6\theta\cos\theta + 5(2\sin 4\theta\cos\theta) + 12(2\sin 2\theta\cos\theta)}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$\frac{2\cos\theta\,(\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta)}{(\sin 6\theta + 5\sin 4\theta) + 12\sin 2\theta)}$$

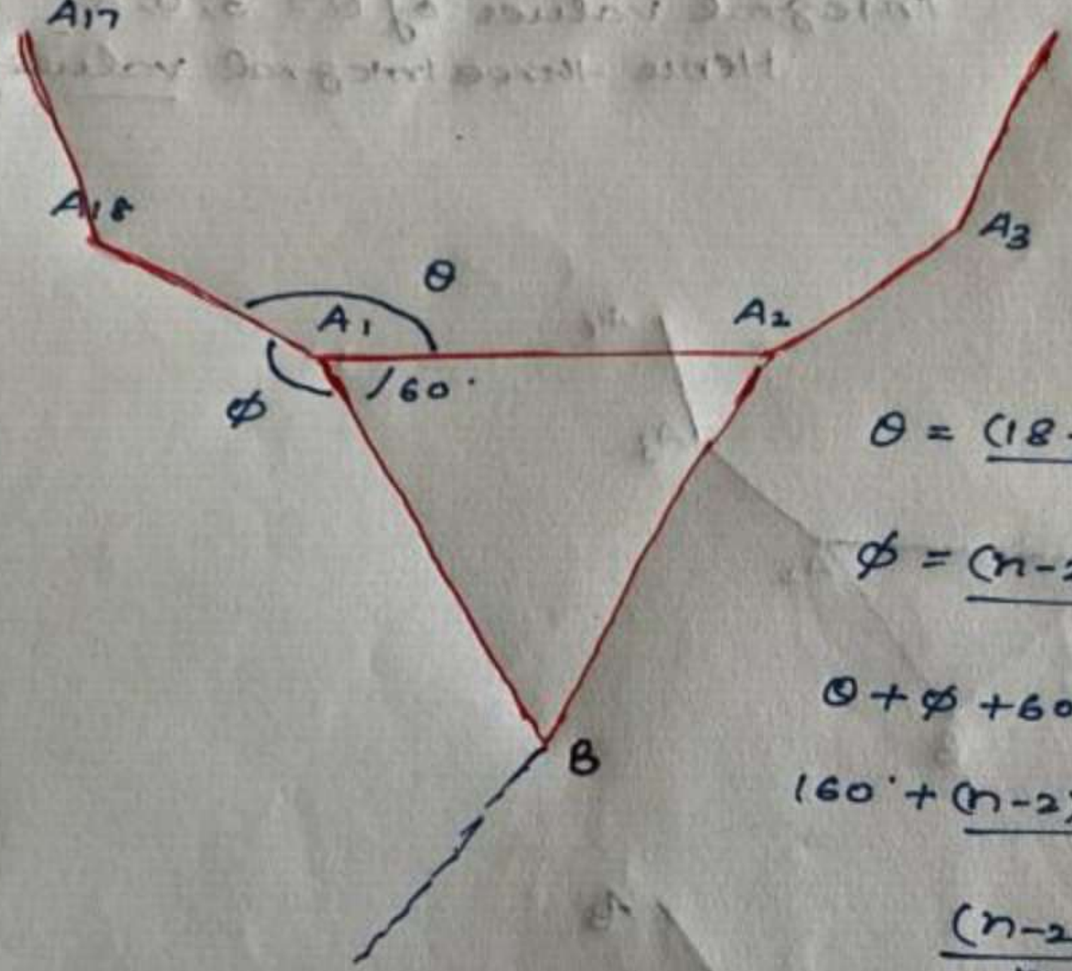
$$2\cos\theta = 2\cos\frac{\pi}{5} = 2\sin\frac{3\pi}{10}$$

QUESTION

(RPP 2)

$A_1A_2A_3$ ..... A_{18} is a regular 18 sided polygon. B is an external point such that A_1A_2B is an equilateral triangle. If $A_{18}A_1$ and A_1B are adjacent sides of a regular n sided polygon, then n =

Ans. 9



$$\theta = \frac{(18-2)\times 180^\circ}{18}$$

$$\phi = \frac{(n-2)\times 180^\circ}{n}$$

$$\theta + \phi + 60^\circ = 360^\circ$$

$$160^\circ + \frac{(n-2)\times 180}{n} = 300$$

$$\frac{(n-2)}{n} \times 180 = 140$$

$$18n - 36 = 14n$$

$$4n = 36$$

$$n = 4$$

QUESTION

(RPP 3)

If λ be the minimum value of

$y = (\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2 + (\tan x + \cot x)^2$ where $x \in R$.

Find $\lambda - 6$.

Ans. 7

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RPP-3

Qe. If λ be the minimum value of
$y = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 +$
$(\tan\theta + \cot\theta)^2$
find $\lambda - 6$

Soln $y = \sin^2\theta + \operatorname{cosec}^2\theta + 2\cos^2\theta + \sec^2\theta + 2 +$
$\tan^2\theta + \cot^2\theta + 2.$

$y = 7 + \operatorname{cosec}^2\theta + \sec^2\theta + \tan^2\theta + \cot^2\theta +$
$= 7 + 1 + \cot^2\theta + 1 + \tan^2\theta + \tan^2\theta + \cot^2\theta$
$y = 9 + 2(\tan^2\theta + \cot^2\theta)$
$y = 9 + 2(\tan\theta - \cot\theta)^2 + 2)$

$y|_{min} = 9 + 2(2) = 13 = \lambda$

$\lambda = 13$ Value of $\lambda - 6$
$= 13 - 6 = 7$ Answer.

Rpp3

Ayush From Jharkhand ❤️

$$y = \left(\sin x + \frac{1}{\sin x}\right)^2 + \left(\cos x + \frac{1}{\cos x}\right)^2 + \left(\tan x + \frac{1}{\tan x}\right)^2$$

$$= \sin^2 x + \frac{1}{\sin^2 x} + 2 + \cos^2 x + \frac{1}{\cos^2 x} + 2 + \tan^2 x + \frac{1}{\tan^2 x} + 2$$

$$= 6 + \sin^2 x + \cos^2 x + \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\sin^2 x}$$

$$= 6 + 1 + \frac{1}{(\sin x \cos x)^2} + \frac{\sin^4 x + \cos^4 x}{(\sin x \cos x)^2}$$

$$= 7 + \frac{4}{\sin^2 2x} + \frac{4(1 - 2\sin^2 x \cos^2 x)}{\sin^2 2x}$$

$$= 7 + \frac{4}{\sin^2 2x} + \frac{4\left(1 - \frac{\sin^2 2x}{2}\right)}{\sin^2 2x}$$

$$7 + \frac{4}{\sin^2 2x} + \frac{4}{\sin^2 2x} - \frac{2\sin^2 2x}{\sin^2 2x}$$

$$= 7 - 2 + \frac{8}{\sin^2 2x} = 5 + \frac{8}{\sin^2 2x}$$

$\sin^2 2x \in [-1, 1]$

$\sin^2 2x \in [0, 1]$

$\frac{8}{\sin^2 2x} \in [8, \infty)$

$5 + \frac{8}{\sin^2 2x} \in [13, \infty)$

$\therefore y_{min} = 13$

$\Rightarrow \boxed{\lambda = 13}$

So, $\lambda - 6 = 7$ Ans

QUESTION

(RPP 4)

If $\cos 20^\circ + 2\sin^2 55^\circ = 1 + \sqrt{2}\sin K^\circ$, $K \in (0, 90)$, then $K =$

Ans. 65

Rpp4

$$\cos 20^\circ + 2\sin^2 55 = 1 + \sqrt{2}\sin k^\circ$$

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LHS $\Rightarrow \cos 20^\circ + 1 - \cos 110$

$\Rightarrow (\cos 20 - \cos 110) + 1$

$\Rightarrow + 2\sin 65 \sin 45 + 1$

$\Rightarrow \sqrt{2}\sin 65^\circ + 1 \equiv 1 + \sqrt{2}\sin k^\circ$

$\therefore \boxed{k = 65^\circ}$ Ans

Basic Practice Problems(BPP)

Single Correct Problems

1. A function f(x) is defined as below
f(x) = $\frac{\cos(\sin x) - \cos x}{x^2}$, $x \neq 0$ and f(0) = a, f(x) is continuous at x = 0 if a equals
(A) 0 (B) 4 (C) 5 (D) 6

2. $f(x) = \begin{cases} \frac{\sqrt{(1+px)} - \sqrt{(1-px)}}{x} & ,-1 \le x < 0 \\ \frac{2x+1}{x-2} & ,0 \le x \le 1 \end{cases}$ is continuous in the interval [–1, 1], then 'p' is equal to:
(A) –1 (B) – 1/2 (C) 1/2 (D) 1

3. Let f(x) = $\left|\left(x + \frac{1}{2}\right)[x]\right|$ when $-2 \le x \le 2$. Then (where [$*$] represents greatest integer function)
(A) f(x) is continuous at x = 2
(B) f(x) is continuous at x = 1
(C) f(x) is continuous at x = –1
(D) f(x) is discontinuous at x = 0

4. Let f(x) = sgn (x) and g(x) = x (x^2 – 5x + 6). The function f(g(x)) is discontinuous at
(A) infinitely many points (B) exactly one point
(C) exactly three points (D) no point

5. If y = $\frac{1}{t^2 + t - 2}$ where t = $\frac{1}{x-1}$, then the number of points of discontinuities of y = f(x), $x \in R$ is
(A) 1 (B) 2 (C) 3 (D) infinite

6. The equation 2 tan x + 5x – 2 = 0 has
(A) no solution
(B) at least one real solution in [0, π/4]
(C) two real solution in [0, π/4]
(D) None of these

7. If f(x) = x $(\sqrt{x} - \sqrt{x+1})$, then indicate the correct alternative(s)
(A) f(x) is continuous but not differentiable at x = 0
(B) f(x) is differentiable at x = 0
(C) f(x) is not differentiable at x = 0
(D) None of these

8. If $f(x) = \begin{cases} \frac{x(3e^{1/x}+4)}{2-e^{1/x}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ then f(x) is

(A) continuous as well differentiable at x = 0
(B) continuous but not differentiable at x = 0
(C) neither differentiable at x = 0 not continuous at x = 0
(D) none of these

9. If $f(x) = \frac{x}{\sqrt{x+1}-\sqrt{x}}$ be a real valued function then

(A) f(x) is continuous, but f′(0) does not exist
(B) f(x) is differentiable at x = 0
(C) f(x) is not continuous at x = 0
(D) f(x) is not differentiable at x = 0

10. The function $f(x) = \sin^{-1}(\cos x)$ is

(A) discontinuous at x = 0 (B) continuous at x = 0
(C) differentiable at x = 0 (D) none of these

11. If $f(x) = \begin{cases} \frac{x^2-1}{x^2+1} & , 0 < x \leq 2 \\ \frac{1}{4}(x^3 - x^2) & , 2 < x \leq 3 \\ \frac{9}{4}(|x-4|+|2-x|) & , 3 < x < 4 \end{cases}$, then

(A) f(x) is differentiable at x = 2 & x = 3
(B) f (x) is non–differentiable at x = 2 & x = 3
(C) f(x) is differentiable at x = 3 but not at x = 2
(D) f(x) is differentiable at x = 2 but not at x = 3.

12. Let f(x) be defined in [–2, 2] by

$f(x) = \begin{cases} \max(\sqrt{4-x^2}, \sqrt{1+x^2}) & , -2 \leq x \leq 0 \\ \min(\sqrt{4-x^2}, \sqrt{1+x^2}) & , 0 < x \leq 2 \end{cases}$ then f(x)

(A) is continuous at all points
(B) is not continuous at more than one point
(C) is not differentiable only at one point
(D) is not differentiable at more than one point.

13. The number of points at which the function f(x) = max. {a – x, a + x, b} $-\infty < x < \infty$, 0 < a < b cannot be differentiable is

(A) 1 (B) 2
(C) 3 (D) none of these

14. If f(x) is differentiable everywhere, then

(A) | f | is differentiable everywhere
(B) $|f|^2$ is differentiable everywhere
(C) f | f | is not differentiable at some point
(D) f + | f | is differentiable everywhere

15. Let $f(x + y) = f(x) f(y)$ all x and y. Suppose that $f(3) = 3$ and $f'(0) = 11$ then $f'(3)$ is given by
(A) 22 (B) 44 (C) 28 (D) 33

16. If $f : R \to R$ be a differentiable function, such that $f(x + 2y) = f(x) + f(2y) + 4xy \ \forall \ x, y \in R$, then
(A) $f'(1) = f'(0) + 1$ (B) $f'(1) = f'(0) - 1$
(C) $f'(0) = f'(1) + 2$ (D) $f'(0) = f'(1) - 2$

17. Let $f(x) = x - x^2$ and $g(x) = \begin{cases} \max f(t), 0 \le t \le x, 0 \le x \le 1 \\ \sin \pi x, \quad x > 1 \end{cases}$.
Then in the interval $[0, \infty)$
(A) g(x) is everywhere continuous except at two points
(B) g(x) is everywhere differentiable except at two points
(C) g(x) is everywhere differentiable except at x = 1
(D) none of these

18. Let $[x]$ denote the integral part of $x \in R$ and $g(x) = x - [x]$. Let f(x) be any continuous function with $f(0) = f(1)$ then the function $h(x) = f(g(x))$
(A) has finitely many discontinuities
(B) is continuous on R
(C) is discontinuous at some x = c
(D) is a constant function.

19. The function f(x) is defined by

$$f(x) = \begin{cases} \log_{(4x-3)}(x^2 - 2x + 5) & \text{if } \frac{3}{4} < x < 1 \ \& \ x > 1 \\ 4 & \text{if } x = 1 \end{cases}$$

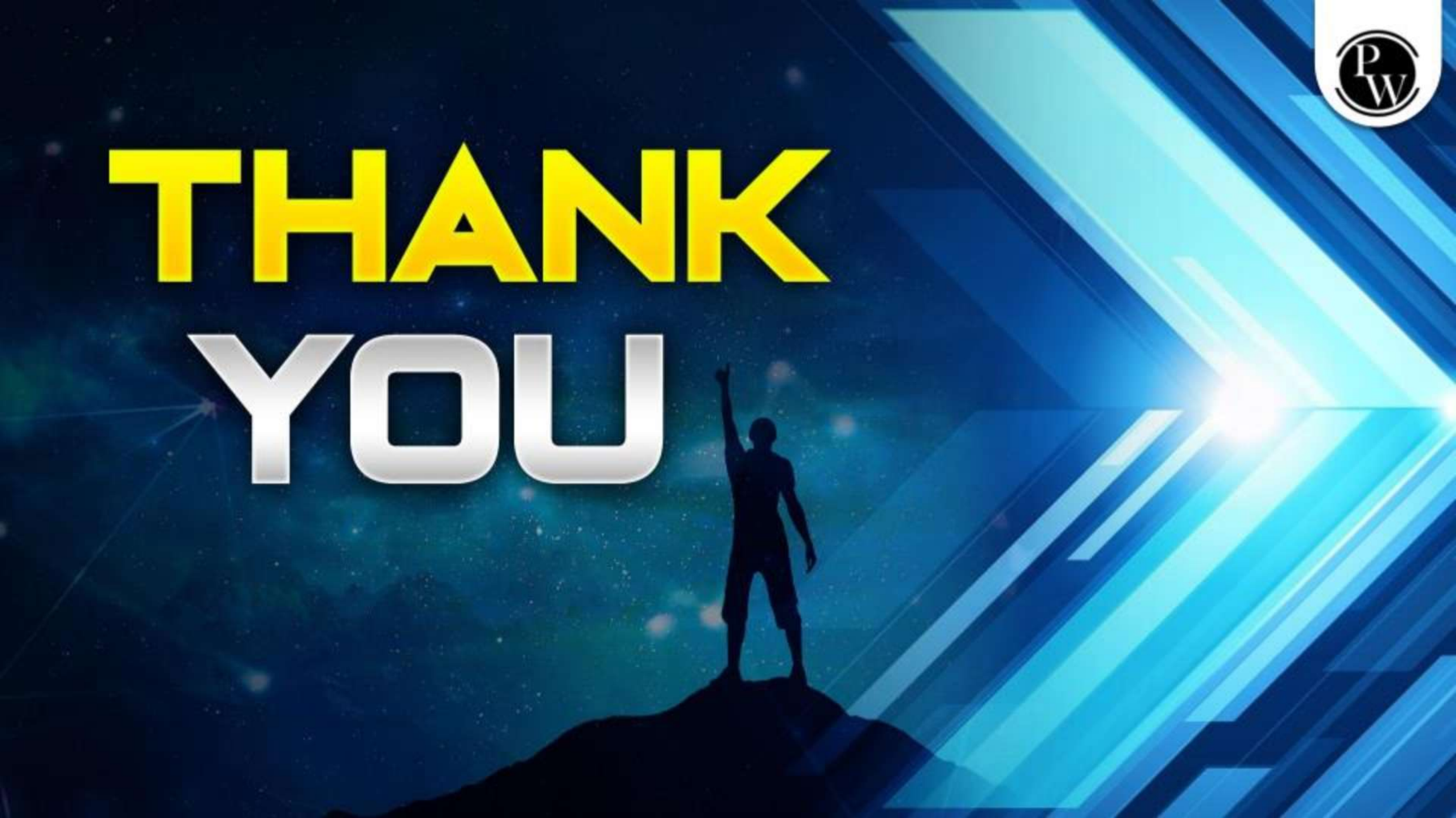
(A) is continuous at x = 1
(B) is discontinuous at x = 1 since $f(1^+)$ does not exist though $f(1^-)$ exists
(C) is discontinuous at x = 1 since $f(1^-)$ does not exist though $f(1^+)$ exists
(D) is discontinuous since neither $f(1^-)$ nor $f(1^+)$ exists.

20. Let $f(x) = \begin{cases} x^2 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$ then
(A) f(x) is discontinuous for all x
(B) discontinuous for all x except at x = 0
(C) discontinuous for all x except at x = 1 or – 1
(D) none of these

21. A point where function f(x) is not continuous where $f(x) = [\sin [x]]$ in $(0, 2\pi)$; is
([$*$] denotes greatest integer $\le x$)
(A) (3, 0) (B) (2, 0) (C) (1, 0) (D) (4, – 1)

Answers

1. A	**2.** B	**3.** D	**4.** C	**5.** C	**6.** B	**7.** B	**8.** B
9. B	**10.** B	**11.** B	**12.** D	**13.** B	**14.** B	**15.** D	**16.** D
17. C	**18.** B	**19.** D	**20.** C	**21.** D			


THANK
YOU

PRAYAS

JEE 2025

Lecture-14 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Parametric Differentiation, Derivative of a Function w.r.t Another Function
2. Derivative of ITFs, Implicit Differentiation
3. Practice Problems
4.

Recap

1. $y = \frac{1}{f^3(2^{5\sin^{-1}x+x^4})}$ then $\frac{dy}{dx} =$ _____

$\frac{-3(f(2^{5\sin^{-1}x+x^4}))^{-4} \cdot f'(2^{5\sin^{-1}x+x^4}) 2^{5\sin^{-1}x+x^4} \cdot \ln 2 \cdot \left(\frac{5}{\sqrt{1-x^2}} + 4x^3\right)}{(f(2^{5\sin^{-1}x+x^4}))^{-3}}$

2. $y = x^{x^x}$ then $\frac{dy}{dx} =$ _____

$y = x^{x^x}$

$\ln y = \ln x^{x^x} = x^x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x^x \cdot \frac{1}{x} + x^x(1+\ln x)\cdot \ln x$

$\frac{dy}{dx} = x^{x^x}\left(x^{x-1} + x^x \ln x(1+\ln x)\right)$

3. $F(x, y) = x^3 + y^3 + \frac{x+y}{3} + 2xy + 7$

find partial derivative of F(x, y) with respect to

(i) x (ii) y

$\frac{\partial F}{\partial x} = 3x^2 + \frac{1}{3} + 2y$

$\frac{\partial F}{\partial y} = 3y^2 + 1/3 + 2x$.

Recap

4. $\frac{d}{dx}|x| = \frac{|x|}{x} = \text{sgn}\,x,\ x \neq 0$

5. $\frac{d}{dx}\text{sgn}(x) = 0,\ x \neq 0$

6. If for $y = f(x)$, $f(a) = 0$ & $f'(a) = 0$ then the curve $y = f(x)$ touches x-axis at $x = a$ & the function $|f(x)|$ is derivable. at $x = a$.

7. $\{h\} = h$ $\quad \{-h\} = 1 - h.$ (where $0 < h < 1$)

$h - [h]$ $\quad -h - [-h] = -h - (-1)$

Question

#Q. For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds?

A. if $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

B. if $f(x) > 0$ for all real $x \Rightarrow f'(x) < 0$

C. if $f'(x) > 0$ for all real $x \Rightarrow f(x) < 0$

D. if $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

$f'(x) = (x^2+bx+c)e^x + (2x+b)e^x$

$= e^x(x^2+(b+2)x+b+c)$.

$f'(x) > 0 \ \forall \ x \in R$ if

$e^x(x^2+(b+2)x+b+c) > 0 \ \forall \ x \in R$

$x^2+(b+2)x+b+c > 0 \ \forall \ x \in R$

$D_2 < 0 \Rightarrow (b+2)^2 - 4(b+c) < 0$.

$b^2+4-4c < 0$.

$b^2-4c < -4$

$\Rightarrow f'(x) > 0 \ \forall \ x \in R$ if $b^2 - 4c < -4$

Now $f(x) > 0 \ \forall \ x \in R$

$e^x (x^2 + bx + c) > 0 \ \forall \ x \in R$

$x^2 + bx + c > 0 \ \forall \ x \in R$

$D_1 < 0 \Rightarrow b^2 - 4c < 0$.

$\Rightarrow f(x) > 0 \ \forall \ x \in R$ if $b^2 - 4c < 0$

Now if $b^2 - 4c < 0 \Rightarrow b^2 - 4c < -4$

if $b^2 - 4c < -4 \Rightarrow b^2 - 4c < 0$.

$f'(x) > 0 \Rightarrow f(x) > 0 \ \forall \ x \in R$

Question

#Q. If $y = \sin x \cdot \sin 2x \cdot \sin 3x \cdot \ldots\ldots \sin nx$. Find y'.

$$\ln y = \ln \sin x + \ln \sin 2x + \ldots + \ln \sin nx$$

$$\frac{1}{y}\frac{dy}{dx} = \cot x + 2\cot 2x + \ldots + n\cot nx$$

$$\frac{dy}{dx} = y(\cot x + 2\cot 2x + \ldots + n\cot nx)$$

Question

#Q. If $f(x)=\frac{(x+1)^7\sqrt{1+x^2}}{(x^2-x+1)^6}$, then the value of $f'(0)$ is equal to

A 10

B 11

C 13

D 15

$$\ln f(x)=\ln\left(\frac{(x+1)^7\sqrt{1+x^2}}{(x^2-x+1)6}\right)=7\ln(x+1)+\frac{1}{2}\ln(1+x^2)-6\ln(x^2-x+1)$$

$$\frac{1}{f(x)}f'(x)=\frac{7}{x+1}+\frac{2x}{2(1+x^2)}-\frac{6\cdot(2x-1)}{x^2-x+1}$$

$$f'(x)=f(x)\left(\frac{7}{x+1}+\frac{x}{1+x^2}-\frac{6(2x-1)}{x^2-x+1}\right)$$

$$f'(0)=f(0)\left(7+0+\frac{6}{1}\right)$$

$$=13.$$

Question

$(f(x))^{g(x)} = e^{\ln(f(x))^{g(x)}}$

#Q. Let $f(x) = \lim_{h\to 0} \frac{(\sin(x+h)^{\ln(x+h)} - (\sin x)^{\ln x}}{h}$ then $f\left(\frac{\pi}{2}\right)$ is

A equal to 0

B equal to 1

C $\ln \pi/2$

D non existent

$$f(x) = \frac{d}{dx}(\sin x)^{\ln x} = \frac{d}{dx} e^{\ln \sin x^{\ln x}} = \frac{d}{dx} e^{\ln x \cdot \ln \sin x}$$

$$f(x) = e^{\ln x \cdot \ln \sin x} \left(\frac{\ln \sin x}{x} + \ln x \cdot \cot x\right)$$

$$f(\pi/2) = e^0 (0+0)$$

$$f(\pi/2) = 0$$

Question

#Q. If $f(x) = y = \pi^2 + 2^x + x^2 + x^{1/x}$, then find the slope of the line perpendicular to the graph of $y = f(x)$ at $x = 1$.

$$\frac{dy}{dx} = 0 + 2^x \cdot \ln 2 + 2x + \frac{d}{dx} x^{\frac{1}{x}}$$

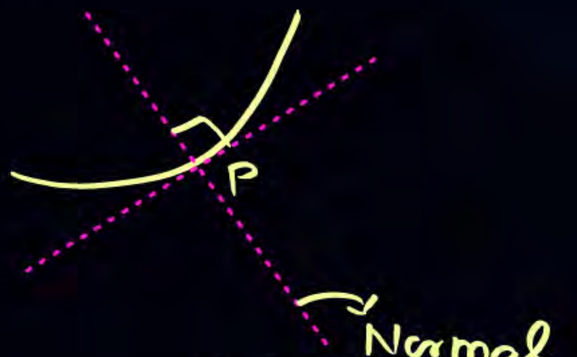
$$= 2^x \cdot \ln 2 + 2x + \frac{d}{dx}\left(e^{\ln x^{\frac{1}{x}}}\right)$$

$$= 2^x \cdot \ln 2 + 2x + \frac{d}{dx}\left(e^{\frac{\ln x}{x}}\right)$$

$$\frac{dy}{dx} = 2^x \cdot \ln 2 + 2x + e^{\frac{\ln x}{x}} \cdot \left(\frac{x \cdot 1/x - 1 \cdot \ln x}{x^2}\right)$$

$$m_T = \left.\frac{dy}{dx}\right|_{x=1} = 2 \cdot \ln 2 + 2 + e^0\left(\frac{1-0}{1}\right)$$

$$= 2\ln 2 + 3$$



Ans. $m = -\frac{1}{2\ln 2 + 3}$

Slope of line $\perp$ to graph at $x=1$ $= \dfrac{-1}{3+2\ln 2}$

Question

#Q. If $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$, then $\left.\frac{dy}{dx}\right]_{x=1}$ is

Tan 1@

A 4

B 1

C 0

D not defined

Question

#Q. Let $f(x) = x + (1 - x)x^2 + (1 - x)(1 - x^2)x^3 + \ldots\ldots$

$+ (1 - x)(1 - x^2) \ldots\ldots (1 - x^{n-1})x^n$; $(n \geq 4)$ then

A $f(x) = -\prod_{r=1}^{n}(1 - x^r)$

B $f(x) = 1 - \prod_{r=1}^{n}(1 - x^r)$

C $f'(x) = (1 - f(x))\left(\sum_{r=1}^{n} \frac{rx^{r-1}}{(1-x^r)}\right)$

D $f'(x) = f(x)\left(\sum_{r=1}^{n} \frac{rx^{r-1}}{(1-x^r)}\right)$

$f(x) = 1-(1-x)+(1-x)x^2+(1-x)(1-x^2)x^3+\cdots$

$+(1-x)(1-x^2)\cdots(1-x^{n-1})x^n$

$= 1-(1-x)(1-x^2)+(1-x)(1-x^2)x^3+(1-x)(1-x^2)(1-x^3)x^4$

$\cdots+(1-x)(1-x^2)\cdots(1-x^{n-1})x^n$

$= 1-(1-x)(1-x^2)(1-x^3)+(1-x)(1-x^2)(1-x^3)x^4+\cdots\cdots$

$\cdots\cdots\cdots+(1-x)(1-x^2)\cdots(1-x^{n-1})x^n$

$f(x) = 1-(1-x)(1-x^2)\cdots(1-x^n)$

$$f(x) = 1 - \prod_{r=1}^{n}(1-x^r)$$

$$1 - f(x) = \prod_{r=1}^{n}(1-x^r)$$

Taking log to base e on both sides

$$\ln(1-f(x)) = \sum_{r=1}^{n} \ln(1-x^r)$$

$$-\frac{1}{1-f(x)} \cdot f'(x) = \sum_{r=1}^{n} \frac{1}{1-x^r}(-rx^{r-1})$$

$$f'(x) = (1-f(x)) \sum_{r=1}^{n} \frac{rx^{r-1}}{1-x^r}.$$

$$\ln(1 \cdot 2 \cdot 3 \cdot 4)$$
$$= \ln 1 + \ln 2 + \ln 3 + \ln 4$$
$$\ln\left(\prod_{r=1}^{4} r\right) = \sum_{r=1}^{4} \ln r.$$

Topic : Parametric Differentiation

$d(\sin x) = \cos x\, dx$.

If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{g'(t)}{f(t)}$

derivable fns of t

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta t} \cdot \frac{\Delta t}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta t \to 0} \left(\frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}} \right) = \frac{\frac{dy}{dt}}{\frac{dx}{dx}}, \left(\text{where } \frac{dx}{dt} \neq 0 \right)$$

$y = g(t)$

$x = f(t)$

$\Downarrow$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0.$$

$$y = f(x)$$

$$y + \Delta y = f(x + \Delta x)$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$$

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

Question

#Q. If $x = a(\cos t + t \cdot \sin t)$ and $y = a(\sin t - t \cdot \cos t)$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a(\cos t - (\cos t - t\sin t))}{a(-\sin t + t\cos t + \sin t)} = \tan t$$

Question

#Q. A function is represented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}$ then $\frac{dy}{dx} - x \cdot \left(\frac{dy}{dx}\right)^3$ has the value equal to

A 2

B 0

C -1

D -2

$x = \frac{1}{t^3} + \frac{1}{t^2}$, $y = \frac{3}{2t^2} + \frac{2}{t}$

$\frac{dx}{dt} = \frac{-3}{t^4} - \frac{2}{t^3}$, $\frac{dy}{dt} = -\frac{6}{2t^3} - \frac{2}{t^2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\left(\frac{3}{t^3} + \frac{2}{t^2}\right)}{-\left(\frac{3}{t^4} + \frac{2}{t^3}\right)} = \frac{3+2t}{3+2t} \cdot t = t \cdot$$

$\frac{dy}{dx} = t \Rightarrow \frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3 = t - \left(\frac{1}{t^3} + \frac{1}{t^2}\right) t^3 .$

$= t - 1 - t = -1$

Question

#Q. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$.

Then show that $(x^2 + 4).\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$

$$\frac{dy}{dx} = \frac{n\sec^{n-1}\theta \cdot \sec\theta\tan\theta + n\cos^{n-1}\theta \sin\theta}{\sec\theta \tan\theta + \sin\theta} =$$

$$= \frac{n\sec\theta \cdot \sin\theta\left(\sec^{n-1}\theta \cdot \frac{1}{\cos\theta} + \cos^n\theta\right)}{\sin\theta\left(\sec\theta \cdot \frac{1}{\cos\theta} + 1\right)}$$

($\sec\theta$ → $\frac{1}{\cos\theta}$)

$$(a+b)^2 = (a-b)^2 + 4ab$$

$$\frac{dy}{dx} = n\frac{(\sec^n\theta + \cos^n\theta)}{(\sec\theta + \cos\theta)} \Rightarrow \left(\frac{dy}{dx}\right)^2 = n^2\frac{(\sec^n\theta + \cos^n\theta)^2}{(\sec\theta + \cos\theta)^2}$$

$$\left(\frac{dy}{dx}\right)^2 = n^2\left(\frac{(\sec n\theta - \cos n\theta)^2 + 4\sec n\theta \cdot \cos n\theta}{(\sec\theta - \cos\theta)^2 + 4\sec\theta \cdot \cos\theta}\right)$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$$

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

(H.P)

Question

#Q. For $t \in (0, 1)$, let $x = \sqrt{2^{\sin^{-1} t}}$ and $y = \sqrt{2^{\cos^{-1} t}}$. Then, $1 + \left(\frac{dy}{dx}\right)^2$ equals

Tan 1 (b)

A $\frac{x^2}{y^2}$

B $\frac{y^2}{x^2}$

C $\frac{x^2+y^2}{y^2}$

D $\frac{x^2+y^2}{x^2}$

Topic : Derivative of a function w.r.t. another Function

$y = f(x)$, $u = g(x)$

$$\frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} \quad , \frac{du}{dx} \neq 0$$

Question

#Q. Prove that differential coefficient of $e^{\sin^{-1}x}$ w.r.t. $e^{-\cos^{-1}x}$ is independent of x.

$y = e^{\sin^{-1}x}$

$z = e^{-\cos^{-1}x}$

$\Rightarrow \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}}{e^{-\cos^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}}$

$= e^{\sin^{-1}x + \cos^{-1}x}$

$= e^{\pi/2}$

↓

Which is indp of x.

Question

#Q. If $u = f(\tan x), v = g(\sec x)$ and $f'(x) = \tan^{-1} x, g'(x) = \operatorname{cosec}^{-1} x$ then $\frac{du}{dv}$ at $x = \frac{\pi}{4}$ is equal to

A $1/\sqrt{2}$

B 1

C $\sqrt{2}$

D 2

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{f'(\tan x)\cdot \sec^2 x}{g'(\sec x)\cdot \sec x \tan x}$$

$$= \frac{\tan^{-1}(\tan x)\cdot \sec x}{\operatorname{cosec}^{-1}(\sec x)\cdot \tan x}\Bigg|_{x=\pi/4}$$

$$= \frac{\tan^{-1}(1)\cdot \sqrt{2}}{\operatorname{cosec}^{-1}\sqrt{2}\cdot 1}$$

$$= \frac{\pi/4\cdot\sqrt{2}}{\pi/4\cdot 1} = \sqrt{2}$$

Question

#Q. Let $f(x) = \begin{cases} x^2 + a, & 0 \le x < 1 \\ 2x + b, & 1 \le x \le 2 \end{cases}$ and $g(x) = \begin{cases} 3x + b, & 0 \le x < 1 \\ x^3, & 1 \le x \le 2 \end{cases}$.

If derivative of f(x) w. r. t. g(x) at x = 1 exists, find the value of a, b and also its value.

$\left.\frac{d f(x)}{d g(x)}\right|_{x=1}$ Exists. $\Rightarrow$ $\frac{df}{dx}$, $\frac{dg}{dx}$ both Exist at $x=1$

$\Downarrow\Downarrow$

f, g are derivable at $x=1$

For f(x) at $x=1$

LHL = 1 + a

RHL = 2 + b

$\Rightarrow a - b = 1$

$LHD = \lim_{x \to 1} 2x + 0 = 2$

$RHD = \lim_{x \to 1} 2$

For g(x)

LHL = 3 + b

RHL = 1

$\Rightarrow b = -2$

$a = -1, b = -2$

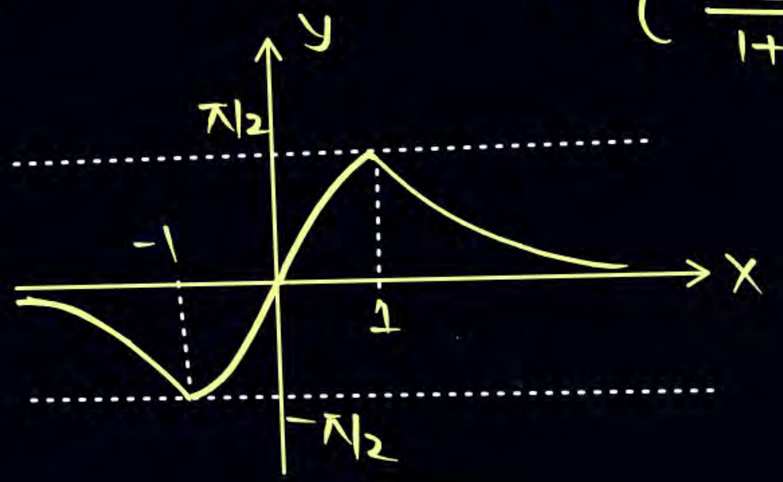
Topic : Derivative of ITFs

In order to differentiate a function involving ITF we can

1. **Differentiate directly or**
2. **Use substitution to simplify**

(a) $\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} -\pi - 2\tan^{-1}x & x<-1 \\ 2\tan^{-1}x & -1\le x\le 1 \\ \pi - 2\tan^{-1}x & x>1 \end{cases}$

$\Rightarrow \frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} \frac{-2}{1+x^2} & x<-1,\ x>1 \\ \frac{2}{1+x^2} & -1<x<1 \end{cases}$

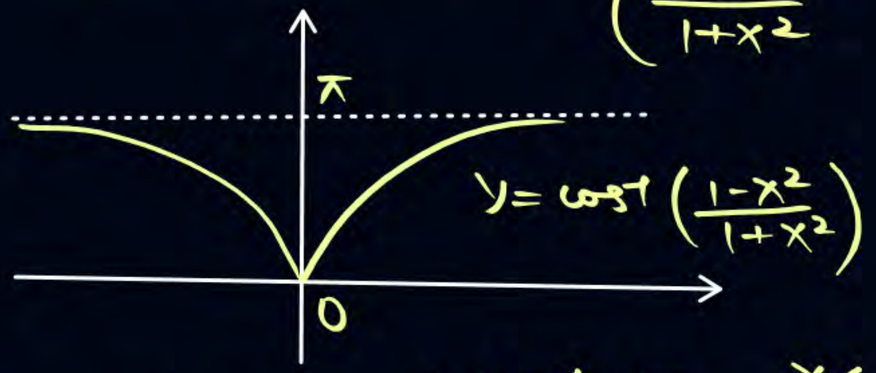


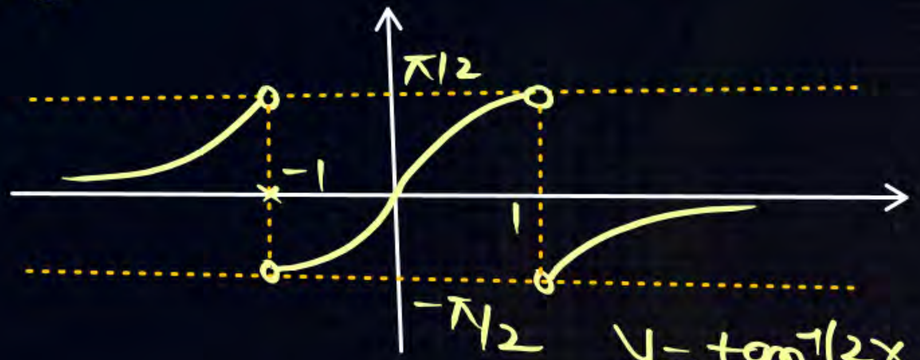
(b) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} -2\tan^{-1}x & x<0 \\ 2\tan^{-1}x & x\geq 0 \end{cases}$

Ex: $\frac{d}{dx}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\Big|_{x=-1}$

$$\frac{d}{dx}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} \frac{-2}{1+x^2} & x<0 \\ \frac{2}{1+x^2} & x>0 \end{cases}$$

$\frac{-2}{1+(-1)^2} = -1$





(c) $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} 2\tan^{-1}x+\pi & x<-1 \\ 2\tan^{-1}x & -1<x<1 \\ 2\tan^{-1}x-\pi & x>1 \end{cases} = \frac{2}{1+x^2} \quad x\neq -1,1$

H.W

(2)(a) $f(x) = \cos^{-1}(4x^3 - 3x)$

(2)(b) $g(x) = \sin^{-1}(3x - 4x^3)$

simplify & find $\frac{dy}{dx}$

also draw their graph

Question

#Q. If $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is :

[JEE Mains 2021]

A -1/2

B -1

C 1/2

D 0

$$y = \cot^{-1}\left(\frac{1+\sin x + 1-\sin x + 2\sqrt{1-\sin^2 x}}{1+\sin x - (1-\sin x)}\right)$$

$$y = \cot^{-1}\left(\frac{2+2|\cos x|}{2\sin x}\right)$$

$$= \cot^{-1}\left(\frac{1+|\cos x|}{\sin x}\right)$$

we have to find derivative at $x = 5\pi/6$ in nbd of $x = 5\pi/6$ cos x is -ve.

$$= \cot^{-1}\left(\frac{1-\cos x}{\sin x}\right)$$

$$= \cot^{-1}\left(\frac{2\sin^2 x/2}{2\sin x/2 \cos x/2}\right) = \cot^{-1}(\tan x/2) = \pi/2 - \tan^{-1}(\tan x/2)$$

$$y = \frac{\pi}{2} - \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{-1}{2}$$

Question

$x=\sin^2\theta \quad \theta\in(0,\frac{\pi}{2})$

#Q. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right);\ 0<x<1$. Then: [JEE Mains 2021]

A $(1-x)^2f'(x)-2(f(x))^2=0$

B $(1+x)^2f'(x)+2(f(x))^2=0$

C $(1-x)^2f'(x)+2(f(x))^2=0$

D $(1+x)^2f'(x)-2(f(x))^2=0$

$f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-\sin^2\theta}{\sin^2\theta}}\right)\right)$

$=\cos(2\tan^{-1}(\sin\cot^{-1}(\cot\theta))$

$=\cos(2\tan^{-1}(\sin\theta))$

$=\cos(2\tan^{-1}\sqrt{x})$ $\quad \tan^{-1}\sqrt{x}=\phi$

$=\cos 2\phi$ $\quad \tan\phi=\sqrt{x}$

$=\frac{1-\tan^2\phi}{1+\tan^2\phi}=\frac{1-x}{1+x}$.

$f'(x)=\frac{(1+x)(-1)-(1-x)}{(1+x)^2}=\frac{-2}{(1+x)^2}$

$$f'(x) = \frac{-2}{(1+x)^2}$$

$$(1-x)^2 f'(x) = -2\left(\frac{1-x}{1+x}\right)^2$$

$$(1-x)^2 f'(x) = -2f^2(x)$$

$$(1-x)^2 f'(x) + 2f^2(x) = 0$$

Question

#Q. If $f(x) = \cos^{-1}(2x^2 - 1)$ and $g(x) = \sqrt{1 - x^2}$, then

A $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \frac{10}{3}$

B $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \frac{-10}{3}$

C $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \frac{-10}{3}$

D $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \frac{10}{3}$

$$\frac{df(x)}{d(g(x))} = \frac{\frac{df(x)}{dx}}{\frac{dg(x)}{dx}} = \frac{\frac{-1}{\sqrt{1-(2x^2-1)^2}} \cdot 4x}{\frac{1}{2\sqrt{1-x^2}} \cdot (-2x)}$$

$$\frac{df(x)}{dg(x)} = \frac{2}{\sqrt{4x^2 - 4x^4}} \cdot 2\sqrt{1-x^2}$$

$$= \frac{2}{2|x|\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}$$

$$\frac{df(x)}{g(x)} = \frac{2}{|x|}$$

$$\left.\frac{df(x)}{dg(x)}\right|_{x=3/5} = \frac{10}{3}, \quad \left.\frac{df(x)}{dg(x)}\right|_{x=-3/5} = 10/3$$

Question

#Q. If $f(x) = \cos^{-1}(2x^2 - 1)$ and $g(x) = \sqrt{1 - x^2}$, then

A $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \frac{10}{3}$

B $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{3}{5}} = \frac{-10}{3}$

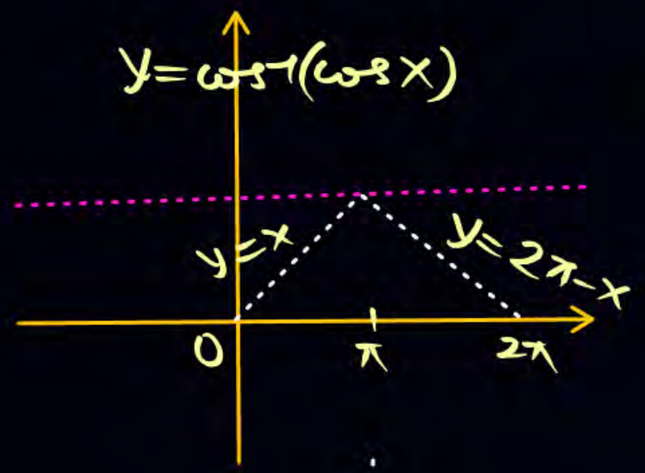
C $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \frac{-10}{3}$

D $\left.\frac{d(f(x))}{d(g(x))}\right|_{x=\frac{-3}{5}} = \frac{10}{3}$

$f(x) = \cos^{-1}(2x^2-1)$, $g(x) = \sqrt{1-x^2}$

$x = \cos\theta$, $\theta \in [0, \pi]$

$f(x) = \cos^{-1}(\cos 2\theta)$ → $[0, 2\pi]$



$$= \begin{cases} 2\theta & 0 \le 2\theta \le \pi \\ 2\pi - 2\theta & \pi < 2\theta \le 2\pi \end{cases}$$

$$= \begin{cases} 2\cos^{-1}x & 0 \le \theta \le \pi/2 \Rightarrow 0 \le \cos\theta \le 1 \\ 2\pi - 2\cos^{-1}x & \frac{\pi}{2} < \theta \le \pi \Rightarrow -1 \le \cos\theta < 0 \end{cases}$$

$$f(x)=\cos^{-1}(2x^2-1)=\begin{cases}2\pi-2\cos^{-1}x & -1\le x<0\\ 2\cos^{-1}x & 0\le x\le 1\end{cases}$$

$$f'(x)=\begin{cases}\frac{2}{\sqrt{1-x^2}} & -1<x<0\\ \frac{-2}{\sqrt{1-x^2}} & 0<x<1\end{cases}$$

$$g(x)=\sqrt{1-x^2}\Rightarrow g'(x)=\frac{-2x}{2\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$$

$$\frac{df(x)}{dg(x)}=\frac{\frac{df}{dx}}{\frac{dg}{dx}}=\frac{f'(x)}{g'(x)}=\begin{cases}-\frac{2}{x} & -1<x<0\\ \frac{2}{x} & 0<x<1\end{cases}$$

$$\left.\frac{df(x)}{dg(x)}\right|_{x=3/5}=\frac{2}{3/5}=10/3$$

$$\left.\frac{df(x)}{dg(x)}\right|_{x=-3/5}=\frac{-2}{-3/5}=10/3.$$

Question

#Q. Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $\forall x \in R$.

Tah 2© (By 2 different methods)

Question

#Q. If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ and $x = \sec^{-1}\frac{1}{2u^2-1}$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

prove that $2\frac{dy}{dx} + 1 = 0$.

Tan (3)

Question

#Q. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $\frac{b}{a}\log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is

[JEE Mains 2021 (March)]

$$f(x) = \sin\left(\cos^{-1}\left(\frac{1-(2^x)^2}{1+(2^x)^2}\right)\right)$$

$$= \sin\left(\cos^{-1}\left(\frac{1-t^2}{1+t^2}\right)\right)$$

$$= \sin(2\tan^{-1}t).$$

$$= \sin(2\theta) \quad \text{where } \theta = \tan^{-1}t$$

$$= \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2t}{1+t^2}.$$

Derivative at $x=1$ is to be found.

$\Rightarrow t = 2^x$ lies in nbd of $x=2$.

& we know

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x, x>0$

$$f(x)=\frac{2\cdot 2^x}{1+(2^x)^2}$$

$$f'(x)=2\cdot\left[\frac{(1+(2^x)^2)\,2^x\ln 2-2^x\cdot 2\cdot 2^x\cdot 2^x\ln 2}{(1+(2^x)^2)^2}\right]$$

put $x=1$

$$f'(1)=2\left[\frac{10\ln 2-16\ln 2}{25}\right]$$

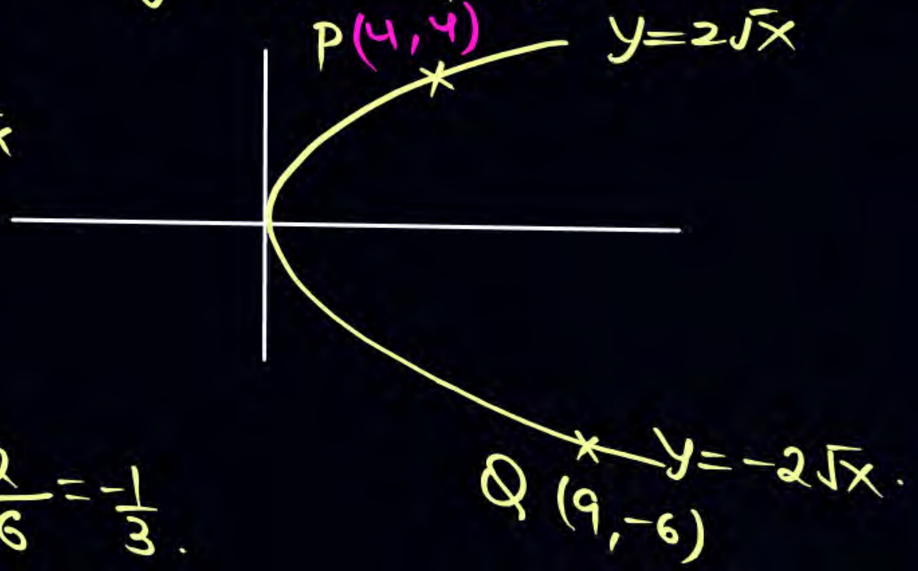
$$f'(1)=\frac{-12}{25}\ln 2.$$

$$|a^2-b^2|=|625-144|=481$$

Topic : Derivative of Implicit Function

In case of implicit functions we will differentiate each term w.r.t. 'x' regarding 'y' as a function of x.

$y^2 = 4x$ — (Implicit fn)



M① $y^2 = 4x \begin{cases} y = 2\sqrt{x} \\ y = -2\sqrt{x} \end{cases}$

AT P : $\frac{dy}{dx} = 2 \cdot \frac{1}{2\sqrt{x}}\Big|_{x=4} = \frac{1}{2}$.

AT Q $\frac{dy}{dx} = -2 \cdot \frac{1}{2\sqrt{x}}\Big|_{x=9} = -\frac{2}{6} = -\frac{1}{3}$.

M② $y^2 = 4x$

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$ $\begin{cases} \text{AT P} & \frac{dy}{dx} = \frac{2}{4} = 1/2 \\ \text{AT Q} & \frac{dy}{dx} = \frac{2}{-6} = -1/3 \end{cases}$

Implicit fn Kaa Derivative.

$f(x,y)=0 \rightsquigarrow$ Implicit fn.

① diff w.r.t x treating y as a fn of x

② collect terms containing $\frac{dy}{dx}$ together on one side while rest of the terms on another side.

③ Take $\frac{dy}{dx}$ common

④ find $\frac{dy}{dx}$.

M③ $y^2-4x=0$

$f(x,y)=y^2-4x$

$\frac{dy}{dx}=-\frac{\frac{\partial f}{\partial x}}{\partial f/\partial y}=-\frac{(-4)}{2y}=\frac{2}{y}$

Note: Derivative of Implicit fn $f(x,y)=0$ can also be found using $\frac{dy}{dx}=-\frac{\frac{\partial f}{\partial x}}{\partial f/\partial y}$

Question

#Q. If $x^y = e^{x-y}$. Then prove that $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$.

M①

$\ln x^y = \ln e^{x-y}$

$y \ln x = x - y \longrightarrow y(1+\ln x) = x$

$y = \frac{x}{1+\ln x}$

$y \cdot \frac{1}{x} + \ln x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\ln x \cdot \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{y}{x}$.

$\frac{dy}{dx}(1+\ln x) = 1 - \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{1-\frac{y}{x}}{1+\ln x} = \frac{1-\frac{x}{(1+\ln x)\cdot x}}{1+\ln x} = \frac{1+\ln x - 1}{(1+\ln x)^2} = \frac{\ln x}{(1+\ln x)^2}$

M②

$x^y = e^{x-y}$

$y \ln x = (x-y)$

$y = \frac{x}{1+\ln x}$

$\frac{dy}{dx} = \frac{(1+\ln x)\cdot 1 - x\cdot(1/x)}{(1+\ln x)^2}$

$\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

M③ $f(x,y) = x^y - e^{x-y} = 0 \rightsquigarrow x^y = e^{x-y}$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \frac{-\left(y \cdot x^{y-1} - e^{x-y}\right)}{x^y \ln x - e^{x-y}(-1)}$$

$$= \frac{-\left(y \cdot x^{y-1} e^{x-y}\right)}{x^y \ln x + e^{x-y}}$$

$$= \frac{-\left(\frac{y}{x} \cdot x^y - e^{x-y}\right)}{x^y \ln x + e^{x-y}} = \frac{-\left(\frac{x}{1+\ln x} \cdot \frac{1}{x} \cdot e^{x-y} - e^{x-y}\right)}{e^{x-y} \cdot \ln x + e^{x-y}}$$

$$= \frac{-\left(\frac{1}{1+\ln x} - 1\right)}{\ln x + 1} = \frac{\ln x}{(1+\ln x)^2}$$

$\frac{\partial}{\partial y} x^y = x^y \cdot \ln x$

$\frac{\partial}{\partial y} e^{x-y} = e^{x-y} \cdot (0-1)$

Question

#Q. If $y = \cfrac{x}{1+\cfrac{x}{2+\cfrac{x}{1+\cfrac{x}{2+}}}}\ldots\ldots\infty$. Find $\frac{dy}{dx}$.

$$y = \cfrac{x}{1+\cfrac{x}{2+\cfrac{x}{1+\cfrac{x}{2+}}}}\ldots\infty$$

repeating unit

$$y = \frac{x}{1+\frac{x}{2+y}} = \frac{x(2+y)}{2+x+y}$$

$$2y + xy + y^2 = 2x + xy$$

$$2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{1}{1+y}$$

Question

#Q. $f(x) = x + \cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+1....}}}$, find $f(100) \cdot f'(100)$

$\tan 4@$

Question

#Q. If $x\sqrt{1+y}+y\sqrt{1+x}=0$. Then prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$.

$$x\sqrt{1+y}=-y\sqrt{1+x}$$

$$x^2(1+y)=y^2(1+x)$$

$$x^2+x^2y=y^2+xy^2$$

$$(x^2-y^2)+x^2y-xy^2=0$$

$$(x-y)(x+y)+xy(x-y)=0$$

$$(x-y)(x+y+xy)=0$$

$$x+y+xy=0 \quad \text{or} \quad x-y=0$$

(reject)

$$y(1+x)=-x$$

$$y=-\frac{x}{1+x}$$

$$\frac{dy}{dx}=-\frac{1+x-x\cdot}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

If $x-y=0 \Rightarrow y=x$.

$$x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$2x\sqrt{1+x}=0$$

$$x=0,-1 \Rightarrow (0,0),(-1,-1)$$

if $y=-\frac{x}{1+x}$

$$x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$x\sqrt{1-\frac{x}{1+x}}-\frac{x}{1+x}\sqrt{1+x}=0$$

$$\frac{x}{\sqrt{1+x}}-\frac{x}{\sqrt{1+x}}=0$$

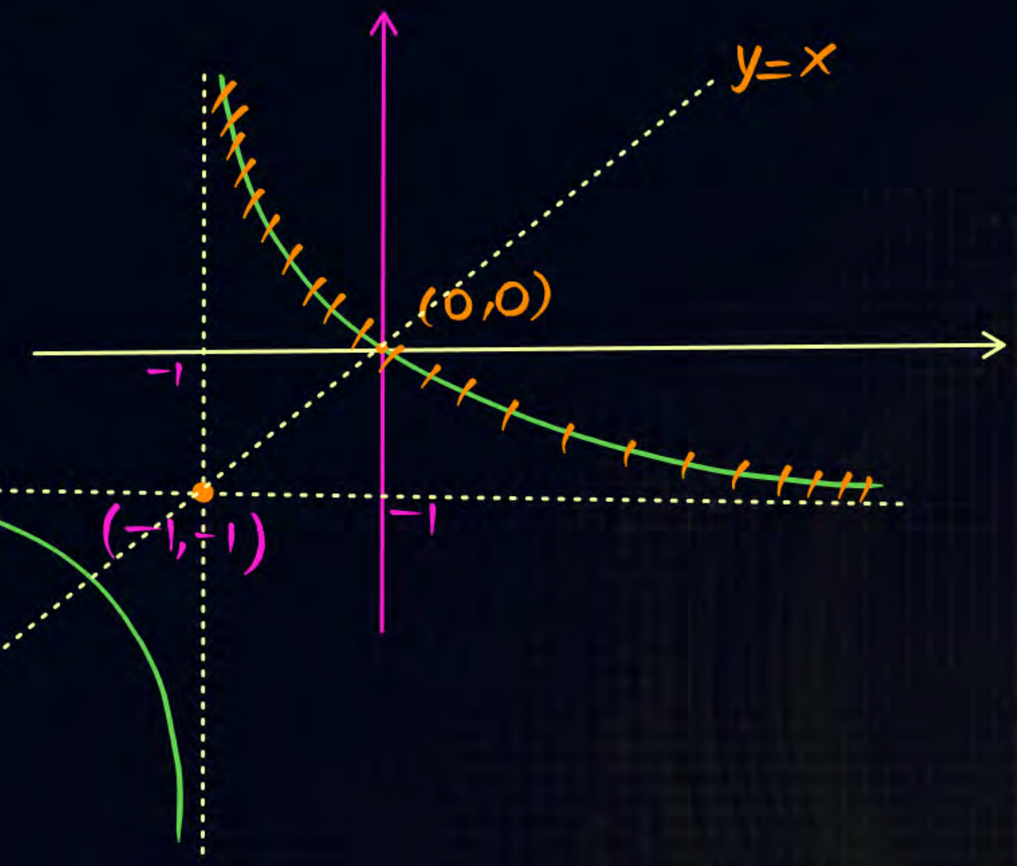
$y(1+x) = -x$

$y(1+x) + x + 1 = 0 + 1$

$(x+1)(y+1) = 1$

↓

Rectangular hyp with centre $(-1,-1)$



Question

#Q. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$. Then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

$x=\sin\theta,\ y=\sin\phi \qquad \theta,\phi\in[-\frac{\pi}{2},\pi/2]$

$\cos\theta+\cos\phi=a(\sin\theta-\sin\phi)$

$2\cos\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2}=2a\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$

$\cos\left(\frac{\theta+\phi}{2}\right)\left[\cos\frac{\theta-\phi}{2}-a\sin\frac{(\theta-\phi)}{2}\right]=0.$

$\cos\left(\frac{\theta+\phi}{2}\right)=0 \quad \text{or} \quad \cot\left(\frac{\theta-\phi}{2}\right)=a$

$\frac{\theta+\phi}{2}=(2n+1)\pi/2 \qquad \frac{\theta-\phi}{2}=n\pi+\cot^{-1}a$

$[-\pi,\pi]\curvearrowright\ \theta+\phi=(2n+1)\pi$

$\theta + \phi = \pi, -\pi$

$\theta = \pi - \phi \quad -\pi - \phi$

$\sin\theta = \sin\phi$

$y = x$

→ (reject)

$\sin^{-1}x - \sin^{-1}y = 2n\pi + 2\omega t + a$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$$

Solution to TAH

QUESTION

KCLS

2. $\lim_{n\to\infty} n \sin\left(2\pi\sqrt{1+n^2}\right)\,(n \in N)$ equals

Tan 1

Priyanka Keshri W.B

Lec 13

Q Tah 1 $\lim_{n\to\infty} n \sin(2\pi\sqrt{1+n^2}) \quad n \in N$

$\lim_{n\to\infty} n \sin\left(2\pi n \left(1+\frac{1}{n^2}\right)^{1/2}\right)$

$\lim_{n\to\infty} n \sin\left(2\pi n \left(1 + \underbrace{\frac{1}{2}\cdot\frac{1}{n^2} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{1}{n^2}\right)^2 + \cdots}\right)\right)$

$\lim_{n\to\infty} n \sin(2\pi n(1+\theta)) = \lim_{n\to\infty} n \sin(2\pi n\theta)$

$\lim_{n\to\infty} \frac{\sin 2\pi\left(\cancel{n} + \frac{n}{2}\cdot\frac{1}{n^2} + n^{\frac{1}{2}}\frac{\left(\frac{1}{2}-1\right)}{2!}\cdot\frac{1}{n^4} + \cdots\right)}{\pi/n \cdot \frac{1}{\pi}}$ ($n^{\frac{1}{2}}\frac{\left(\frac{1}{2}-1\right)}{2!}\cdot\frac{1}{n^4} \to 0$)

$\lim_{n\to\infty} = \pi$

QUESTION

TAH 1b

If $f(x) = \sin\left(\frac{5\pi}{3}[x] - x\right)$, then $f'(\pi)$ is equal to

[Note : $[\alpha]$ denotes the greatest integer less than or equal to α.]

A 0

B 1

C -1

D $1/2$

In nbd of $x=\pi$ $[x]=3$

$$f(x) = \sin\left(\frac{5\pi}{3}[x] - x\right) \text{ becomes}$$

$$f(x) = \sin\left(\frac{5\pi}{3}\cdot 3 - x\right)$$

$$f(x) = \sin(5\pi - x)$$

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$\Rightarrow f'(\pi) = -1 \text{ Ans}$$

QUESTION [JEE Mains 2023 (29 Jan)]



Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1, \forall\, x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to

Ans. 3

Take $f: R \to R$

$f(x+y) = f(x) + f(y) - 1 \quad \forall x, y \in R \quad f'(0) = 2 \quad |f(-2)| = ?$

Partially diff wrt y. → $x=0, y=0 \Rightarrow f(0) = 1$

$f'(x+y)(0+1) = 0 + f'(y)$

Put $y=0 \Rightarrow f'(x) = f'(0) = 2$

$\Rightarrow f(x) = 2x + c.$

$\Rightarrow f(0) = 0 + c$) $x=0$.

$\boxed{c = 1}$.

$\Rightarrow f(x) = 2x + 1$

$\Rightarrow |f(-2)| = |-2 \cdot 2 + 1| = 3.$

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QUESTION [JEE Mains 2024 (30 Jan)]

Tan3

Let $f : \mathbb{R} - \{0\} \to \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all $x, y, f(y) \neq 0$. If $f'(1) = 2024$, then

(A) $x f'(x) + 2024 f(x) = 0$

(B) $x f'(x) - 2023 f(x) = 0$

(C) $x f'(x) - 2024 f(x) = 0$

(D) $x f'(x) + f(x) = 2024$

Ans. C

Tah3 $f: R-\{0\} \to R$ $\quad f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ $\quad x, y, f(x) \neq 0$

$f'(1) = 2024$

↳ $x = 1 = y$

$f(1) = \frac{f(1)}{f(1)} \Rightarrow \boxed{f(1) = 1}$

f' Partially diff wrt y.

$f'\left(\frac{x}{y}\right)\left(+\frac{x}{y^2}\right) = +\frac{f(x) \cdot f'(y)}{f^2 y}$

put $y = 1$

$f'(x) \cdot x = \frac{f(x) \cdot f'(1)}{f^2(1)}$ ($f'(1) \to 2024$, $f^2(1) \to 1^2$)

$x f'(x) - 2024 f(x) = 0$

Priyanka
Keshri
W.B

Question

#Q. If $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$, find $\frac{dy}{dx}\Big]_{x=\pi/4}$

TAH 1 (A)

Q- $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$, find $\frac{dy}{dx}\Big]_{x=\pi/4}$

Sol~

Simplifying y

$$y = \frac{\sec x + \tan x - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1}$$

$$y = \frac{\sec x + \tan x - [(\sec x - \tan x)(\sec x + \tan x)]}{\tan x - \sec x + 1}$$

$$= \frac{(\sec x + \tan x)[1 - (\sec x - \tan x)]}{\tan x - \sec x + 1}$$

$$= \frac{(\sec x + \tan x)[(1 + \tan x - \sec x)]}{(1 + \tan x - \sec x)}$$

$$y = \sec x + \tan x$$

$$\left.\frac{dy}{dx}\right|_{x=\pi/4} = \sec x \tan x + \sec^2 x$$

$$= \sec x [\tan x + \sec x]$$

$$= \sec \pi/4 [\tan \pi/4 + \sec \pi/4]$$

$$= \sec \pi/4 [1 + \sqrt{2}]$$

$$= \sqrt{2}[1 + \sqrt{2}]$$

$$= \sqrt{2} + 2 \quad \text{Ans.}$$

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(8) $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$ $\quad y'|_{x=\pi/4}$

$\{\because \sec^2 x - \tan^2 x = 1\}$

$$y = \frac{\sec x + \tan x - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1}$$

$$y = \frac{\sec x + \tan x - (\sec x + \tan x)(\sec x - \tan x)}{\tan x - \sec x + 1}$$

$$y = \frac{\sec x + \tan x (1 - (\sec x - \tan x))}{(\tan x - \sec x + 1)}$$

$$y = \frac{\sec x + \tan x (1 - \sec x + \tan x)}{(\tan x - \sec x + 1)}$$

$y = \sec x + \tan x \Rightarrow y' = \sec x \tan x + \sec^2 x$

$y'|_{x=\pi/4} = \sqrt{2} \times 2 \Rightarrow \boxed{y' = 2 + \sqrt{2}}$

Kamal Sharma Gurugram

Question

#Q. For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds?

A if $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

B if $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

C if $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

D if $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

Question

#Q. If $f(x) = \sin\left(\frac{5\pi}{3}[x] - x\right)$, then $f'(\pi)$ is equal to

[Note : $[\alpha]$ denotes the greatest integer less than or equal to α.]

A 0

B 1

C -1

D 1/2

Tah-1(b)

$$f(x) = \sin\left(\frac{5\pi}{3}[x] - x\right)$$

In nbd of x

$$f(x) = \sin\left(\frac{5\pi}{3}[3.1\pi] - x\right)$$

$$f(x) = \sin(5\pi - x)$$

$$f'(x)\Big|_{x=\pi} = \cos(5\pi - x)(-1)$$

$$f'(x) = \cos(5\pi - \pi)(-1)$$

$$= \cos(4\pi)(-1)$$

$$\boxed{f'(\pi) = -1}$$ Ans

Komal Sharma Gurugram

Question

#Q. Let $F(x) = f(x) \cdot g(x) \cdot g(x) \cdot h(x)$. If for some $x = x_0$, $F'(x_0) = 21\, F(x_0)$; $f'(x_0) = 4f(x_0)$, $g'(x_0) = -7 \cdot g(x_0)$ and $h'(x_0) = k \cdot h(x_0)$. Then find the value of k.

TAH - 1(c)

(Q) Let $F(x) = f(x)\,g(x)\,h(x)$

Sol $F(x) = f(x)\,g(x)\,h(x)$

$F'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

Replace $x = x_0$

$F'(x_0) = f'(x_0)g(x_0)h(x_0) + f(x_0)g'(x_0)h(x_0) + f(x_0)g(x_0)h'(x_0)$

$\alpha_1 F(x_0) = 1 f(x_0)g(x_0)h(x_0) - 7 f(x_0)g'(x_0)h(x_0) + K f(x_0)g(x_0)h(x_0)$

$\alpha_1 f(x_0) = (1 - 7 + K) f(x_0)g(x_0)h(x_0)$

Replace $x_0 = x$

$\alpha_1 f(x) = (-3 + K) f(x)g(x)h(x)$

$\alpha_1 f(x)g(x)h(x) = (-3+K) f(x)g(x)h(x)$

$\alpha_1 = -3 + K$

$\boxed{K = 24}$

Kamal Sharma Gurugram

Question

Paragraph

#Q. People living at Mars define the derivative of $f(x)$ in an unusual manner as $D * f(x) = \lim_{h \to 0} \frac{f^2(x+h) - f^2(x)}{h}$ where $D * f(x)$ denotes the derivative of $f(x)$ at Mars. Now answer the following.

If $u = f(x), v = g(x)$ then the value of $D * \left(\frac{u}{v}\right)$ is

A $\frac{u^2 D * v - v^2 D * u}{v^4}$

B $\frac{uD * v - vD * u}{v^2}$

C $\frac{v^2 D * u - u^2 D * v}{v^4}$

D $\frac{vD * u - uD * v}{v^2}$

TAH-2(a)

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(6) If $u = f(x)$, $v = g(x)$ then the value of $D^*\left(\frac{u}{v}\right)$ is

$$D^* f(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h}\right)\left(f(x+h) + f(x)\right)$$

$$D^*(f(x)) = 2f(x)f'(x)$$

$$D^*\left(\frac{u}{v}\right) = 2\left(\frac{u}{v}\right)\left(\frac{u}{v}\right)' = 2\frac{u}{v}\left(\frac{u'v - v'u}{v^2}\right)$$

$$\frac{2u'uv - 2u^2v'}{v^3}$$

$$\frac{2u'uv^2}{v^4} - \frac{2u^2vv'}{v^4}$$

$$\frac{v^2D^*u - u^2D^*v}{v^4}$$ Ans

$\because u'u = D^*u$

$\because vv' = D^*v$

Question

#Q. Let $h(x) = f(g(\sin 4x))$, find $h'\left(\frac{\pi}{24}\right)$. If $g\left(\frac{1}{2}\right) = 2; f'(2) = 4, g'\left(\frac{1}{2}\right) = \sqrt{3}$.

Q $h(x) = f(g(\sin 4x))$, $h'\left(\frac{\pi}{24}\right) = ?$ $g\left(\frac{1}{2}\right) = 2$; $f'(2) = 4$

$g'\left(\frac{1}{2}\right) = \sqrt{3}$

Sol $h'(x) = f'(g(\sin 4x))\, g'(\sin 4x) \cos 4x \cdot 4$

$h'\left(\frac{\pi}{24}\right) = f'\left(g\left(\frac{1}{2}\right)\right) g'\left(\frac{1}{2}\right) \frac{\sqrt{3}}{2} \cdot 4$

$h'\left(\frac{\pi}{24}\right) = f'(2)\, \sqrt{3}\, \sqrt{3} \cdot 2$

TAH-3(a) $h'\left(\frac{\pi}{24}\right) = 4 \cdot 2 \cdot 3 = 24$ Ans

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MATH 3(Q)

$h(x) = f(g(\sin 9x))$ find $h'\left(\frac{\pi}{24}\right)$

$g\left(\frac{1}{2}\right) = 2$

$f'(2) = 9$

$g'(1/2) = \sqrt{3}$

$h(x) = f(g(\sin 9x))$

$h'(x) = f'(g(\sin 9x)) \cdot g'(\sin 9x) \cdot \cos 9x \cdot 9$

$h'(\pi/20) = f'\left(g\left(\sin\frac{\pi}{6}\right)\right) \cdot g'\left(\sin\frac{\pi}{6}\right) \cdot \cos\frac{\pi}{6} \cdot 9$

$= f'\left(g\left(\frac{1}{2}\right)\right) \cdot g'\left(\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} \times 9 \times 2$

$= f'(2) \cdot 9\sqrt{3} \cdot \sqrt{3} \cdot 2$

$= 4 \times 3 \times 2$

$= \underline{\underline{24}}$

Question

#Q. Suppose f and g are derivable functions such that $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$. find $g'(x)$.

TAH-3(b)

$F(g(x)) = x$ & $f'(x) = 1 + [f(x)]^2$ $g'(x) = ?$

$F'(g(x))\, g'(x) = 1$

$g'(x) = \dfrac{1}{F'(g(x))}$ $\quad \therefore F(g(x)) = 1$

$g'(x) = \dfrac{1}{1 + (f(g(x)))^2}$

$\boxed{g'(x) = \dfrac{1}{1 + (x)^2}}$ Ans

Kamal Sharma Gangaran

TAH 8(b)

$f(g(x)) = x$ & $f'(x) = 1 + [f(x)]^2$ find $g'(x)$

differentiate

$f'(g(x)) \cdot g'(x) = 1$ ... (1)

we know $f'(x) = 1 + [f(x)]^2$

$f'(g(x)) = 1 + [f(g(x))]^2$

$= \underline{\underline{1 + x^2}}$

from (1) $\rightarrow$ $1 + x^2 \cdot g'(x) = 1$

$g'(x) = \frac{1}{1+x^2}$

Ved joshi
Gujarat

Question

#Q. Let $f:(-2,2) \to R$ be a differentiable function such that $f(0) = -1$ and $f'(0) = 1$. If $g(x) = (f(2f(x) + 2))^2$ then $g'(0)$ is equal to

A -4

B 0

C -2

D 4

Q- $f:(-2,2) \to R$ (differentiable fno)
such that $f(0) = -1$ $f'(0) \neq 1$

$g(x) = (f(2f(x)+2))^2$ $g'(0) = ?$

Ans-

Soln:- $g'(x) = 2(f(2f(x)+2)) \times f'(2f(x)+2) \times 2f'(x)$

put $x = 0$

$g'(0) = 2(f(2f(0)+2) \times f'(2f(0)+2) * 2f'(0)$

$g'(0) = 2(f(2x-1+2) \times f'(2x-1+2) * 2f'(x)$

$g'(0) = 2(f(0)) \times f'(0) * 2$

$g'(0) = -2 \times 1 \times 2$

$= -4$ Ans

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TAH3(C)

TAH-3(C)

$g(x) = (f(2f(x)+2))^2$ $g'(0) = ?$

$g'(x) = 2(f(2f(x)+2)) \cdot f'(2f(x)+2)$

$f'(x)$

$g'(0) = 2(f(2f(0)+2)) f'(2f(0)+2)$

$2f'(0)$

$\because f(0) = -1, f'(0) = 1$

$g'(0) = 2(f(-2+2)) f'(-2+2) \cdot 2$

$g'(0) = -2 \cdot 1 \cdot 2$

$\boxed{g'(0) = -4}$

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THH 8©

$f:(-2,2) \to R$ be diff. fn s.t $f(0)=-1$ $f'(0)=1$

$g(x)=(f(2f(x)+2))^2$ $g'(0)$ is

$$g'(x) = 2(f(2f(x)+2)) \cdot f'(2f(x)+2) \cdot (2f'(x)+0)$$

$$g'(0) = 2(f(2f(0)+2)) \cdot f'(2f(0)+2) \cdot 2f'(0)$$

$$= 2(f(-2+2)) \cdot f'(-2+2) \cdot 2 \times 1$$

$$= 2f(0) \cdot f'(0) \cdot 2$$

$$= 2 \times -1 \times 1 \times 2$$

$$= \underline{\underline{-4}}$$

Ved joshi.
Gujarat

Question

#Q. Let $g(x) = x^3 \ln\left(x^2 f(x)\right)$, where $f(x)$ is a differentiable positive function on $(0, \infty)$ satisfying $f(2) = \frac{1}{4}$ and $f'(2) = -3$, then $g'(2)$ equals

A 77

B - 88

C 88

D - 77

TAH 3(a)

$g(x) = x^3 \ln(x^2 f(x))$ where $f(x)$ is differentiable +ve f^n on $(0,\infty)$, $f(2) = 1/4$ $f'(2) = -3$ $g'(2)$

$g(x) = x^3 \ln(x^2 f(x))$

$g'(x) = 3x^2 \cdot \ln(x^2 (f(x)) + x^3 \cdot \frac{1}{x^2 \cdot f(x)} \cdot [2x \cdot f(x) + x^2 \cdot f'(x)]$

$g'(2) = 3\times 4 \cdot \ln(4 \cdot f(2)) + \frac{2^3}{2^2 \cdot f(2)} [2\times 2 \cdot f(2) + 4 \cdot f'(2)]$

$= 12 \cdot \ln 1 + 2\times 4 [4 \times \frac{1}{4} + 4(-3)]$

$= 0 + 8[1 - 12]$

$= -88$ (B)

Solution to KTK

Question KTK 1

#Q. Let $f(x)$ be a differentiable function on R such that $2f(x+y)+f(x-y)=3f(x)+3f(y)+2xy$ for all $x, y \in R$. If $f'(0)=0$ then find the value of $f(3)$.

KTK-1

$2f(x+y)+f(x-y)=3f(x)+3f(y)+2xy$ —①

Put $x=y=0$

$2f(0)+f(0)=3f(0)+3f(0)+0$

$f(0)=0$

partial differentiate eqn ① w.r.t y

$2f'(x+y)\times(0+1)+f'(x-y)\times(0-1)=0+3f'(y)+2x$

$2f'(x+y)-f'(x-y)=3f'(y)+2x$

Put $y=0$

$2f'(x)-f'(x)=3f'(0)+2x$

$f'(x)=2x$

$\int f'(x)\,dx=\int 2x\,dx \Rightarrow f(x)=x^2+c$

Put $x=0 \Rightarrow f(0)=0+c \Rightarrow \boxed{c=0}$

so, $f(x)=x^2$

hence, $f(3)=9$ Ans.

Good Write

ASHISH FROM DELHI

KTK (5)

Shashi Bhai

KTK 1 :- $f(x)$ is differentiable for all $x\in R$ s.t. $2f(x+y)+f(x-y)=3f(x)+3f(y)+2xy$

for all $x,y\in R$; $f'(0)=0$

find $f(3)$

$2f(x+y)+f(x-y)=3f(x)+3f(y)+2xy$

differentiating Both sides w.r.t. y

$2f'(x+y)-f'(x-y)=3f'(y)+2x$

putting $y=0$

$2f'(x)-f'(x)=3f'(0)+2x$

$f'(0)=0$

$f'(x)=2x$

$\frac{d(f(x))}{dx}=2x$

$\int d(f(x))=\int 2x\,dx$

$f(x)=x^2+C$

By functional equation given above

$f(0)=0$

putting in f^n; $f(0)=(0)^2+C \Rightarrow C=0$

$\therefore f(x)=x^2$

$f(3)=3^2 \Rightarrow 9$

Question KTK2

#Q. Let f be a differentiable function satisfying the functional rule $f(xy) = f(x) + f(y) + \frac{x+y-1}{xy} \forall x, y > 0$ and $f'(1) = 2$. Find the value of $[f(e^{100})]$.

Note : $[k]$ denotes the greatest integer less than or equal to k.

KTK-2

#Q Let f be a differentiable function such that $f(xy) = f(x) + f(y) + \frac{x+y-1}{xy}$ $\forall x, y > 0$ and $f'(1) = 2$

Find the value of $[f(e^{100})]$

$f(xy) = f(x) + f(y) + \frac{x+y-1}{xy}$

Partial differentiation w.r.t y

$f(xy)x = 0 + f'(y)\frac{-1}{y^2} + 0 + \frac{1}{xy^2}$

Put $y = 1$

$[f'(x)x = 0 + f'(1) - \frac{1}{1} + 0 + \frac{1}{x}]$

$[x f'(x) = 2 + \frac{1}{x} - 2]$

$f'(x) = \frac{1}{x} + \frac{1}{x^2}$

$\int f'(x)dx = \left[\int \frac{1}{x}dx + \int \frac{1}{x^2}dx\right]$

$f(x) = \left[\ln x + \frac{x^{-2+1}}{-2+1} + C\right]$

$f(x) = [\ln x - \frac{1}{x} + C]$ —①

Nitarika Delhi

from que
$f(1) = 2f(1) + 2$
$-f(1) = 2$
$f(1) = -1$
from ① $x = 1$
$-1 = 0 - 1 + C$
$C = 0$

$f(x) = \ln x - \frac{1}{x}$

now $f(e^{100}) = (\ln e^{100} - e^{-100})$

$f[e^{100}] = [100 - \frac{1}{e^{100}}]$ [99+]

Large no

$[f(e^{100}) = 100]$ Ans

$f(e^{100}) = [99+] = 99$ Ans

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KTK2

KTK(2)

Q- $f(xy) = f(x) + f(y) + \frac{x+y-1}{xy}$ $\forall\ x, y > 0$ and $f'(1) = 1$

$[f(e^{100})]$

diff wrt y

Soln. $f'(xy) \cdot x = f'(y) + \frac{xy(1-0) - x(x+y-1)}{x^2y^2}$

Putting $y = 1$

$f'(x)\,x = f'(1) + \frac{x - x^2}{x^2}$

$f'(x) = \frac{f'(1)}{x} + \frac{1-x}{x^2}$

$f'(x) = \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x}$

$\int f'(x) = \int \left(\frac{1}{x^2}\right)$

$f(x) = \ln x + \frac{x^{-1}}{-1} + c$

$f(x) = \ln x - \frac{1}{x}$

Putting $x = e^{100}$

$[f(e^{100})] = \left[\ln(e^{100}) - \frac{1}{e^{100}}\right]$

$[f(e^{100})] = \left[100 - \frac{1}{e^{100}}\right]$

$= 99$ Ans

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Question

KTK 3

#Q. Let a_1 and a_2 be two values of a for which

$$f(x) = \begin{cases} x \cdot \dfrac{\ln(1+x) + \ln(1-x)}{\sec x - \cos x}, & x \in (-1, 0) \\ (a^2 - 3a + 1)x + x^2. & x \in [0, \infty) \end{cases}$$

is differentiable at $x = 0$, then find the value of $(a_1^2 + a_2^2)$.

KTH-1

Since $f(x)$ is differentiable at $x=0$

RHD : $\lim_{h\to 0} \frac{(a^2-3a+1)h+h^2}{h} = \lim_{h\to 0} (a^2-3a+1)+h$

$= a^2-3a+1$

LHD : $\lim_{h\to 0} -h\left(\frac{\ln(1-h)+\ln(1+h)}{\sin(-h)-\cos(-h)}\right)$

$\lim_{h\to 0} -h\left(\frac{\left[-h-\frac{h^2}{2}-\frac{h^3}{3}-\cdots\right]+\left[h-\frac{h^2}{2}+\frac{h^3}{3}-\cdots\right]}{\frac{\sin^2(-h)}{\cos(-h)}-h}\right)$

$\lim_{h\to 0} \cos(-h)\left(\frac{\left[-\frac{h^2}{2}-\frac{h^3}{3}-\cdots\right]+\left[-\frac{h^2}{2}+\frac{h^3}{3}-\cdots\right]}{\left(\frac{\sin^2(-h)}{(-h)^2}\right)\times h^2}\right)$

$\lim_{h\to 0} \cos(-h)\left(\frac{\left[-\frac{1}{2}-\frac{h}{3}-\cdots\right]+\left[-\frac{1}{2}+\frac{h}{3}-\cdots\right]}{(\quad)}\right)$

$\Rightarrow -\frac{1}{2}+\frac{-1}{2} = -1$

So, LHD = RHD $\Rightarrow a^2-3a+1 = -1$

$\Rightarrow a^2-3a+2=0$

$\Rightarrow a=1,2$

Hence $a_1^2+a_2^2 = 1^2+2^2$

$= 5$ Ans.

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Question

KTK 4

#Q. If the function $f(x)$ defined as $f(x) = \begin{cases} -\frac{x^2}{2} \text{ for } x \leq 0 \\ x^n \sin\frac{1}{x} \text{ for } x > 0 \end{cases}$ is continuous but not derivable at $x = 0$, then find the range of n.

KTK-6

$$f(x)=\begin{cases}\frac{-x^2}{2}, & x\le 0\\ x^n\sin\frac{1}{x}, & x>0\end{cases}$$

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At $x=0$

LHL: $\lim_{h\to 0}\frac{-h^2}{2}=0$

RHL: $\lim_{h\to 0} h^h\left(\sin\frac{1}{h}\right)$ — $\sin\frac{1}{h}$ is bounded

h^h: $x>0$, RHL $=0$

$x=0$, RHL $=$ ONE

$x<0$, RHL $=$ ONE

At $x=0$

RHD: $\lim_{h\to 0}\frac{h^n\sin\frac{1}{h}-0}{h}=\lim_{h\to 0} h^{n-1}\sin\frac{1}{h}$

$n-1>0 \Rightarrow$ RHD $=0$

$n-1\le 0 \Rightarrow$ RHD $=$ ONE

LHD: $\lim_{h\to 0}\frac{\frac{-h^2}{2}}{-h}=0$

Now, for continuity and non differentiability at $x=0$

$n>0$ and $n-1\le 0$

$\Rightarrow \boxed{n\in(0,1]}$ $= n\le 1$

n

QUESTION

KTK 5

If $\lim_{x\to 0^+} f(x) = p$ and $\lim_{x\to 0^-} f(x) = q$ then the correct statement(s) is/are true?

(A) $\lim_{x\to 0^+} f(x^3 - x) = q$

(B) $\lim_{x\to 0^-} f(x^3 - x) = p$

(C) $\lim_{x\to 0^+} f(x^2 - x^4) = q$

(D) $\lim_{x\to 0^-} f(x^2 - x^4) = p$

Ans. A, B, D

KTKS $\lim_{x\to 0^+} f(x) = p$ & $\lim_{x\to 0^-} f(x) = q$ then which of the statement(s) is/are true?

(a) $\lim_{x\to 0^+} f(x^3 - x) = q$ $\hookrightarrow \lim_{h\to 0} f(h^3 - h) = q$. (chota badaa; $h^3 - h$: -ve)

(b) $\lim_{x\to 0^-} f(x^3 - x) = p$ $\hookrightarrow \lim_{h\to 0} f(-h^3 + h) = p$ (chhota badaa; $-h^3 + h$: +ve)

(c) $\lim_{x\to 0^+} f(x^2 - x^4) = q$ $\hookrightarrow \lim_{h\to 0} f(h^2 - h^4) = p$ (baada chhota; $h^2 - h^4$: +ve)

(d) $\lim_{x\to 0^-} f(x^2 - x^4) = p$ $\hookrightarrow \lim_{h\to 0} f(h^2 - h^4) = p$ ($h^2 - h^4$: +ve)

Yaad Rakho $h^4 < h^3 < h^2 < h$

Priyanka Keshri W.B

(Solution to RPP)

QUESTION

(RPP 1)

The value of $\frac{\cos 24^\circ}{2 \tan 33^\circ \sin^2 57^\circ} + \frac{\sin 162^\circ}{\sin 18^\circ - \cos 18^\circ \tan 9^\circ} + \cos 162^\circ$ is equal to:

Ans. 2

RPP1 $\dfrac{\cos 24^\circ}{2\tan 33^\circ \sin^2 57^\circ} + \dfrac{\sin 162^\circ}{\sin 18^\circ - \cos 18^\circ \tan 9^\circ} + \cos 162^\circ = ?$

$\parallel$

$\dfrac{\cos 24^\circ}{2\dfrac{\sin 33}{\cos 33}\cdot \sin(\cos 33^\circ)}$

$\parallel$

$\dfrac{\cos 24^\circ}{2\sin 33 \cos 33} = \dfrac{\cos 24}{\sin 66} = 1$

$\parallel$

$\dfrac{\sin(180-18)}{\dfrac{\sin 18\cdot\cos 9 - \cos 18\cdot\sin 9}{\cos 9}}$

$\parallel$

$\dfrac{\sin 18\cdot\cos 9}{\sin(18-9)}$

$\parallel$

$\dfrac{2\sin 9\cdot\cos 9\cdot\cos 9}{\sin 9}$

$\parallel$

$2\cos^2 9 = 1+\cos 18$

$\parallel$

$\cos(180-18)$

$\parallel$

$-\cos 18$

Priyanka Keshri W.B

Ans: $1+1+\cos 18 - \cos 18 = 2$

QUESTION

(RPP 2)

If $\cos 20^\circ + 2\sin^2 55^\circ = 1 + \sqrt{2}\sin K^\circ, K \in (0, 90)$, then $K =$

Ans. 65

RPP2 $\cos 20^\circ + 2\sin^2 55^\circ = 1 + \sqrt{2}\sin k^\circ$, $k \in (0, 90)$, $k = ?$

$\cos 20 + 1 - \cos 110^\circ = 1 + \sqrt{2}\sin k^\circ$

$1 + \cos 20 + 1 - \cos 110^\circ = 1 + \sqrt{2}\sin k^\circ$

$1 + 2\sin\frac{130}{2} \cdot \sin\frac{90}{2} = 1 + \sqrt{2}\sin k^\circ$

$1 + \sqrt{2}\sin 65^\circ = 1 + \sqrt{2}\sin k^\circ \Rightarrow \boxed{k = 65^\circ}$

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QUESTION

(RPP 3)

Find the value of the expression $\frac{\sin 20^\circ (4 \cos 20^\circ + 1)}{\cos 20^\circ \cos 30^\circ}$.

Ans. 2

RPP 3 $\dfrac{\sin 20^\circ (4\cos 20^\circ + 1)}{\cos 20^\circ \cdot \cos 30^\circ} = \dfrac{4\sin 20 \cdot \cos 20 + \sin 20}{\frac{\sqrt{3}}{2} \cdot \cos 20}$

$\dfrac{2}{\sqrt{3}} \cdot \dfrac{\sin 40 + (\sin 40 + \sin 20)}{\cos 20} = \dfrac{2}{\sqrt{3}} \dfrac{\sin 40 + 2 \cdot \sin 30 \cdot \cos 10}{\cos 20}$

$= \dfrac{2}{\sqrt{3}} \dfrac{\sin 40 + \sin 80}{\cos 20} = \dfrac{2}{\sqrt{3}} \cdot 2\sin 60 \cdot \dfrac{\cos 20}{\cos 20} = 2$

Priyanka
Keshri
W.B

QUESTION

(RPP 4)

Find the value of $\tan\theta(1+\sec 2\theta)(1+\sec 4\theta)(1+\sec 8\theta)$, when $\theta = \frac{\pi}{32}$.

Ans. 1

RPPY $\tan\theta(1+\sec 2\theta)\cdot(1+\sec 4\theta)\cdot(1+\sec 8\theta)\Big|_{\theta=\frac{\pi}{32}} = ?$

$$= \frac{\sin\theta}{\cos\theta}\cdot\frac{(1+\cos 2\theta)}{\cos 2\theta}\cdot\frac{(1+\cos 4\theta)}{\cos 4\theta}\cdot\frac{(1+\cos 8\theta)}{\cos 8\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\cdot\frac{2\cos^2\theta}{\cos 2\theta}\cdot\frac{2\cos^2 2\theta}{\cos 4\theta}\cdot\frac{2\cos^2 4\theta}{\cos 8\theta}$$

$$= 8\cdot\sin\theta\cdot\cos\theta\cdot\cos 2\theta\cdot\cos 4\theta\cdot\frac{1}{\cos 8\theta}$$

$$= \frac{8\sin\theta\cdot\sin 2^3\theta}{2^3\sin\theta\cdot\cos 8\theta} = \tan 8\theta\Big|_{\theta=\frac{\pi}{32}} = \tan\frac{\pi}{4} = 1.$$

Priyanka Keshri W.B


THANK
YOU

PRAYAS

JEE 2025

Lecture-15 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Successive Differentiation
2. Derivative of Inverse Function
3. Derivative of Determinants
4.

Topic : Successive Differentiation

Let a function $y = f(x)$ be defined on an interval (a, b). If f(x) is differentiable function, then its derivative f(x) [or (dy/dx) or y'] is called the first derivative of y w.r.t. x. If f'(x) is again differentiable function on (a, b), then its derivative f"(x) [or d^2y/dx^2 or y"] is called second derivative of y w.r.t. x. Similarly, the 3rd order derivative of y w.r.t. to x, if it exists, is defined by $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ and denoted by f"'(x) or y"' and so on.

$Dy = f'(x) = y' = y_1 = \frac{dy}{dx}$

$D^2y = f''(x) = y'' = y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$D^3y = f'''(x) = y''' = y_3 = \frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$

$D^4y = f^{(iv)}(x) = y^{(iv)} = y_4 = \frac{d^4y}{dx^4} = \frac{d}{dx}\left(\frac{d^3y}{dx^3}\right)$

$D^ny = f^{(n)}(x) = y^{(n)} = y_n = \frac{d^ny}{dx^n} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$

Question

#Q. If $y = \sin(\sin x)$ then prove that $y_2 + (\tan x)y_1 + y\cos^2 x = 0$.

$$y_1 = \cos(\sin x)\cdot\cos x$$

$$y_2 = -\sin(\sin x)\cdot\cos^2 x - \cos(\sin x)\sin x$$

$$y_2 = -y\cos^2 x - \frac{y_1}{\cos x}\cdot\sin x.$$

$$y_2 + y_1\tan x + y\cos^2 x = 0$$

Question

#Q. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then : [JEE Mains 2022]

A $x^2y'' + xy' - 25y = 0$

B $x^2y'' - xy' - 25y = 0$

C $x^2y'' - xy' + 25y = 0$

D $x^2y'' + xy' + 25y = 0$

$\frac{y}{2} = \cos\left(5\ln\frac{x}{5}\right)$

$y = 2\cos\left(5\ln\frac{x}{5}\right)$

$y' = -2\sin\left(5\ln\frac{x}{5}\right) \cdot 5 \cdot \left(\frac{1}{x/5}\right) \cdot \frac{1}{5}$

$y' = -10\sin\left(5\ln\frac{x}{5}\right) \cdot \frac{1}{x}$

$xy' = -10 \cdot \sin\left(5\ln(x/5)\right)$

Diff w.r.t x

$xy'' + y' = -10\cos\left(5\ln x/5\right) \cdot 5\frac{1}{x/5} \cdot \frac{1}{5}$

$xy'' + y' = -10 \cdot \frac{y}{2} \cdot \frac{5}{x/5}$

$$x^2y'' + xy' = -25y$$

$$x^2y'' + xy' + 25y = 0$$

Question

#Q. If $y = \tan^{-1}\left(\sec x^3 - \tan x^3\right), \frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then [JEE Mains 2022]

A $xy'' + 2y' = 0$

B $x^2y'' - 6y + \frac{3\pi}{2} = 0$

C $x^2y'' - 6y + 3\pi = 0$

D $xy'' - 4y' = 0$

$x^3 = t$

$y = \tan^{-1}(\sec t - \tan t)$

$= \tan^{-1}\left(\frac{1-\sin t}{\cos t}\right)$

$= \tan^{-1}\left(\frac{\cos^2\frac{t}{2} + \sin^2\frac{t}{2} - 2\sin t/2 \cos t/2}{(\cos t/2 - \sin t/2)(\cos t/2 + \sin t/2)}\right)$

$= \tan^{-1}\left(\frac{\cos t/2 - \sin t/2}{\cos t/2 + \sin t/2}\right)$

$= \tan^{-1}\left(\frac{1-\tan t/2}{1+\tan t/2}\right) = \tan^{-1}\left(\tan(\pi/4 - t/2)\right)$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right)$$

$$y = \pi/4 - \frac{x^3}{2}$$

$$y' = \frac{dy}{dx} = -\frac{3x^2}{2}$$

$$y'' = -3x$$

$$\Downarrow$$

$$x^2y'' = -3x^3$$

$$\frac{x^3}{2} \in (\pi/4, \frac{3\pi}{4})$$

$$-\frac{x^3}{2} \in (-3\pi/4, -\pi/4)$$

$$\pi/4 - \frac{x^3}{2} \in (-\pi/2, 0)$$

$$x^2y'' = -3 \cdot 2(\pi/4 - y)$$

$$x^2y'' = -\frac{3\pi}{2} + 6y$$

$$x^2y'' - 6y + 3\pi/2 = 0$$

Question

#Q. If $y^{1/4} + y^{-1/4} = 2x$ and $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$ then $|\alpha - \beta|$ is equal to

[JEE Mains 2021 (August)]

$y^{\frac{1}{4}} + y^{-\frac{1}{4}} = 2x$

$\frac{1}{4}y^{-\frac{3}{4}}\frac{dy}{dx} - \frac{1}{4}y^{-5/4}\frac{dy}{dx} = 2$

$(y^{-3/4} - y^{-5/4})\frac{dy}{dx} = 8$

$\left(\frac{y^{1/4}}{y} - \frac{y^{-1/4}}{y}\right)\frac{dy}{dx} = 8$

$(y^{1/4} - y^{-1/4})y_1 = 8y$

clearly $y_1 \neq 0$

$(y^{1/4} - y^{-1/4})^2 y_1^2 = 64y^2$

$\left((y^{\frac{1}{4}} + y^{-\frac{1}{4}})^2 - 4y^{\frac{1}{4}}\cdot y^{-\frac{1}{4}}\right)y_1^2$

$\| \quad 64y^2$

$(4x^2 - 4)\,y_1^2 = 64y^2$

$4(x^2-1)y_1^2 = 64y^2$

$(x^2-1)y_1^2 = 16y^2$

$(x^2-1)\cdot 2y_1y_2 + 2xy_1^2 = 32yy_1$

$(x^2-1)\,y_2 + xy_1 = 16y$

$(x^2-1)y_2 + xy_1 - 16y = 0$

$\alpha = 1, \beta = -16.$

$|\alpha - \beta| = 17$

Question

#Q. If $x = 2\cos t - \cos 2t$; $y = 2\sin t - \sin 2t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

$\frac{d^2y}{dx^2} \neq \frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$ — Yaadho/Yaadhiyoo aisaa naa karo!!

$$\frac{dy}{dx} = \frac{2\cos t - 2\cos 2t}{-2\sin t + 2\sin 2t} = \frac{\cos t - \cos 2t}{\sin 2t - \sin t} = \frac{2\sin\frac{3t}{2}\sin t/2}{2\sin t/2 \cdot \cos\frac{3t}{2}}$$

$$\frac{dy}{dx} = \tan\frac{3t}{2}$$

M① $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left(\tan 3t/2\right) = \sec^2 3t/2 \cdot \frac{3}{2}\frac{dt}{dx}$ (Nahi Bhulnaa!!)

M② $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\tan\frac{3t}{2}\right)$

$= \frac{d}{dt}\left(\tan\frac{3t}{2}\right) \cdot \frac{dt}{dx}$

$= \frac{3}{2}\sec^2 3t/2 \; dt/dx$

$$\frac{d^2y}{dx^2} = \frac{3}{2}\sec^2\frac{3t}{2}\cdot\frac{dt}{dx}$$

$$= \frac{3}{2}\sec^2\frac{3t}{2}\cdot\frac{1}{\frac{dx}{dt}}$$

$$= \frac{3}{2}\sec^2\frac{3t}{2}\cdot\frac{1}{2\sin 2t - 2\sin t}\Bigg|_{t=\pi/2}$$

$$= \frac{3}{2}\sec^2\frac{3\pi}{4}\cdot\frac{1}{2\sin\pi - 2\sin\pi/2}$$

$$= \frac{3}{2}\cdot 2\cdot\frac{1}{-2} = \underline{-3/2}$$

#Q. Let $g(x) = \log f(x)$, where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$.

Then for $N = 1, 2, 3, \ldots, g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$ [IIT-JEE 2008 (Paper 2)]

A $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$

B $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$

C $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N+1)^2}\right\}$

D $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N+1)^2}\right\}$

$g(x) = \ln f(x)$, $f(x+1) = x f(x)$

$g(x+1) = \ln(f(x+1)) = \ln(x f(x)) = \ln x + \underbrace{\ln f(x)}_{g(x)}$

$g(x+1) = \ln x + g(x)$

$g(x+1) - g(x) = \ln x$

$g''(x+1) - g''(x) = \frac{d}{dx}(1/x) = -1/x^2$

$g''(x+1) - g''(x) = \frac{-1}{x^2}$

$x \to x - 1/2$ $\quad g''(x+1/2) - g''(x-1/2) = \frac{-1}{(x-1/2)^2} = \frac{-4}{(2x-1)^2}$

$$g''\left(x+\tfrac{1}{2}\right)-g''\left(x-\tfrac{1}{2}\right)=\frac{-4}{(2x-1)^2}$$

$x=1$ $\quad g''(3/2)-g''(1/2)=-\frac{4}{1^2}$

$x=2$ $\quad g''(5/2)-g''(3/2)=-\frac{4}{3^2}$

$x=3$ $\quad g''(7/2)-g''(5/2)=-\frac{4}{5^2}$

$x=N$ $\quad g''(N+1/2)-g''(N-1/2)=-\frac{4}{(2N-1)^2}$

$$g''(N+1/2)-g''(1/2)=-4\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\frac{1}{(2N-1)^2}\right)$$

Question

#Q. Let $f: S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g: S \to R$ be defined as $g(x) = \log_4 f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to: [JEE Mains 2021 (March)]

TAHI@

A $\frac{205}{144}$

B $\frac{197}{144}$

C $\frac{187}{144}$

D 1

Question

[JEE Mains 2023 (1 Feb)]

#Q. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + x\,f'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to _____

$f(x) = x^2 + ax + b$

where $a = g'(1)$

$b = g''(2)$

$f'(x) = 2x + a$

$f''(x) = 2$

$f(1) = 1 + a + b$

$g(x) = (1+a+b)x^2 + x(2x+a) + 2$

$\downarrow$ Diff w.r.t x

$g'(x) = 2x(1+a+b) + 4x + a$

$g''(x) = 2(1+a+b) + 4$

$a = g'(1) = 2(1+a+b) + 4 + a = 3a + 2b + 6 \Rightarrow 2a + 2b + 6 = 0$

$a + b + 3 = 0$ — (I)

$b = g''(2) = 2a + 2b + 6 \Rightarrow 2a + b + 6 = 0$ — (II)

$a+b+3=0$

$2a+b+6=0$

$a=-3$

$b=0$

$f(x)=x^2-3x$

$g(x)=-2x^2+x(2x-3)+2$

$g(x)=-3x+2$

$f(1)-g(4)=4+10=14.$

$$2a+2b+4=0$$
$$2a+b+4=0$$

$$b=0$$
$$a=-2$$

$f(x)=x^2-2x$

$g(x)=$

Question

#Q. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f(2)$ equals : [JEE Mains 2019]

Tah ! (b)

A -4

B 30

C -2

D 8

Question

[JEE Mains 2020 (5 Sept)]

#Q. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is :

Tan (2)

A $\frac{2\sqrt{3}}{3}$

B $\frac{2\sqrt{3}}{5}$

C $\frac{\sqrt{3}}{10}$

D $\frac{\sqrt{3}}{12}$

Question

#Q. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$. Find $\frac{dy}{dx}$.

(MCQ)

Tah (3)

A $\frac{\cos x}{2y - 1}$

B $\frac{\cos x}{\sqrt{1 + 4\sin x}}$

C $\frac{y\cos x}{y^2 + \sin x}$

D $\frac{y\cos x}{y^2 - \sin x}$

Question

#Q. Use the substitution $x = \tan\theta$ to show that the equation, $\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{y}{(1+x^2)^2} = 0$ changes to $\frac{d^2y}{d\theta^2} + y = 0$.

$$\frac{dy}{d\theta} = \frac{dy}{dx}\cdot\frac{dx}{d\theta}, \quad x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$$

$$\frac{dy}{dx}\cdot\sec^2\theta = \frac{dy}{d\theta}$$

$$\frac{dy}{dx} = \cos^2\theta\frac{dy}{d\theta} \quad \text{—— (1)}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\cos^2\theta\frac{dy}{d\theta}\right) = \frac{d}{d\theta}\left(\cos^2\theta\frac{dy}{d\theta}\right)\cdot\frac{d\theta}{dx}$$

$$= \left(\cos^2\theta\frac{d^2y}{d\theta^2} - 2\cos\theta\sin\theta\frac{dy}{d\theta}\right)\cdot\frac{1}{\frac{dx}{d\theta}}$$

$$\frac{d^2y}{dx^2} = \left(\cos^2\theta \frac{d^2y}{d\theta^2} - \sin 2\theta \frac{dy}{d\theta}\right) \cdot \frac{1}{\sec^2\theta}$$

$$= \cos^4\theta \frac{d^2y}{d\theta^2} - \cos^2\theta \sin 2\theta \frac{dy}{d\theta} \quad \text{——} (11)$$

Now

$$\frac{d^2y}{dx^2} + \frac{2x}{1+x^2}\frac{dy}{dx} + \frac{y}{(1+x^2)^2} = 0$$

$$\left(\cos^4\theta \frac{d^2y}{d\theta^2} - \cos^2\theta \sin 2\theta \frac{dy}{d\theta}\right) + \frac{2\tan\theta}{1+\tan^2\theta} \cos^2\theta \frac{dy}{d\theta} + \frac{y}{(1+\tan^2\theta)^2} = 0$$

$$\cos^4\theta \frac{d^2y}{d\theta^2} - \cos^2\theta \sin 2\theta \frac{dy}{d\theta} + \sin 2\theta \cos^2\theta \frac{dy}{d\theta} + y\cos^4\theta = 0$$

$$\frac{d^2y}{d\theta^2} + y = 0$$

Question

#Q. Starting with $\frac{dx}{dy} = \frac{1}{dy/dx}$. Prove that $\frac{d^2x}{dy^2} = -\frac{d^2y/dx^2}{(dy/dx)^3}$.

$$\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$$

Diff. w.r.t y

$$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{dy}{dx}\right)^{-1}$$

$$\frac{d^2x}{dy^2} = \left(\frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\right)\cdot\frac{dx}{dy} = \left(-1\cdot\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\right)\cdot\frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{1}{\frac{dy}{dx}} = -\frac{\frac{d^2y}{dx^2}}{(dy/dx)^3}$$

$$\frac{d^2x}{dy^2} = -\left(\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}\right)$$

Yaad Rakhe!!

Question

#Q. If a curve is represented parametrically by the equations $x = f(t)$ and $y = g(t)$ then $\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{d^2x}{dy^2}\right)}$ is equal to (where $f'(t)\ 0, g'(t) \neq 0$)

A. 1

B. $\frac{g'(t)}{f'(t)}$

C. $-\left(\frac{g'(t)}{f'(t)}\right)^2$

D. $-\left(\frac{g'(t)}{f'(t)}\right)^3$

We know

$$\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^3 = -\frac{\frac{d^2y}{dx^2}}{\frac{d^2x}{dy^2}}$$

$$\left(\frac{\frac{d^2y}{dx^2}}{d^2x/dy^2}\right) = -\left(\frac{dy}{dx}\right)^3 = -\left(\frac{\frac{dy}{dt}}{dx/dt}\right)^3 = -\left(\frac{g'(t)}{f'(t)}\right)^3$$

Question

[JEE Mains 2022 (27 June)]

$(a^m)^n = a^{m\cdot n}$

$(x^x)^x = x^{x^2}$

#Q. If $y(x) = (x^x)^x$, $x > 0$, then $\frac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to _____

$$y = x^{x^2} = e^{\ln x^{x^2}} = e^{x^2 \ln x}$$

$$\frac{dy}{dx} = e^{x^2\ln x}\cdot\left(2x\cdot\ln x + x^2\cdot\frac{1}{x}\right)$$

$$\frac{dy}{dx} = e^{x^2\ln x}\cdot(2x\ln x + x)$$

M① Now

$$\frac{d^2y}{dx^2} = e^{x^2(\ln x)}\left(2\cdot x\cdot 1/x + 2\ln x + 1\right) + e^{x^2\ln x}(2x\cdot\ln x + x)^2$$

Put $x=1$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1} = e^0(2+0+1) + e^0(0+1)^2 = 4.$$

$$\left.\frac{dy}{dx}\right|_{x=1} = e^0(0+1) = 1$$

$$\frac{d^2x}{dy^2} = -\left.\frac{\frac{d^2y}{dx^2}}{(dy/dx)^3}\right|_{x=1} = -4 \Rightarrow \frac{d^2x}{dy^2} + 20 = 16.$$

M(2)

$$\frac{dy}{dx} = e^{x^2\ln x}(2x\ln x + x)$$

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{e^{x^2\ln x}\cdot(2x\ln x + x)}$$

Diff. w.r.t y

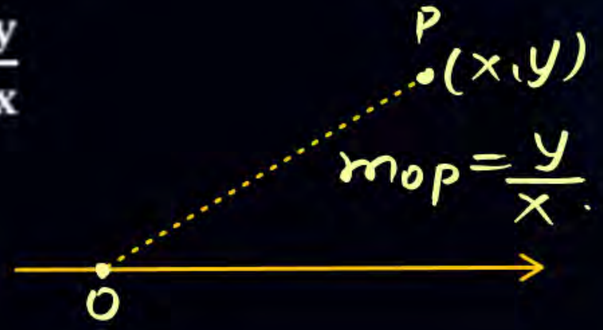
$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\left(e^{x^2\ln x}(2x\ln x + x)\right)^{-1}\right) = \frac{d}{dx}\left(\left(e^{x^2\ln x}(2x\ln x + x)\right)^{-1}\right)\cdot\frac{dx}{dy}$$

$$= \frac{-1}{\left(e^{x^2\ln x}(2x\ln x + x)\right)^2}\cdot\frac{\left[e^{x^2\ln x}(2x\ln x + x)^2 + e^{x^2\ln x}\cdot(2 + 2\ln x + 1)\right]}{\frac{dy}{dx}}$$

$$\left.\frac{d^2x}{dy^2}\right|_{x=1} = -4$$

Question

#Q. If $ax^2 + 2hxy + by^2 = 0$. Then prove that $\frac{dy}{dx} = -\left(\frac{ax+hy}{hx+by}\right) = \frac{y}{x}$



$$2ax + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(hx+by) = -(ax+hy)$$

$$\frac{dy}{dx} = -\frac{(ax+hy)}{hx+by} = -\frac{(ax+hy)}{hx+by}\cdot\frac{x}{y}\cdot\frac{y}{x}$$

Now $ax^2 + 2hxy + by^2 = 0$

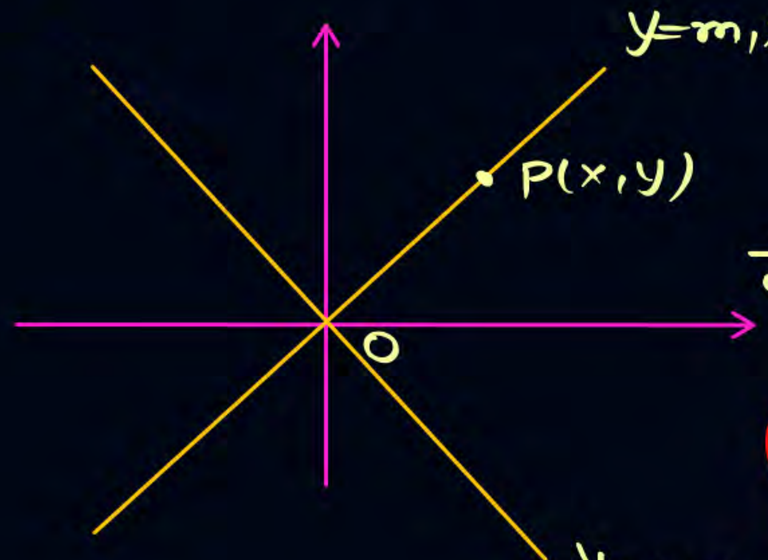
$ax^2 + hxy = -hxy - by^2$

$$= -\frac{(ax^2+hxy)}{(hxy+by^2)}\cdot\frac{y}{x}$$

$$\frac{dy}{dx} = -(-1)\cdot\frac{y}{x} = \frac{y}{x}$$

Kaam Ki Baat

$y = ax^2 + 2hxy + by^2$



$\frac{dy}{dx} = m_{OP} = \frac{y}{x}$.

$\Downarrow$

for any homogeneous Eqn in x, y of degree n: $\frac{dy}{dx} = \frac{y}{x}$.

$\frac{d^2y}{dx^2} = 0$.

Note :

In case of homogeneous function $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = \frac{d^3y}{dx^3}$ ..... = 0

Question

#Q. If $x^3 + 3x^2y - 6xy^2 + 2y^3 = 0$. Find $\frac{d^2y}{dx^2}$ at (1, 1).

Homogenous of degree 3

$\frac{dy}{dx} = \frac{1}{1} = 1$

$\frac{d^2y}{dx^2} = 0$

How to Find the Derivative of Inverse Function without actually finding the Inverse

Let $y=f(x)$ be a derivable fn such that 'g' is its inverse fn.

i.e $g(y)=x$ then $g'(y)=\frac{1}{f'(x)}$ provided $f'(x)\neq 0$ i.e $\frac{dx}{dy}=\frac{1}{\frac{dy}{dx}}$

provided $\frac{dy}{dx}\neq 0$

we know $g(f(x))=x$.

$$g'(f(x))\cdot f'(x)=1$$

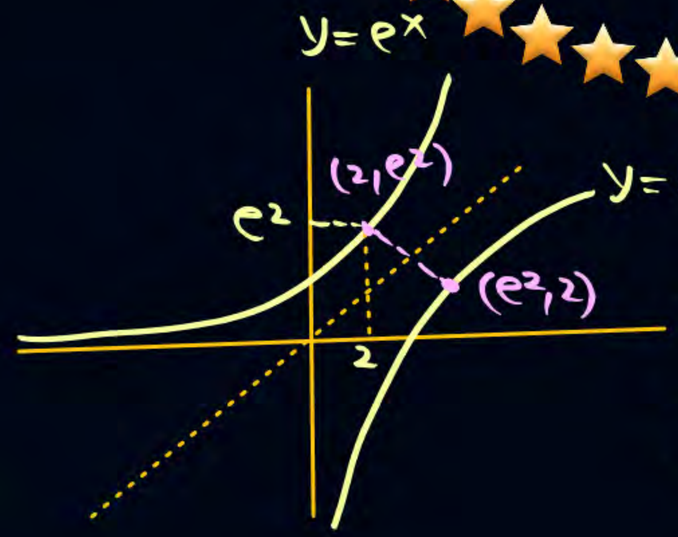
$$g'(f(x))=\frac{1}{f'(x)}$$

$$g'(y)=\frac{1}{f'(x)} \rightarrow \frac{dx}{dy}=\frac{1}{\frac{dy}{dx}}$$

NICHOD

if $\beta = f(\alpha)$ then $g'(\beta) = \frac{1}{f'(\alpha)}$

$f(2) = e^2$



$f(x) = e^x$

$f'(x) = \frac{dy}{dx} = e^x \Big|_{x=2} = e^2 = f'(2)$

$g(x) = \ln x$

$g'(x) = \frac{1}{x}$

$g'(e^2) = \frac{1}{e^2} = \frac{1}{f'(2)}$

Question

#Q. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value $g'(1)$ is

[JEE Advance 2009]

$f(x) = x^3 + e^{x/2}$ $\quad \beta = 1$

$\beta = f(\alpha) = \alpha^3 + e^{\alpha/2} = 1$

$\Downarrow$

$\alpha = 0$.

$\Rightarrow g'(\beta) = \frac{1}{f'(\alpha)}$

$g'(1) = \frac{1}{f'(0)} = \frac{1}{3x^2 + \frac{1}{2}e^{x/2}}\Big|_{x=0} = 2$

Question

#Q. Let $f(x) = \exp(x^3 + x^2 + x)$ for any real number x and let g be the inverse function for f. The value of $g'(e^3)$ is –

A $\frac{1}{6e^3}$

B $\frac{1}{6}$

C $\frac{1}{34e^{39}}$

D 6

$f(x) = e^{x^3+x^2+x}$ $\quad \beta = e^3$

$\beta = f(\alpha) = e^3 = e^{x^3+x^2+x}$

$x^3+x^2+x = 3$

$x = 1$

$g'(\beta) = \frac{1}{f'(\alpha)}$

$g'(e^3) = \frac{1}{f'(1)} = \frac{1}{e^{x^3+x^2+x}\cdot(3x^2+2x+1)}\Big|_{x=1} = \frac{1}{6e^3}$

Question

#Q. If $y = f(x) = x^3 + x^5$ and g is the inverse of f. Find g'(2) and g"(2).

$\beta = 2 \quad f(\alpha) = \beta$

$x^3 + x^5 = 2$

$x = 1$

$g'(2) = \frac{1}{f'(1)} = \frac{1}{3x^2 + 5x^4}\Big|_{x=1} = \frac{1}{8}$

Chomu !!

$g'(2) = \frac{1}{8}$

$g''(2) = 0$

$\Rightarrow$

$x = g(y), \quad y = f(x)$

$g''(y) = \frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} = -\frac{f''(x)}{(f'(x))^3}$

$g''(\beta) = -\frac{f''(\alpha)}{(f'(\alpha))^3}$

$$g''(2) = \frac{-f''(1)}{(f'(1))^3} = -\frac{26}{8^3} = \frac{-13}{256}.$$

$f'(x) = 3x^2 + 5x^4$

$f''(x) = 6x + 20x^3$

Question

#Q. Let $f : (0, \infty) \to (-\infty, \infty)$ be defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$, then

Tan (4)

A $g''(e) = \frac{1-e}{(1+e)^3}$

B $g''(e) = \frac{e-1}{(1+e)^3}$

C $g'(e) = e + 1$

D $g'(e) = \frac{1}{e+1}$

Question

#Q. If $f(x) = \begin{cases} e^x + b; & x \le 2 \\ kx^3; & x > 2 \end{cases}$ be a twice differentiable function with inverse function g, find the value of $\left|36\, k\, b\, g''\left(\frac{2e^2}{3}\right)\right|$.

Tan(5)

Question

#Q. Let $f : R \to R$, $g : R \to R$ and $h : R \to R$ be differentiable function such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $g(g(g(x))) = x$ for all $x \in R$. Then-

[JEE Advance 2016]

A $g'(2) = 1/15$

B $g'(1) = 666$

C $h(0) = 16$

D $h(g(3)) = 36$

Topic : Derivative of Functions Expressed in Determinant Form

$$\Delta(x)=\begin{vmatrix} f & g & h \\ \ell & m & n \\ p & q & r \end{vmatrix}$$

$f, g, h, p, q, r, \ell, m, n$ are all derivable fns of x.

$$\Delta'(x)=\begin{vmatrix} f' & g' & h' \\ \ell & m & n \\ p & q & r \end{vmatrix}+\begin{vmatrix} f & g & h \\ \ell' & m' & n' \\ p & q & r \end{vmatrix}+\begin{vmatrix} f & g & h \\ \ell & m & n \\ p' & q' & r' \end{vmatrix}$$

can be differentiated one by one along Row or column.

$$\Delta(x)=\begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix}=f(x)\cdot k(x)-g(x)\cdot h(x)$$

$$\Delta'(x)=f'(x)\cdot k(x)+f(x)\cdot k'(x)-g'(x)h(x)-g(x)\cdot h'(x)$$

$$\Delta'(x) = f'(x)k(x) - g'(x)\cdot h(x) + f(x)\cdot k'(x) - g(x)\cdot h'(x)$$

$$= \begin{vmatrix} f'(x) & g'(x) \\ h(x) & k(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ h'(x) & k'(x) \end{vmatrix}$$

Always try to simplify the determinant before Differentiation.

Question

#Q. If $f(x)=\begin{vmatrix}\cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2)\\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2)\\ \sin 2x & 0 & \sin 2x^2\end{vmatrix}$, then find $f'(x)$.

$A=x+x^2$

$B=x-x^2$

$A+B=2x$

$A-B=2x^2$

$$f(x)=\begin{vmatrix}\cos A & \sin A & -\cos A\\ \sin B & \cos B & \sin B\\ \sin(A+B) & 0 & \sin(A-B)\end{vmatrix}$$

Expand along R_3

$=\sin(A+B)(\sin A\sin B+\cos A\cos B)$

$+\sin(A-B)(\cos A\cos B-\sin A\sin B)$

$=\sin(A+B)\cos(A-B)+\sin(A-B)\cos(A+B)$

$=\sin(A+B+A-B)=\sin(2A)$

$$f(x) = \sin(2(x+x^2))$$

$$f(x) = \sin(2x+2x^2)$$

$$f'(x) = \cos(2x+2x^2) \cdot (2+4x)$$

Solution to TAH

Question

#Q. If $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$, then $\left.\frac{dy}{dx}\right]_{x=1}$ is

Tan 1@

A 4

B 1

C 0

D not defined

TAH-1[Q]

$$y = 2^{\log_2 x^{2x}} + (\tan \tfrac{\pi x}{4})^{1/\pi x}$$

$$\log y = \log 2 \log_2 x^{2x} + \log (\tan \tfrac{\pi x}{4})^{1/\pi x}$$

$$\frac{1}{y}\frac{dy}{dx}\Big|_{x=1} = 2x \log_2 x \log 2 + \frac{1}{\pi x}\left(\frac{1}{\tan \frac{\pi x}{4}}\right)\left(\sec \tfrac{\pi x}{4}\right)\frac{\pi}{4}$$

$$= y\left(2x \log_2 x \log 2 + \frac{1}{\pi x}\left(\frac{1}{\tan \frac{\pi x}{4}}\right)\left(\sec \tfrac{\pi x}{4}\right)\left(\tfrac{\pi}{4}\right)\right)$$

$$y|_{x=1} = 2^0 + 1 = 2$$

$$-\frac{dy}{dx}\Big|_{x=1} = 2\left(0 + \frac{1}{\pi} 2 \cdot \frac{\pi}{4}\right) = 2(2) = 4$$

Shrestha yadav
Lucknow

SHASHIKANT TAH (S)

(19) If $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$

, then $\left.\frac{dy}{dx}\right]_{x=1}$ is ?

Let $f(x) = 2^{\log_2 x^{2x}}$

$g(x) = \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$

$\frac{dy}{dx} = \frac{d(f(x))}{dx} + \frac{d(g(x))}{dx}$

$f(x) = 2^{\log_2 x^{2x}} \Rightarrow f(x) = x^{2x}$

taking $\log_e$ Both sides

$\ln(f(x)) = 2x \ln x$

$\frac{d(\ln f(x))}{dx} = \frac{d(2x\ln x)}{dx}$

$\frac{1}{f(x)} \times f'(x) = 2(\ln x + 1)$

$f'(x) = 2x^{2x}(\ln x + 1)$

at $x=1; f'(x) = 2$

$g(x) = \left(\tan\frac{\pi}{4}x\right)^{\frac{4}{\pi x}}$

$\ln(g(x)) = \frac{4}{\pi} x \ln\left(\tan\frac{\pi}{4}x\right)$

differentiating Both sides wrt. x

$\frac{1}{g(x)} g'(x) = \frac{4}{\pi}\left(\ln\left(\tan\frac{\pi}{4}x\right) + x\frac{\pi}{4}\cdot\frac{\sec^2\frac{\pi}{4}x}{\tan\frac{\pi}{4}x}\right)$

at $x=1 \quad g'(x) \Rightarrow 2$

hence $\frac{dy}{dx} = 2+2 \Rightarrow 4$

Option (A)

Ans (a) $y = 2^{\log_2 x^{2x}} + \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$ Then $\left.\frac{dy}{dx}\right|_{x=1}$

$f(x) = 2^{\log_2 x^{2x}} \quad g(x) = \left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$

taking $\log_e$ both side

$\log_e f(x) = \ln 2^{\log_2 x^{2x}} = x^{2x} = 2x\ln x$

$\frac{1}{f(x)} f'(x) = 2\ln x + \frac{2x}{x}$

$f'(x) = x^{2x} \cdot 2(\ln x + 1)$

$f'(1) = (1)^{2(1)} \cdot 2(\ln 1 + 1) = 2$

$\ln g(x) = \ln\left(\tan\frac{\pi x}{4}\right)^{\frac{4}{\pi x}}$

$\frac{1}{g(x)} g'(x) = \frac{4}{\pi x}\ln\left(\tan\frac{\pi x}{4}\right)$

$\frac{g'(x)}{g(x)} = \left(\frac{-4}{\pi x^2}\ln\left(\tan\frac{\pi x}{4}\right)\right) + \frac{4}{\pi x}\frac{1}{\tan\left(\frac{\pi x}{4}\right)}\cdot\sec^2\frac{\pi x}{4}\cdot\frac{\pi}{4}$

$g'(x) = g(x)\left[\frac{-4}{\pi x^2}\ln\left(\tan\frac{\pi x}{4}\right) + \frac{4}{\pi x}\frac{1}{\tan\left(\frac{\pi x}{4}\right)}\sec^2\frac{\pi x}{4}\frac{\pi}{4}\right]$

$g'(1) = (1)^{\frac{4}{\pi}}\left[\frac{-4}{\pi}\ln(1) + \frac{4}{\pi}\frac{1}{\tan\left(\frac{\pi}{4}\right)}\sec^2\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{4}\right]$

$= 1\left[0 + \frac{1}{1}(\sqrt{2})^2\right] = 2$

$f'(x) + g'(x) = 2+2 = 4$ Ans

पूजा सैनिया
कॉसी (U.P.)

Question

#Q. For $t \in (0, 1)$, let $x = \sqrt{2^{\sin^{-1} t}}$ and $y = \sqrt{2^{\cos^{-1} t}}$. Then, $1 + \left(\frac{dy}{dx}\right)^2$ equals

A $\frac{x^2}{y^2}$

B $\frac{y^2}{x^2}$

C $\frac{x^2+y^2}{y^2}$

D $\frac{x^2+y^2}{x^2}$

Riyanshu Sharma
Jharkhand.....

Date __/__/____

TAH-1(b)

for $t \in (0,1)$, let $x = \sqrt{2^{\sin^{-1}t}}$ and $y = \sqrt{2^{\cos^{-1}t}}$. Then $1 + \left(\frac{dy}{dx}\right)^2$

$x^2 = 2^{\sin^{-1}t}$ $\quad$ $y^2 = 2^{\cos^{-1}t}$

$2\ln x = \sin^{-1}t \cdot \ln 2$ $\quad$ $2\ln y = \cos^{-1}t \cdot \ln 2$

$\frac{2}{x}\cdot\frac{dx}{dt} = \frac{\ln 2}{\sqrt{1-t^2}}$ $\quad$ $\frac{2}{y}\cdot\frac{dy}{dt} = -\frac{\ln 2}{\sqrt{1-t^2}}$

$\frac{dx}{dt} = \frac{x}{2}\cdot\frac{\ln 2}{\sqrt{1-t^2}}$ $\quad$ $\frac{dy}{dt} = -\frac{y}{2}\cdot\frac{\ln 2}{\sqrt{1-t^2}}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x\cdot\ln 2}{2\sqrt{1-t^2}} \cdot \frac{2\sqrt{1-t^2}}{y\cdot\ln 2} = \frac{x}{y}$$

Now,

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{x^2}{y^2} = \frac{x^2+y^2}{y^2}$$ (c) Ans

TAH-1.[b]

$x = \sqrt{2^{\sin^{-1}t}}$ $\quad$ $y = \sqrt{2^{\cos^{-1}t}}$ $\quad$ then $1+\left(\frac{dy}{dx}\right)^2$.

$x^2 = 2^{\sin^{-1}t}$ $\quad$ $y^2 = 2^{\cos^{-1}t}$

Multiply

$x^2y^2 = 2^{\sin^{-1}t + \cos^{-1}t}$

$x^2y^2 = 2^{\pi/2}$

$x^2y^2 - 2^{\pi/2} = 0$

using partial differentiation

$$\frac{dy}{dx} = \frac{2x\,y^2}{x^2\,2y} = \frac{y}{x}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{y^2}{x^2}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{y^2}{x^2} = \frac{x^2+y^2}{x^2}$$

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Question

#Q. Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ $\forall x \in R$.

TAN 2 ⓒ

M-1

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $g = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\frac{dy}{dg} = \frac{\frac{dy}{dx}}{\frac{dg}{dx}} = \frac{\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \left(\frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}\right)}{\frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \left(\frac{(1+x)(-2x) - (1-x^2)2x}{(1+x^2)^2}\right)}$$

$$= \frac{\frac{-1}{\sqrt{\frac{1+x^4+2x^2-4x^2}{(1+x^2)^2}}}\left(2+2x^2-4x^2\right)}{\frac{1}{\sqrt{\frac{1+x^4+2x^2-1-x^4+2x^2}{(1+x^2)^2}}}\left(-2x-2x^3-2x+2x^3\right)}$$

$$= \frac{\frac{2}{|1-x^2|}(1-x^2)}{\frac{4x}{2|x|}} = \frac{|x|(1-x^2)}{x|(1-x^2)|}$$

$$\frac{dy}{dg} = \begin{cases} \frac{-x(1-x^2)}{-x(1-x^2)} = 1 & , x<-1 \\ \frac{-x(1-x^2)}{x(1-x^2)} = -1 & , -1<x<0 \\ 1 & , 0<x<1 \\ -1 & , x>1. \end{cases}$$ Ans

TAN 2 ⓒ

M-2

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x & , x<-1 \\ 2\tan^{-1}x & , -1\le x\le 1 \\ \pi - 2\tan^{-1}x & , x>1 \end{cases}$$

$$y' = \begin{cases} \frac{-2}{1+x^2} & , x>1, x<-1 \\ \frac{2}{1+x^2} & , -1<x<1. \end{cases}$$

$$g = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} -2\tan^{-1}x & , x<0 \\ 2\tan^{-1}x & , x\ge 0 \end{cases}$$

$$g' = \begin{cases} \frac{-2}{1+x^2} & , x<0 \\ \frac{2}{1+x^2} & , x>0 \end{cases}$$

So,

$$\frac{dy}{dg} = \begin{cases} \frac{\frac{-2}{1+x^2}}{\frac{-2}{1+x^2}} & , x<-1 \\ \frac{\frac{2}{1+x^2}}{\frac{-2}{1+x^2}} & , -1<x<0 \\ \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}} & , 0<x<1 \\ \frac{\frac{-2}{1+x^2}}{\frac{2}{1+x^2}} & , x>1. \end{cases}$$

$$\frac{dy}{dg} = \begin{cases} 1 & , x<-1 \\ -1 & , -1<x<0 \\ 1 & , 0<x<1 \\ -1 & , x>1. \end{cases}$$ Ans.

Question

#Q. If $y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ and $x = \sec^{-1}\frac{1}{2u^2-1}$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

prove that $2\frac{dy}{dx} + 1 = 0$.

Riyanshu Sharma
Jharkhand....

Date ___/___/_____

TAH-3

$y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$ and $x = \sec^{-1}\frac{1}{2u^2-1}$, $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$

prove that $2.\frac{dy}{dx} + 1 = 0$

$y = \tan^{-1}\frac{u}{\sqrt{1-u^2}}$

let $u = \sin\theta$

$y = \tan^{-1}\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$y = \tan^{-1}(\tan\theta)$

$y = \theta$

$y = \sin^{-1}u$

$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

$x = \sec^{-1}\frac{1}{2u^2-1}$

let $u = \cos\theta$

$x = \sec^{-1}\frac{1}{2\cos^2\theta - 1}$

$x = \sec^{-1}(\sec 2\theta)$

$x = 2\theta$

$x = 2\cos^{-1}u$

$\frac{dx}{du} = \frac{-2}{\sqrt{1-u^2}}$

$\frac{dy}{dx} = \frac{dy/du}{dx/du} = -1/2$

Now,

$2.\frac{dy}{dx} + 1 = 2 \times \frac{-1}{2} + 1 = 0$ Ans

TAH-3

$y = \tan^{-1}\left(\frac{u}{\sqrt{1-u^2}}\right)$

$u = \sin\theta$

$y = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = y = \sin^{-1}(\tan\theta) = \theta$

$y = \sin^{-1}u$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}}$ —(I)

Put $u = \cos\theta$

$x = \sec^{-1}\left(\frac{1}{2u^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

$= \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = 2\theta$

$x = 2\cos^{-1}u$

$\frac{dx}{du} = \frac{-2}{\sqrt{1-u^2}}$ —(II)

from (I) & (II) $\frac{dy}{dx} = \frac{1}{-2}$

$2\frac{dy}{dx} = -1$

$2\frac{dy}{dx} + 1 = 0$

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TAH 2@

$f(x) = \cos^{-1}(4x^3 - 3x)$

Let, $x = \cos\theta$. , $\theta \in [0,\pi] \Rightarrow 3\theta \in [0,3\pi]$

$f(x) = \cos^{-1}(\cos 3\theta)$

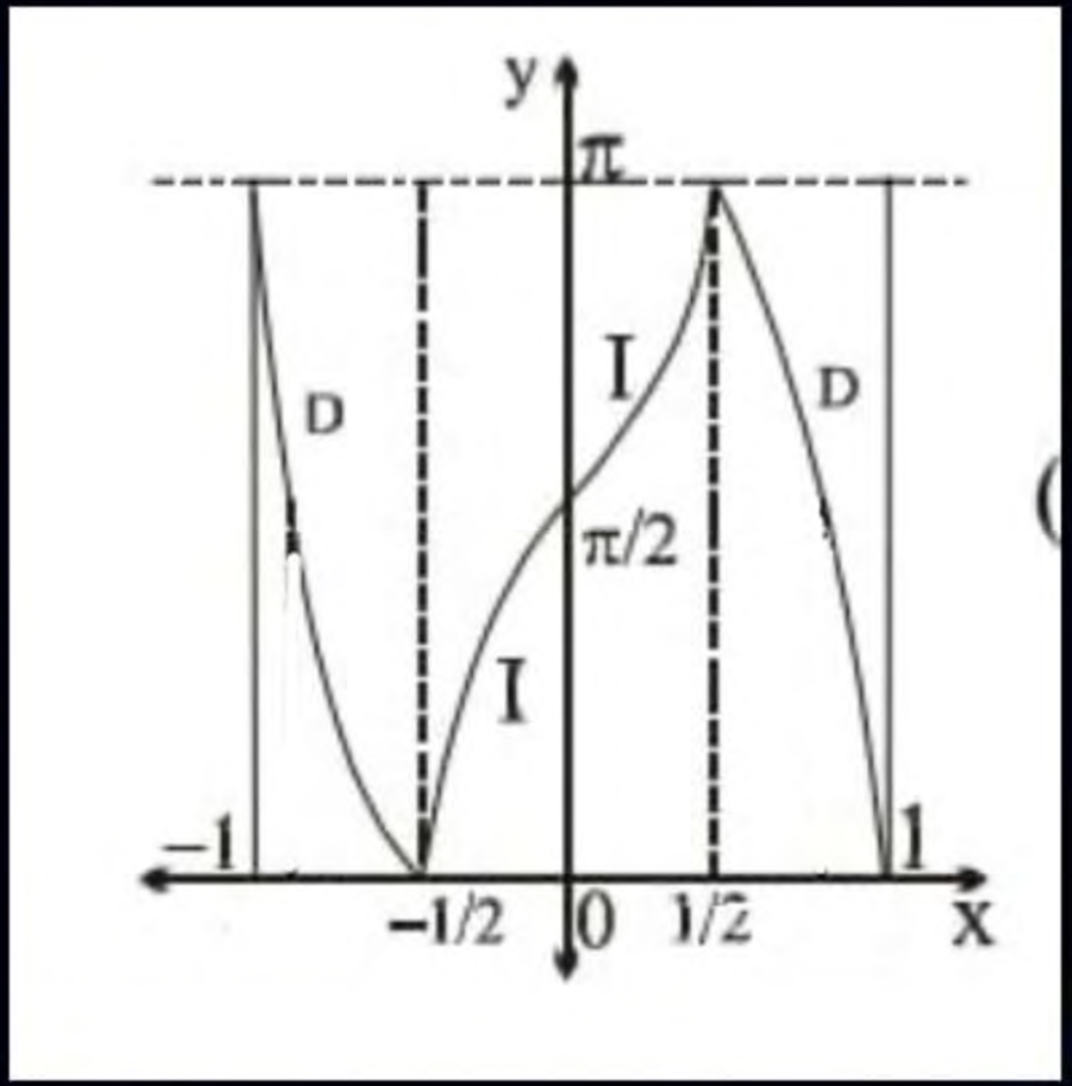
$$\cos^{-1}(\cos 3\theta) = \begin{cases} 3\theta , & 0 \le 3\theta \le \pi \\ 2\pi - 3\theta , & \pi < 3\theta < 2\pi \\ 3\theta - 2\pi , & 2\pi \le 3\theta \le 3\pi \end{cases}$$

$$f(x) = \cos^{-1}(4x^3 - 3x) = \begin{cases} 3\cos^{-1}x , & \frac{1}{2} \le x \le 1 \\ 2\pi - 3\cos^{-1}x , & \frac{-1}{2} < x < \frac{1}{2} \\ 3\cos^{-1}x - 2\pi , & -1 \le x \le \frac{-1}{2} \end{cases}$$

&

$$f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} , & \frac{1}{2} < x < 1 \ \& \ -1 < x < \frac{-1}{2} \\ \frac{3}{\sqrt{1-x^2}} , & \frac{-1}{2} < x < \frac{1}{2} \end{cases}$$

Ans



TAH 2(b)

$g(x) = \sin^{-1}(3x - 4x^3)$

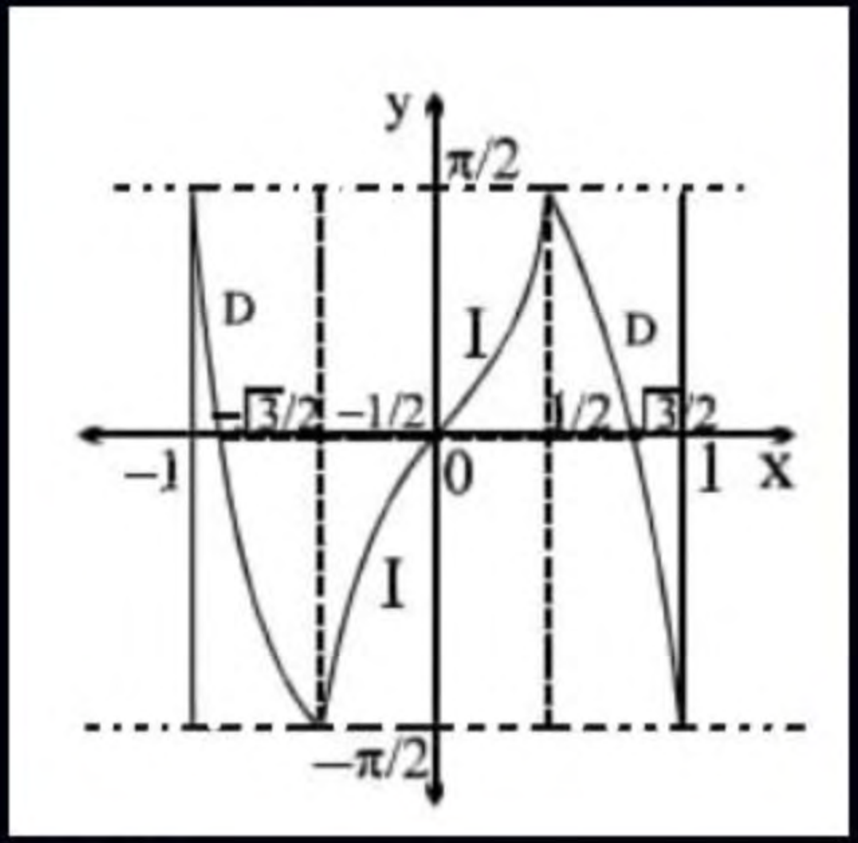
Let, $x = \sin\theta$, $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

$g(x) = \sin^{-1}(\sin 3\theta)$, $3\theta \in \left[\frac{-3\pi}{2}, \frac{3\pi}{2}\right]$

$$g(x) = \sin^{-1}\sin 3\theta = \begin{cases} -(3\theta + \pi), & \frac{-3\pi}{2} \le 3\theta \le \frac{-\pi}{2} \\ 3\theta, & -\frac{\pi}{2} < 3\theta < \frac{\pi}{2} \\ \pi - 3\theta, & \frac{\pi}{2} \le 3\theta \le \frac{3\pi}{2} \end{cases}$$

$$\sin^{-1}(3x - 4x^3) = \begin{cases} -(3\sin^{-1}x + \pi), & \frac{-\pi}{2} \le \theta \le \frac{-\pi}{6} \\ 3\sin^{-1}x, & \frac{-\pi}{6} < \theta < \frac{\pi}{6} \\ \pi - 3\sin^{-1}x, & \frac{\pi}{6} \le \theta \le \frac{\pi}{2} \end{cases}$$

$$g(x) = \begin{cases} -(\pi + 3\sin^{-1}x), & -1 \le x \le \frac{-1}{2} \\ 3\sin^{-1}x, & \frac{-1}{2} < x < \frac{1}{2} \\ \pi - 3\sin^{-1}x, & \frac{1}{2} \le x \le 1. \end{cases}$$



TAH 2(b) Continuing

$$g'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}}, & -1 < x < \frac{-1}{2} \text{ \& } \frac{1}{2} < x < 1 \\ \frac{3}{\sqrt{1-x^2}}, & \frac{-1}{2} < x < \frac{1}{2}. \end{cases}$$

Ans

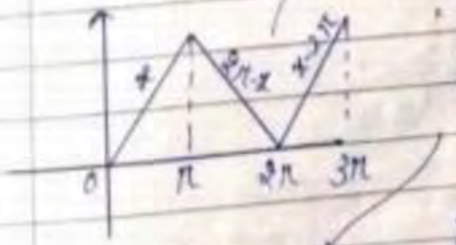
TAH-2(a) $f(x) = \cos^{-1}(4x^3 - 3x)$ $\xrightarrow{x = \cos\theta}$ $\theta \in [0, \pi]$, $3\theta \in [0, 3\pi]$

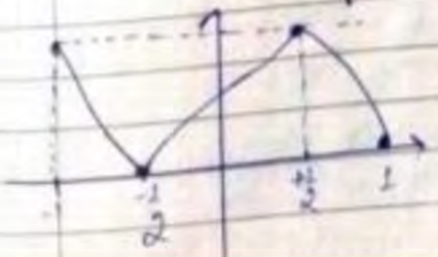
$f(x) = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$$f(x) = \cos^{-1}(\cos 3\theta) = \begin{cases} 3\theta & 0 \le 3\theta \le \pi \\ 2\pi - 3\theta & \pi < 3\theta \le 2\pi \\ 3\theta - 2\pi & 2\pi < 3\theta \le 3\pi \end{cases}$$

$$= \begin{cases} 3\cos^{-1}x & x \in \left[\frac{1}{2}, 1\right] \\ 2\pi - 3\cos^{-1}x & x \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ 3\cos^{-1}x - 2\pi & x \in \left[-1, -\frac{1}{2}\right] \end{cases}$$





$$f'(x) = \begin{cases} \frac{-3}{\sqrt{1+x^2}} & x \in \left[\frac{1}{2}, 1\right], x \in \left[-1, -\frac{1}{2}\right] \\ \frac{3}{\sqrt{1+x^2}} & x \in \left[-\frac{1}{2}, \frac{1}{2}\right] \end{cases}$$

TAH-2(b) $g(x) = \sin^{-1}(3x - 4x^3)$

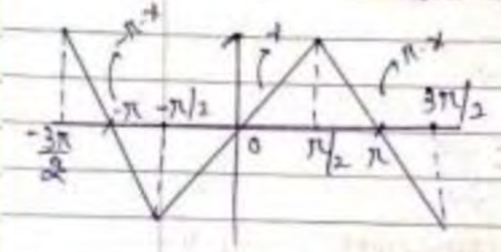
$x = \sin\theta$, $\theta = \sin^{-1}x$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$3\theta \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

$g(x) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$$= \begin{cases} -\pi - 3\theta & -\frac{3\pi}{2} \le 3\theta < -\frac{\pi}{2} \\ 3\theta & -\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2} \\ \pi - 3\theta & \frac{\pi}{2} < 3\theta \le \frac{3\pi}{2} \end{cases}$$



$$g(x) = \sin^{-1}(\sin 3\theta) = \begin{cases} -\pi - 3\sin^{-1}x & -\frac{\pi}{2} \le \theta < -\frac{\pi}{6} & -1 < x < -\frac{1}{2} \\ 3\sin^{-1}x & -\frac{\pi}{6} \le \theta \le \frac{\pi}{6} & -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - 3\sin^{-1}x & \frac{\pi}{6} < \theta \le \frac{\pi}{2} & \frac{1}{2} < x \le 1 \end{cases}$$

$$\frac{d}{dx}(g(x)) = \frac{-3}{\sqrt{1+x^2}} \quad x \in \left(-1, -\frac{1}{2}\right), \; x \in \left(\frac{1}{2}, 1\right)$$

$$\frac{3}{\sqrt{1+x^2}} \quad x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$

पूजा सैनिया
कॉंसी (U.P)

Question

#Q. $f(x) = x + \cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+1....}}}$, find $f(100) \cdot f'(100)$

$$f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}$$

$$f(x) = x + \frac{1}{x + f(x)}$$

$$\Rightarrow f(x) \times [x + f(x)] = x \times [x + f(x)] + 1$$

$$\Rightarrow f(x) \times x + f^2(x) = x^2 + x f(x) + 1$$

$$\Rightarrow f^2(x) = x^2 + 1$$

$$\Rightarrow 2 f(x) \times f'(x) = 2x$$

$$\Rightarrow \boxed{f(100) \times f'(100) = 100}$$

Riyanshu Sharma
Jharkhand....

Date __/__/____

TAH-4

$f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}$, find $f(100) \cdot f'(100)$

$$f(x) = y = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}$$

$$y - x = \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \cdots}}}$$ → Repeating unit

$$y - x = \frac{1}{2x + y - x}$$

$$y - x = \frac{1}{x + y}$$

$$(x + y)(y - x) = 1$$

$$y^2 - x^2 = 1$$

$$y^2 = 1 + x^2$$

$$2y\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$f'(x) = \frac{x}{y}$$

$$f(x) f'(x) = x$$

$f(100) \cdot f'(100) = 100$ Ans


THANK
YOU

PRAYAS

JEE 2025

Lecture-16 | Mathematics

Limit, Continuity and Differentiability



By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. L'Hopital Rule
2. Practice Probelms
3. Derivative of Determinants & Repeated Roots
4.

Recap

1. If f is an invertible function such that $f'(\alpha) \neq 0$ where $f(\beta) = \alpha$ then $g'(\beta) = \frac{1}{f'(\alpha)}$ and $g''(\beta) = \frac{-f''(\alpha)}{(f'(\alpha))^3}$

2. $\frac{d^2x}{dy^2} = \frac{-d^2y/dx^2}{\left(\frac{dy}{dx}\right)^3}$

3. $X = f(t), y = g(t)$ then $\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$

4. The derivative of a differentiable periodic function is a function with the same period. (True)

5. Derivative of an even differentiable function is an odd function and derivative of an odd differentiable function is an even function. (True)

JEE **Recap**

6. **Which of the following functions are not derivable at x = 0 ?**

A $f(x) = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

B $g(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x \cdot 2}{1+(2^x)^2}\right) = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ $2^x = t$ Is not derivable at $t=1$

C $h(x) = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \pi/2 - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

D $k(x) = \sin^{-1}(\cos x) = \pi/2 - \cos^{-1}(\cos x)$

@ $f(x) = \sin^{-1}(\sin 2\theta)$ $x = \sin\theta$ $\theta \in [-\pi/2, \pi/2]$ $2\theta \in [-\pi, \pi]$

$$= \begin{cases} -\pi - 2\theta & -\pi \le 2\theta < -\pi/2 \\ 2\theta & -\frac{\pi}{2} \le 2\theta \le \pi/2 \\ \pi - 2\theta & 2\theta > \pi/2 \end{cases}$$

$$f(x)=\begin{cases}-\pi-2\sin^{-1}x & -\frac{\pi}{2}\le\theta<-\pi/4\\ 2\sin^{-1}x & -\pi/4\le\theta\le\pi/4\\ \pi-2\sin^{-1}x & \frac{\pi}{4}<\theta\le\pi/2\end{cases}$$

$$=\begin{cases}-\pi-2\sin^{-1}x & -1\le x<-1/\sqrt{2}\\ 2\sin^{-1}x & -\frac{1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}\\ \pi-2\sin^{-1}x & \frac{1}{\sqrt{2}}< x\le 1\end{cases}$$

Question

#Q. Let $f : R \to R$, $g : R \to R$ and $h : R \to R$ be differentiable function such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then- [JEE Advance 2016]

~~A~~ $g'(2) = 1/15$

~~B~~ $h'(1) = 666$

~~C~~ $h(0) = 16$

~~D~~ $h(g(3)) = 36$

$f : R \to R$, $g : R \to R$ $h : R \to R$

$f(x) = x^3 + 3x + 2$, $g(f(x)) = x$

$f(x)$ ↓ (Bijective fn)

$g(x) = f^{-1}(x)$ — (I)

Since Range of $f = R$

$h(g(g(x))) = x \quad \forall x \in R$

replace x by $f(x)$

$h(g(g(f(x)))) = f(x)$

$h(g(x)) = f(x) \quad \forall x \in R$ — (II)

replace $x \to f(x)$

$$g(f(x)) = f(f(x)) \quad \forall x \in R$$

$$h(x) = f(f(x)) \longrightarrow (11)$$

Now $f(x) = x^3 + 3x + 2$

$\beta = 2$, $f(\alpha) = 2$

$$\alpha^3 + 3\alpha + 2 = 2$$

$$\downarrow$$

$$\alpha = 0$$

$$g'(2) = \frac{1}{f'(0)} = \frac{1}{3x^2+3}\Big|_{x=0} = 1/3.$$

from (3) $h(g(x)) = f(x)$

$h(g(3)) = f(3) = 27 + 9 + 2 = 38.$

$h'(x) = f'(f(x)) \cdot f'(x)$, $h(0) = f(f(0))$

$h'(1) = f'(f(1)) \cdot f'(1)$

$= f'(6) \quad 3x^2+3\big|_{x=1}$

$= 3x^2+3\big|_{x=6} \cdot 6.$

$= (108+3) \cdot 6 = 666.$

$= f(2)$

$= 8 + 6 + 2$

$= 16$

Question

#Q. Find a polynomial function f(x) such that $f(2x) = f'(x)\, f''(x)$. (ASKQ)

Let degree of f be n.

$n = (n-1) + (n-2)$

$n = 2n-3 \Rightarrow n = 3 \Rightarrow f(x) = ax^3 + bx^2 + cx + d$

Now $f(2x) = f'(x) \cdot f''(x)$

$8ax^3 + 4bx^2 + 2cx + d = (3ax^2 + 2bx + c)(6ax + 2b)$

Constant term

$d = 2bc$

$d = 0$

Coeff of x^3: $8a = 18a^2$

$a = 0, \frac{4}{9}$

Coeff of x^2: $4b = 6ab + 12ab$

$4b = 18 \cdot \frac{4}{9} b$

$b = 0$

Coeff of x

$2c = 6ac + 4b^2$

$2c = 6ac + 0$

$c = 0$.

$f(x) = \frac{4}{9}x^3$ (Ans)

Question

#Q. If $y = y(x)$ and it follows the relation $e^{xy} + y\cos x = 2$, then find
(i) $y'(0)$ and (ii) $y''(0)$

(i) $e^{xy} + y\cos x = 2$ if $x=0$, $e^0 + y\cdot\cos 0 = 2 \Rightarrow y = 1$

$e^{xy}(xy' + y) + y'\cos x - y\sin x = 0$ — (1)

put $x=0, y=1$ $\quad e^0(0 + 1) + y'\cos 0 - (1)\cdot 0 = 0$

$1 + y' = 0$

$y' = -1$

(ii) Diff (1) Again

$e^{xy}(xy'+y)^2 + e^{xy}(xy'' + y' + y') + y''\cos x - y'\sin x - (y'\sin x + y\cos x) = 0$ (m+z)

$x=0, y=1, y'=-1$

$1^2 + 1\cdot(0-1-1) + y''\cdot 1 - 0 - (0 + 1\cdot 1) = 0$

$y'' - 2 = 0$

$y''(0) = 2$

Question

#Q. If f is twice differentiable such that $f''(x) = -f(x)$, $f'(x) = g(x)$, $h'(x) = [f(x)]^2 + [g(x)]^2$ and $h(0) = 2$, $h(1) = 4$ then the equation $y = h(x)$ represents :

A a curve of degree 2

B a curve passing through the origin

C a straight line with slope 2

D a straight line with y intercept equal to –2

$f''(x) = -f(x) \longrightarrow g'(x) = -f(x)$

$f'(x) = g(x) \Rightarrow f''(x) = g'(x)$

$h'(x) = f^2(x) + g^2(x)$

$h''(x) = 2f(x)\cdot f'(x) + 2g(x)g'(x)$

$= 2f(x)g(x) + 2g(x)(-f(x))$

$h''(x) = 0$.

$h'(x) = k$

$h(x) = kx + c$

$h(0) = 2 \Rightarrow c = 2$

$h(1) = 4 \Rightarrow k + 2 = 4 \Rightarrow k = 2$

$y = h(x) = 2x + 2$

Question

#Q. If f, g, h are differentiable function of x, $D = \begin{vmatrix} f & g & h \\ (x.f)' & (x.g)' & (x.h)' \\ (x^2f)'' & (x^2.g)'' & (x^2.h)'' \end{vmatrix}$

Then prove the $D' = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3.f'')' & (x^3.g'')' & (x^3.h'')' \end{vmatrix}$.

$(xf)' = xf' + f$

$(x^2f)'' = (2xf + x^2f')' = 2(xf)' + (x^2f')' = 2(xf' + f) + 2xf' + x^2f'' = x^2f'' + 4xf' + 2f$.

$$D = \begin{vmatrix} f & g & h \\ xf' + f & xg' + g & xh' + h \\ x^2f'' + 4xf' + 2f & x^2g'' + 4xg' + 2g & x^2h'' + 4xh' + 2h \end{vmatrix}$$

$R_3 \to R_3 - 4R_2 + 2R_1$

$$D=\begin{vmatrix} f & g & h \\ xf'+f & xg'+g & xh'+h \\ x^2f'' & x^2g'' & x^2h'' \end{vmatrix} \quad R_2 \to R_2 - R_1$$

$$D=\begin{vmatrix} f & g & h \\ xf' & xg' & xh' \\ x^2f'' & x^2g'' & x^2h'' \end{vmatrix} \Rightarrow D=\begin{vmatrix} f & g & h \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}$$

$$D'=\begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}+\begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}+\begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$$

0 $R_1=R_2$ 0

$$D' = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3 f'')' & (x^3 g'')' & (x^3 h'')' \end{vmatrix}$$

Question

[JEE Mains 2020 (9 Jan)]

#Q. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

A $\frac{3}{8}$

B $\frac{3}{2}$

C $\frac{3}{4}$

D $-\frac{3}{4}$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2\sin\theta + 2\sin 2\theta}{2\cos\theta - 2\cos 2\theta}$$

$$\frac{dy}{dx} = \frac{2\sin\theta/2\cos 3\theta/2}{2\sin\theta/2\sin 3\theta/2} = \cot 3\theta/2$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\frac{3\theta}{2}\cdot\frac{3}{2}\frac{d\theta}{dx}$$

$$= -\frac{3}{2}\operatorname{cosec}^2\frac{3\theta}{2}\cdot\frac{1}{dx/d\theta}$$

$$= -3/2\operatorname{cosec}^2 3\theta/2 \left.\frac{1}{2\cos\theta - 2\cos 2\theta}\right|_{\theta=\pi}$$

$$= -3/2(-1)^2\frac{1}{-2-2} = 3/8$$

Question

[JEE Mains 2022 (28 July)]

#Q. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t \in \left(0, \frac{\pi}{2}\right)$.

Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t = \frac{\pi}{4}$ is equal to:

Tan 1@

A $\frac{-2\sqrt{2}}{3}$

B $\frac{2}{3}$

C $\frac{1}{3}$

D $\frac{-2}{3}$

L' Hopital Rule

used for finding limit of indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$. If f, g are two derivable fns in deleted nbd of $x=a$ & $g'(x) \neq 0$ in this nbd.

also $\lim_{x \to a} f(x) = 0 = \lim_{x \to a} g(x)$

OR

$\lim_{x \to a} f(x) = \infty$ & $\lim_{x \to a} g(x) = \infty$.

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists finitely or is infinite.

$$\lim_{x\to 0}\frac{\sin x}{x}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\cos x}{1}=1$$

,

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{\frac{1}{1+x}}{1}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\sin x}{2x}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{\cos x}{2}=\frac{1}{2}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f''(x)}{g''(x)}=\cdots$$

$\frac{f(x)}{g(x)}$ → $0/0, \frac{\infty}{\infty}$; $\frac{f'(x)}{g'(x)}$ → $0/0, \frac{\infty}{\infty}$; $\frac{f''(x)}{g''(x)}$ → $0/0, \infty/\infty$

till indeterminate form is resolved.

Topic : L' Hôpital's Rule

(3) **Indeterminate forms of the type 1^{∞}, ∞^0 or 0° are reduced to forms of the type $0 \cdot \infty$ by taking logarithms**

$$[f(x)]^{\phi(x)} = e^{\phi(x) \cdot \ln f(x)}$$

1^{∞} $\downarrow \log$ $\infty \times 0$ $\downarrow$ $\frac{\infty}{\infty}$

∞^0 $\downarrow \log$ $0 \times \infty$ $\downarrow$ $\frac{\infty}{\infty}$

0° $\downarrow \log$ $0 \times \infty$ $\downarrow$ $\frac{\infty}{\infty}$

Question

#Q. Find a and b if $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} = 1$

$$\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0} \frac{(1+a\cos x)+x(-a\sin x)-b\cos x}{3x^2}$$

$$\lim_{x\to 0} \frac{-a\sin x - a\sin x - x\cdot a\cos x + b\sin x}{6x}$$

$$\frac{1}{6}\lim_{x\to 0} \frac{(-2a+b)\sin x}{x} - \frac{ax\cos x}{x} = -\frac{3a+b}{6} = 1 \text{ (given)}$$

$$b-3a=6 \quad (II)$$

for $\left(\frac{0}{0}\right)$ form

$$1+a-b=0 \quad (I)$$

$a = -5/2$

$b = -3/2$

Ans. a = - 5/2, b = - 3/2

Question

#Q. $\lim_{x\to 0} \dfrac{1+\sin x-\cos x+\ln(1-x)}{x\tan^2 x}$ $\left(\frac{0}{0}\right)$

$\frac{d}{dx}(1-x)^{-1} = -1\cdot(1-x)^{-2}(-1) = (1-x)^{-2}$

A. $-1/2$

B. $-1/3$

C. $1/6$

D. DNE

$$\lim_{x\to 0} \frac{1+\sin x-\cos x+\ln(1-x)}{x^3}\cdot\left(\frac{x^2}{\tan^2 x}\right) \quad (0/0)$$

$$\downarrow$$

$$\lim_{x\to 0} \frac{\cos x+\sin x-\frac{1}{1-x}}{3x^2}\cdot(\quad) \quad (0/0)$$

$$\lim_{x\to 0} \frac{-\sin x+\cos x-\frac{1}{(1-x)^2}}{6x}(\quad) \quad (0/0)$$

$$\lim_{x\to 0} \frac{-\cos x-\sin x-\frac{2}{(1-x)^3}}{6}(\quad) = -\frac{3}{6}\cdot 1 = -1/2$$

Question

#Q. $\lim_{x\to 0} \dfrac{\log_{\sec\frac{x}{2}}(\cos x)}{\log_{\sec x}(\cos(x/2))}$

A 1

B 16

C 4

D 2

$$= \lim_{x\to 0} \frac{\ln\cos x \cdot \ln\sec x}{\ln\sec\frac{x}{2} \cdot \ln\left(\cos\frac{x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{-(\ln\cos x)^2}{-\left(\ln\cos\frac{x}{2}\right)^2}$$

$$L = \lim_{x\to 0}\left(\frac{\ln\cos x}{\ln\cos x/2}\right)^2$$

let $\ell = \lim_{x\to 0}\left(\frac{\ln\cos x}{\ln\cos x/2}\right)\left(\frac{0}{0}\right)$

$$= \lim_{x\to 0}\left(\frac{-\tan x}{-\frac{1}{2}\tan x/2}\right) = \lim_{x\to 0} 2\frac{\tan x}{x}\cdot\frac{x/2}{\tan x/2}\cdot 2 = 4$$

$\ln\frac{1}{\cos x} = \ln(\cos x)^{-1} = -\ln\cos x$

$\log_a^b = \dfrac{\log_e^b}{\log_e^a}$

#Q. If $\lim_{x\to 0} \frac{1-\cos x\cdot\cos 2x\cdot\cos 3x\ldots\cos nx}{x^2}$ has the value equal to 253, find the value of n (where $n \in N$)

$$\lim_{x\to 0} \frac{1-\cos x\cdot\cos 2x\cdot\cos 3x - - \cos nx}{x^2} = 253.$$

$$253 = \lim_{x\to 0} \frac{0+\sin x(\quad)+2\sin 2x(\quad)+3\sin 3x(\quad)+\cdots+n\sin nx(\quad)}{2x}$$

$$253 = \frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x}\cdot(\quad)+2^2\cdot\frac{\sin 2x}{2x}(\quad)+3^2\frac{\sin 3x}{3x}(\quad)+\cdots+n^2\frac{\sin nx}{nx}(\quad)$$

$$= \frac{1}{2}(1+2^2+3^2+\cdots+n^2) = \frac{n(n+1)(2n+1)}{12}$$

$$n(n+1)(2n+1) = 12\times 253 = 11\times 23\times 12 = 11\times 12(2\cdot 11+1) \Rightarrow n=11$$

Question

#Q. $\lim\limits_{x\to 0} \dfrac{1-\cos x\cdot\cos 2x\cdot\cos 3x}{x^2}$

Tan 15

Ans. 7

Question

#Q. $\lim\limits_{x\to 0} \dfrac{1-\cos(ax)\cdot\cos(bx)\cdot\cos(cx)\cdot\cos(dx)}{x^2}$

Tan 1©

where, a = 1, b = 2, c = 3, d = 4

Ans. $\dfrac{a^2+b^2+c^2+d^2}{2}$

Question

#Q. $\lim_{k\to 0} \frac{\ln(\ln(1+k^4))}{\ln(\ln(1+k^2))} \left(\frac{\infty}{\infty}\right)$

$$= \lim_{k\to 0} \frac{\frac{1}{\ln(1+k^4)} \cdot \frac{4k^3}{1+k^4}}{\frac{1}{\ln(1+k^2)} \cdot \frac{1}{1+k^2} 2k}$$

$$= \lim_{k\to 0} \frac{\ln(1+k^2)}{\ln(1+k^4)} \cdot \frac{1+k^2}{1+k^4} \cdot 2k^2 \cdot$$

$$= \lim_{k\to 0} \frac{\frac{\ln(1+k^2)}{k^2}}{\frac{\ln(1+k^4)}{k^4}} \cdot 2 \cdot \left(\frac{1+k^2}{1+k^4}\right) = 2.$$

Ans. 2

Question

#Q. $\lim_{x \to 0} \frac{(\cos a x)^{1/m} - (\cos b x)^{1/n}}{x^2}$

— Tan 1@

Ans. $\frac{b^2 m - a^2 n}{2mn}$

Question

#Q. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim\limits_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$. if $f(x) = 1$, then x is equal to : [JEE Mains 2020]

(codomain)

(x constant)

A 2e

B 1/2e

C 1/e

D e

$$\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} \left(\frac{0}{0}\right)$$

$$\lim_{t \to x} \frac{2t f^2(x) - x^2 \cdot 2 f(t) \cdot f'(t)}{1}$$

$$0 = 2x \cdot f^2(x) - x^2 \cdot 2 f(x) \cdot f'(x)$$

$$f(x) = x f'(x)$$

$$\frac{f'(x)}{f(x)} = 1/x \Rightarrow \int \frac{f'(x)}{f(x)} dx = \int \frac{1}{x} dx \Rightarrow \ln f(x) = \ln x + C.$$

$f(1) = e$ (given)

$$\ln f(1) = \ln 1 + C$$

$$C = \ln e = 1$$

$$\Rightarrow \ln f(x) = \ln x + 1$$

$$\ln \frac{f(x)}{x} = 1$$

$$\frac{f(x)}{x} = e.$$

$$f(x) = ex.$$

$$f(x) = 1 = ex$$

$$x = 1/e.$$

Question

#Q. $\lim\limits_{x\to 1} \dfrac{x^x - x}{x - 1 - \ln x}$ $\left(\frac{0}{0}\right)$

A 0

B 1

C 2

D 3

$$\lim_{x\to 1} \frac{x^x(1+\ln x)-1}{1-\frac{1}{x}}$$

$$\lim_{x\to 1} \frac{x^x(1+\ln x)^2 + x^x \cdot 1/x}{1/x^2}$$

$= 1\cdot(1+0)+1 = 2$.

Question

#Q. $\lim\limits_{x\to 0^+} (\text{cosec } x)^{1/\ln x} = \dfrac{1}{e}\ (\infty^0)$

$$\ell = \lim_{x\to 0^+} (\text{cosec}\, x)^{\frac{1}{\ln x}}$$

$$\ln \ell = \lim_{x\to 0^+} \frac{1}{\ln x} \cdot \ln(\text{cosec}\, x) \cdot$$

$$= \lim_{x\to 0^+} \frac{\ln \text{cosec}\, x}{\ln x} \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\text{cosec}\, x}(-\text{cosec}\, x \cdot \cot x)}{}$$

$$= \lim_{x\to 0^+} \frac{-x}{\tan x} = -1 \xrightarrow{1/x} \ln \ell = -1$$

$$\ell = e^{-1} = 1/e.$$

Question

#Q. $\lim_{x \to \pi/2} (\sec x)^{\cot x} (\infty^0)$

$\tan(2)(@)$

Ans. 1

Question

#Q. $\lim_{x\to 0}(\sin x)^x \ (0^0)$

$$\ln l = \lim_{x\to 0} x \ln \sin x \quad (0\times\infty)$$

$$= \lim_{x\to 0} \frac{\ln \sin x}{1/x} \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0} \frac{\cot x}{-1/x^2}$$

$$= \lim_{x\to 0} \frac{-x^2}{\tan x} = 0.$$

$l = 1$

Ans. 1

Question

[JEE 2006]

#Q. For $x > 0$, $\lim_{x \to 0} \left((\sin x)^{1/x} + (1/x)^{\sin x} \right)$ is :

$\to 0$; $(\sin x)^{1/x} \to 0^{\infty}$; $(1/x)^{\sin x} \to \infty^{0}$

(Not indeterminate form)

$= 0 + \ell = 0 + 1 = 1$

Let $\ell = \lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x}$

A 0

B -1

C 1

D 2

$\ell = \lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x}$

$\ln \ell = \lim_{x \to 0} -\sin x \ln x$.

$= \lim_{x \to 0} \frac{-\ln x}{\operatorname{cosec} x} \left(\frac{\infty}{\infty}\right) = \lim_{x \to 0} \frac{-1/x}{-\operatorname{cosec} x \cdot \cot x}$

$= \lim_{x \to 0} \frac{\sin^2 x}{\cos x \cdot x} = 0 \cdot \Rightarrow \ell = 1$

Question

#Q. $\lim_{x \to 0} (\tan x)^{\sin x} (0^0)$

TqH

Ans. 1

Question

#Q. $\lim_{x \to 0^+} (\cot x)^{1/\ln x} (\infty^0)$

Tah

Ans. 1/e

Question

#Q. $\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$ (∞^0)

Tan

Ans. 1

Question

#Q. $\lim_{x \to 4} \frac{\sqrt{2x+1} + \sqrt{x-3} - 4}{\sqrt{(3x+4)} + \sqrt{5x+5-9}} \left(\frac{0}{0}\right)$

Taw

Ans. 20/21

Question

#Q. $\lim_{x \to 1} (1 - x^2)^{1/\ln(1/x)}$ (0^0)

Tan

Ans. e

Question

#Q. $\lim_{x \to 0^-} (1 - 2^x)^{\sin x}$ (0^0)

Ans. 1

Question

#Q. $\lim_{x \to \frac{\pi}{2}} (4 - 2\ln(\cos\theta))\sqrt{\cos\theta}$

Tah

Ans. 0

Question

#Q. If $f(x) = \begin{cases} e^x + b; & x \le 2 \\ kx^3; & x > 2 \end{cases}$ be a twice differentiable function with inverse function g, find the value of $\left|36\ k\ b\ g''\left(\frac{2e^2}{3}\right)\right|$.

f is twice derivable ⟨ f is continuous, f is derivable
f′ is continuous, f′ is derivable.

$$f(x) = \begin{cases} e^x + b & x \le 2 \\ kx^3 & x > 2. \end{cases}$$

At $x = 2$

RHL $= 8k$ RHD $= 3x^2k\big|_{x=2} = 12k$

LHL $= e^2 + b$ LHD $= e^x\big|_{x=2} = e^2$

$\Rightarrow 8k = e^2 + b$

$\frac{2e^2}{3} = e^2 + b \Rightarrow b = -e^2/3$

$k = e^2/12$.

$$f(x)=\begin{cases} e^x-\frac{e^2}{3} & x\le 2 \\ \frac{e^2}{12}x^3 & x>2 \end{cases} \Rightarrow f'(x)=\begin{cases} e^x & x\le 2 \\ \frac{e^2}{4}x^2 & x>2 \end{cases}$$

$$\Downarrow$$

$$f''(x)=\begin{cases} e^x & x\le 2 \\ \frac{e^2}{2}\cdot x & x>2 \end{cases}$$

$$f(2)=e^2-\frac{e^2}{3}=\frac{2e^2}{3}$$

$$\Rightarrow g'(2e^2/3)=\frac{1}{f'(2)}=\frac{1}{e^2}$$

$$g''(2e^2/3)=-\frac{f''(2)}{(f'(2))^3}=\frac{-(e^2)}{e^6}=-\frac{1}{e^4}$$

$$\left|36\,g'(2e^2/3)\,g''(2e^2/3)\right|=\left|36\cdot\frac{e^2}{12}\cdot\frac{-e^2}{3}\cdot\frac{1}{e^4}\right|=\underline{1}$$

Topic : Equal Roots (or Multiple roots or Repeated Roots)

Definition

A root α is said to be a multiple root of an equation f (x) = 0 if α occurs more than once among the list of roots of f (x) = 0. If α is repeated r times, then α is said to be a root of multiplicity r and f (x) = $(x-\alpha)^r g(x)$, where $g(\alpha) \neq 0$.

polynomial

$g(x)$ clearly $g(1) \neq 0$

$$f(x) = (x-1)^3 (x-2)^2 (x-3) = 0$$

roots = 1, 1, 1, 2, 2, 3

multiple roots

multiplicity = 2

multiplicity = 3

$$f(x) = (x-2)^2 (x-1)^3 (x-3)$$

$g(x)$ & clearly $g(2) \neq 0$.

Let f be a polynomial in s.t $(x-\alpha)$ is repeated m times in $f(x)$ i.e $f(x)=(x-\alpha)^m g(x)$ where $g(\alpha)\neq 0$

$$f'(x) = (x-\alpha)^m g'(x) + m(x-\alpha)^{m-1} g(x)$$

$$= (x-\alpha)^{m-1}\big(\underbrace{(x-\alpha)\cdot g'(x) + m g(x)}_{h(x)} \big)$$

$$f'(x) = (x-\alpha)^{m-1} h(x)$$

Now $h(\alpha) = 0 + m g(\alpha) \neq 0$

$\Downarrow$

Clearly $(x-\alpha)$ is repeated $(m-1)$ times in $f'(x)$

If $x=\alpha$ is a repeated root of polynomial $f(x)$ with multiplicity m then

	$(x-\alpha)$ has multiplicity	
$f(x)$	m	
$f'(x)$	$m-1$	
$f''(x)$	$m-2$	
	⋮	
	0	

If $(x-\alpha)$ is repeated m times in $f'(x) \Rightarrow (x-\alpha)$ will be repeated $m+1$ times in $f(x)$

Ex: $f'(x) = 3(x-2)^2 \longrightarrow x=2$ has multiplicity $=2$

$f(x) = (x-2)^3 + 5 \longrightarrow x=2$ is not a repeated root.

$f(x) = (x-\alpha)^{m+1} g(x)$

$f'(x) = (x-\alpha)^m h(x)$

If $(x-\alpha)$ is repeated m times in H.C.F of $f(x)$ & $f'(x)$ then it shall be repeated $(m+1)$ times in $f(x)$

Topic : Equal Roots (or Multiple roots or Repeated Roots)

Theorem :

If α is a root of multiplicity r of the equation f (x) = 0 then α is a root of multiplicity (r – 1) of the equation f'(x) = 0, where f'(x) is the derivative of f(x)

#Q. If $f(x) = (x-1)^4(x-2)^3(x-3)^2(x-4)$, then the value of $(f'''(1) + f''(2) + f'(3) + f'(4))$ equals

A 638

B $3(6^3)$

C 50

D 0

	$x=1$	$x=2$	$x=3$
$f(x)$	4	3	2
$f'(x)$	3	2	1
$f''(x)$	2	1	0
$f'''(x)$	1	0	0

$f'''(1) = 0$

$f''(2) = 0$

$f'(3) = 0$

$f'(4) = (x-4)\cdot\frac{d}{dx}(x-1)^4 + (x-4)\cdot\frac{d}{dx}(x-2)^3 + (x-4)\cdot\frac{d}{dx}(x-3)^2 + (x-1)^4(x-2)^3(x-3)^2\frac{d}{dx}(x-4)\Big|_{x=4}$

$= 3^4 \cdot 2^3 \cdot 1^2 \cdot 1$

Question

#Q. If $x_1, x_1, x_2, x_3, \ldots\ldots x_{n-1}$ be n zero's of the polynomial $P(x) = x^n + \alpha x + \beta$, where $x_i \neq x_j \; \forall \; i \; \& \; j = 1, 2, 3, \ldots\ldots (n-1)$. Prove that the value of $Q(x) = (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \ldots\ldots (x_1 - x_{n-1})$, is equal to ${}^nC_2 x_1{}^{n-2}$.

$P(x) = x^n + \alpha x + \beta = (x-x_1)^2 (x-x_2)(x-x_3) \ldots (x-x_{n-1})$

$x_1, x_1, x_2, x_3, \ldots x_{n-1}$

$(x-x_2)(x-x_3)\ldots = Q(x)$.

$Q(x) = \dfrac{P(x)}{(x-x_1)^2}$; $x = x_1$ multiplicity $= 2$; $x = x_1$ multiplicity $= 1$

$$\lim_{x \to x_1} Q(x) = \lim_{x \to x_1} \frac{P(x)}{(x-x_1)^2} \left(\frac{0}{0}\right) = \lim_{x \to x_1} \frac{P'(x)}{2(x-x_1)} \left(\frac{0}{0}\right) = \lim_{x \to x_1} \frac{P''(x)}{2 \cdot 1} = \frac{P''(x_1)}{2}$$

$$Q(x_1) = \frac{P''(x_1)}{2} = \frac{P''(x)}{2}\Big|_{x=x_1} = \frac{(nx^{n-1}+\alpha)'}{2}\Big|_{x=x_1} = \frac{n(n-1)x^{n-2}+0}{2}\Big|_{x=x_1}$$

$$Q(x_1) = \frac{n(n-1)}{2}x_1^{n-2} = {}^nC_2 x_1^{n-2}.$$

Question

#Q. Let $x_1, x_1, x_1, x_2, x_3, x_4, \ldots\ldots x_8$ be 10 real zeroes of the polynomial $P(x) = x^{10} + ax^2 + bx + c$ where $a, b, c \in R$. If the value of $Q(x_1) = \frac{p}{q}$ where p and q are coprime to each other, $Q(x) = (x - x_2)(x - x_3) \ldots\ldots (x - x_8)$ and $x_1 = \frac{1}{2}$, then find the value of $p + q$.

Ans ①

Question

#Q. If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x)$, $B(x)$, $C(x)$ be the polynomials of degree 3, 4 and 5 respectively, then show $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where dash denotes the derivative.

Since $x=\alpha$ is a repeated root of $f(x)$

$f(x) = a(x-\alpha)^2$

degree = 3, degree = 4, degree = 5

$$\Delta(x) = \begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

$C(\alpha)$ → constants, $C'(\alpha)$ → constants.

$$\Delta'(x) = \begin{vmatrix} A'(x) & B'(x) & C'(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix} + 0 + C$$

$$\Delta'(\alpha)=\begin{vmatrix} A'(\alpha) & B'(\alpha) & C'(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}=0$$

because $R_1 = R_3$

$\Downarrow$

$(x-\alpha)$ is a factor of $\Delta'(x)$

Also $\Delta(\alpha)=\begin{vmatrix} A(\alpha) & B(\alpha) & C(\alpha) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}=0 \Rightarrow (x-\alpha)$ is also a factor of $\Delta(x)$

$\Downarrow$

$(x-\alpha)$ occurs atleast once in HCF of $\Delta(x)$ & $\Delta'(x)$

$\Downarrow$

$(x-\alpha)$ occurs atleast twice in $\Delta(x)$

$\Rightarrow \Delta(x)$ is divisible by $f(x)$

$\Delta(x) = (x-\alpha)^2 g(x)$

$f(x) = a(x-\alpha)^2$

$$a(x^2+1) \overline{)\; x^2+1 \;(\; \frac{1}{a}}$$

$$\underline{x^2+1}$$

$$0$$

$\sin 2x = 2\sin x \cos x$

↓ Diff both sides

$\cancel{2}\cos 2x = \cancel{2}(\cos^2 x - \sin^2 x)$

$\cos 2x = \cos^2 x - \sin^2 x$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_n x^n$

↓ diff both sides w.r.t x

$n(1+x)^{n-1} = {}^nC_1 + 2\cdot{}^nC_2 x + 3\cdot{}^nC_3 x^2 + \cdots + n\cdot{}^nC_n x^{n-1}$

$\underline{x=1}$ $\quad {}^nC_1 + 2\cdot{}^nC_2 + 3\cdot{}^nC_3 + \cdots + n\cdot{}^nC_n = n\cdot 2^{n-1}$

Topic : Deduction of new identities by Differentiating a given Identity

If $\cos\frac{x}{2}\cdot\cos\frac{x}{2^2}\cdot\cos\frac{x}{2^3}\cdot\ldots\ldots\cdot\cos\frac{x}{2^n}=\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}$, then prove that

$$\sum_{r=1}^{n}\frac{1}{2^r}\tan\frac{x}{2^r}=\frac{1}{2^n}\cot\frac{x}{2^n}-\cot x$$

↓ Taking log to base e on both sides.

$$\ln\sin x-\ln\sin\frac{x}{2^n}-n\ln 2=\ln\cos\frac{x}{2}+\ln\cos\frac{x}{2^2}+\ln\cos\frac{x}{2^3}+\ldots+\ln\cos\frac{x}{2^n}$$

↓ Diff both sides w.r.t x

$$\cot x-\frac{\cot(x/2^n)}{2^n}-0=-\frac{1}{2}\tan\frac{x}{2}-\frac{1}{2^2}\tan x/2-\frac{1}{2^3}\tan(x/2^3)-\ldots\ldots-\frac{1}{2^n}\tan\frac{x}{2^n}$$

$$+\left(\frac{\cot(x/2^n)}{2^n}-\cot x\right)=+\sum_{r=1}^{n}\frac{1}{2^r}\tan(x/2^r)$$

Question

#Q. Prove that if $|a_1 \sin x + a_2 \sin 2x + ... + a_n \sin nx| \le |\sin x|$ for $x \in R$, then $|a_1 + 2a_2 + 3a_3 + ... + na_n| \le 1$.

Diff both sides $|a_1 \cos x + 2a_2 \cos 2x + - - + n \cdot a_n \cos nx| \le |\cos x|$

put $x=0$ $|a_1 + 2a_2 + - - + na_n| \le 1$

Gadho/ Gadhiyoo aisaa naa Karo!!

We can not differentiate an inequality

let $P(x) = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + - - + a_n \sin nx$ clearly $P(0)=0$

$|P(x)| \le |\sin x|$ (given)

$$\left|\frac{P(x)-P(0)}{x}\right| \le \left|\frac{\sin x}{x}\right|$$

$$\left|\frac{P(x)-P(0)}{x-0}\right| \le \left|\frac{\sin x}{x}\right|$$

$$\lim_{x\to 0}\left|\frac{P(x)-P(0)}{x-0}\right| \le \lim_{x\to 0}\left|\frac{\sin x}{x}\right|$$

$$|P'(0)| \le 1$$

$$\left|(a_1\cos x + 2a_2\cos 2x + 3a_3\cos 3x + \cdots + na_n\cos nx)\Big|_{x=0}\right| \le 1$$

$$|a_1 + 2a_2 + 3a_3 + \cdots + na_n| \le 1$$

(H.P)

Solution to TAH

Question

#Q. Let $f: S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g: S \to R$ be defined as $g(x) = \log_4 f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to: [JEE Mains 2021 (March)]

A $\frac{205}{144}$

B $\frac{197}{144}$

C $\frac{187}{144}$

D 1

Tah-29

Date
Page No.

Q: $f: S \to S$ where $S = (0, \infty)$ be twice diff. funct such that $f(x+1) = x f(x)$. If $g: S \to R$ be defined as $g(x) = \log_e f(x)$ then value of $|g''(5) - g''(1)| = ?$

a) $\frac{205}{144}$ b) $\frac{197}{144}$ c) $\frac{127}{144}$ d) 1

$f(x+1) = x f(x)$ $\quad g(x) = \log_e f(x)$

$g(x) = \ln f(x)$ $\quad \boxed{g(x) = \ln f(x)} \to (1)$

$g(x+1) = \ln f(x+1)$

$g(x+1) = \ln x f(x)$

$g(x+1) = \ln x + \ln f(x)$

$g'(x+1) = g'(x) + \frac{1}{x}$ $\quad \{$using (1)$\}$

$\boxed{g''(x+1) = g''(x) - \frac{1}{x^2}}$

$x=1 \Rightarrow g''(2) - g''(1) = -\frac{1}{1^2}$

$x=2 \Rightarrow g''(3) - g''(2) = -\frac{1}{4}$

$x=3 \Rightarrow g''(4) - g''(3) = -\frac{1}{9}$

$x=4 \Rightarrow g''(5) - g''(4) = -\frac{1}{16}$

$g''(5) - g''(1) = -\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right) \Rightarrow -\frac{205}{144}$

$|g''(5) - g''(1)| = \frac{205}{144}$ Ans

Rtesh Yadav....

TAH (2) $f(x+1) = x f(x)$ $\quad g(x) = \log f(x)$

Then value of $|g''(5) - g''(1)|$

$g(x+1) = \ln f(x+1) = \ln(x f(x))$

$= \ln x + \ln f(x)$

$g(x+1) = \ln x + g(x)$

$g(x+1) - g(x) = \ln x$

$g''(x+1) - g''(x) = -\frac{1}{x^2}$

$x=1$ $\quad g''(2) - g''(1) = -1$

$x=2$ $\quad g''(3) - g''(2) = -\frac{1}{4}$

$x=3$ $\quad g''(4) - g''(3) = -\frac{1}{9}$

$g''(5) - g''(4) = -\frac{1}{16}$

TEDDY (UP)

$g''(5) - g''(1) = -1 - \frac{1}{4} - \frac{1}{9} - \frac{1}{16}$

$= -\frac{1}{9} + \frac{-4-16-1}{16}$

$= \frac{-189 - 16}{16 \times 9} = -\frac{205}{144}$

$|g''(5) - g''(1)| = \frac{205}{144}$

Experiment No.

Page No.

TAH1A

TAH 1 (a)

Q- $g(x) = \log_4 f(x)$ $f(x+1) = x \cdot f(x)$

$|g''(5) - g''(1)| = ?$

Sol^n. Now, $g(x+1) = \ln f(x+1)$

$g(x+1) = \ln(x \cdot f(x))$

$g(x+1) = \ln x + \ln f(x)$

$g(x+1) = \ln x + g(x)$

$g(x+1) - g(x) = \ln x$

Differentiating w.r.t x.

$g'(x+1) - g'(x) = \frac{-1}{x}$

Double Differentiating w.r.t x

$g''(x+1) - g''(x) = \frac{-1}{x^2}$

Putting values of $x = 1, 2, 3, 4$ in above eqn.

$g''(2) - g''(1) = -1$

$g''(3) - g''(2) = \frac{-1}{4}$

$g''(4) - g''(3) = \frac{-1}{9}$

$g''(5) - g''(4) = \frac{-1}{16}$

$g''(5) - g''(1) = \frac{-205}{144}$

$|g''(5) - g''(1)| = \frac{205}{144}$

Ans- (a) $\frac{205}{144}$

ARYAN MISHRA
BALLIA UP

Sol. $g(x) = \log_4 f(x)$, $f(x+1) = x \cdot f(x)$.

$g(x+1) = \ln f(x+1)$

$g(x+1) = \ln f(x) + \ln(x)$

$g(x+1) = g(x) + \ln(x)$

$g'(x+1) = g'(x) + \frac{1}{x}$

$g''(x+1) = g''(x) - \frac{1}{x^2}$

$x = 1$

$g''(2) - g''(1) = -1$

$g''(3) - g''(2) = -\frac{1}{4}$

$g''(4) - g''(3) = -\frac{1}{9}$

$g''(5) - g''(4) = -\frac{1}{25}$

$g''(5) - g''(1) = -\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right) = -\left(\frac{144+36+16+9}{144}\right)$

$|g''(5) - g''(1)| = \frac{205}{144}$

TAH 1 A

Neha (Gorakhpur)

Question

#Q. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f(2)$ equals : [JEE Mains 2019]

A -4

B 30

C -2

D 8

TAH-1b

Q) Let $f: R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$, Then $f(2)$ equals -

Jee Mains 2019

ans - let $f(x) = x^3 + ax^2 + bx + c$ where $a = f'(1)$ & $c = f'''(3)$

$b = f''(2)$

$\Rightarrow f'(x) = 3x^2 + 2ax + b \Rightarrow f'(1) = 3 + 2a + b$

$\Rightarrow f''(x) = 6x + 2a \Rightarrow f''(2) = 12 + 2a$

$\Rightarrow f'''(x) = 6 \Rightarrow f'''(3) = 6$

from here

$a = 3 + 2a + b$
$b = 12 + 2a$
$c = 6$

on solving

$3 + a + b = 0$
$b - 12 = 2a$

$\Rightarrow 3 + \frac{b-12}{2} + b = 0 \Rightarrow 6 + b - 12 + 2b = 0$

$\Rightarrow 3b = 6 \Rightarrow \boxed{b = 2}$

$\Rightarrow 2 = 12 + 2a \Rightarrow a = \frac{-10}{2} \quad \boxed{a = -5}$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

Now - $f(2) = 8 - 20 + 4 + 6 \Rightarrow -12 + 10 \Rightarrow (-2)$ option (C) ans

Vinamra Kapoor

TAH 1(B)

Prabhu Tiwari
Bareilly (UP)

TAH - 1(B)

Que. Let $f: R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f(2)$ is:

Solution:

$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$f(x) = x^3 + ax^2 + bx + c$ ← $a = f'(1)$, $b = f''(2)$, $c = f'''(3)$

$\therefore f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$f'''(x) = 6 \longrightarrow (f'''(3) = 6)$

Now

$f'(1) = 3 + 2a + b \Rightarrow a = 3 + 2a + b \Rightarrow a + b + 3 = 0 \quad \text{---} (1)$

$f''(2) = 12 + 2a \Rightarrow b = 12 + 2a \Rightarrow 2a - b + 12 = 0 \quad \text{---} (2)$

Now from eq^n (1)+(2)

$3a = -15 \Rightarrow (a = -5)$

$\therefore (b = 2)$

$\therefore f(x) = x^3 + ax^2 + bx + c$

$f(x) = x^3 - 5x^2 + 2x + 6$

$\therefore f(2) = 8 - 20 + 4 + 6 \Rightarrow (f(2) = -2)$ Ans calculate

TAH-1B

$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ $x \in R$ $\boxed{f(2) = ?}$

let consider a cubic eqn similar to $f(x)$

$g(x) = x^3 + ax^2 + bx + c.$

$g'(x) = 3x^2 + 2ax + b$ $\qquad g''(x) = 6x + 2a.$

$g'''(x) = 6$

SOHAM DAS

$g'''(3) = c = f'''(3) = 6.$ $\boxed{c = 6}$

$g''(2) = b = f''(2) = 12 + 2a \Rightarrow \boxed{2a - b = -12} \quad -(I)$

$g'(1) = a = f''(1) = 3 + 2a + b \Rightarrow \boxed{a + b = -3} \quad -(II)$

From (I) and (II) we get

$a = -5 \quad b = 2$

$f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

$\boxed{f(2) = -2}$ (Ans)

Question

[JEE Mains 2020 (5 Sept)]

#Q. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\frac{1}{2}$ is :

A $\frac{2\sqrt{3}}{3}$

B $\frac{2\sqrt{3}}{5}$

C $\frac{\sqrt{3}}{10}$

D $\frac{\sqrt{3}}{12}$

TAH-2

Q) The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=\frac{1}{2}$ is

Ans - Put $\boxed{x=\tan\theta}$, then

$\Rightarrow u=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$\Rightarrow u=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$\Rightarrow u=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) \Rightarrow \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{\theta}{2}\right) \Rightarrow u=\frac{\theta}{2} \Rightarrow \boxed{u=\frac{\tan^{-1}x}{2}}$

$\Rightarrow \frac{du}{dx}=\frac{1}{2(1+x^2)}$

$v=\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ Put $\boxed{x=\sin\theta}$

$\Rightarrow \theta=\sin^{-1}x$

$\Rightarrow v=\tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos2\theta}\right)$

$\Rightarrow v=\tan^{-1}\left(\frac{\sin2\theta}{\cos2\theta}\right) \Rightarrow v=\tan^{-1}(\tan2\theta)$

$\Rightarrow v=2\theta$

$\Rightarrow \boxed{v=2\sin^{-1}x}$

$\Rightarrow \frac{dv}{dx}=\frac{2}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{du/dx}{dv/dx}=\frac{\frac{1}{2(1+x^2)}}{\frac{2}{\sqrt{1-x^2}}}=\frac{dy}{dx}=\frac{\sqrt{1-x^2}}{4(1+x^2)}$ at $x=\frac{1}{2}$

$\Rightarrow \frac{dy}{dx}=\frac{\frac{\sqrt{3}}{2}}{4\times\frac{5}{4}}$

$\Rightarrow \boxed{\frac{dy}{dx}=\frac{\sqrt{3}}{10}}$ option (C)

Viranva Kapoor

Question

#Q. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$. Find $\frac{dy}{dx}$.

A $\frac{\cos x}{2y - 1}$

B $\frac{\cos x}{\sqrt{1 + 4\sin x}}$

C $\frac{y\cos x}{y^2 + \sin x}$

D $\frac{y\cos x}{y^2 - \sin x}$

TAH 3

TAH-3

Que: If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\cdots}}}$ find $\frac{dy}{dx}$

Solution:

$y=\sqrt{\sin x+y} \Rightarrow y^2=\sin x+y$

diff wrt x

$\Rightarrow 2y\frac{dy}{dx}=\cos x+\frac{dy}{dx}$

$(2y-1)\frac{dy}{dx}=\cos x$

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$\frac{dy}{dx}=\frac{\cos x}{2y-1}$ → Agra

Ans...

Now

$y^2=\sin x+y$

$y(y-1)=\sin x$

$y=\sin x+1$

$\frac{dy}{dx}=\frac{y\cos x-\sin x\frac{dy}{dx}}{y^2}$

$\frac{dy}{dx}\left(1+\frac{\sin x}{y^2}\right)=\frac{\cos x}{y}$

$\frac{dy}{dx}=\frac{y\cos x}{y^2+\sin x}$ → Calcutta.

Ans....

$y^2=\sin x+y$

$y^2-y-\sin x=0$

$y=\frac{1+\sqrt{1+4\sin x}}{2}$

$y\neq -ve$

$\Rightarrow 2y=1+\sqrt{1+4\sin x}$

$2\frac{dy}{dx}=\frac{1}{2\sqrt{1+4\sin x}}\cdot 4\cos x$

$\frac{dy}{dx}=\frac{\cos x}{\sqrt{1+4\sin x}}$ → IIT Bombay

Ans...

1. $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\cdots}}}$

$y=\sqrt{\sin x+y}$

$y^2=\sin x+y$

$y^2-y=\sin x$

$2y\frac{dy}{dx}=\cos x\frac{dy}{dx}$

$(2y-1)\frac{dy}{dx}=\cos x$

$\frac{dy}{dx}=\frac{\cos x}{2y-1}$ option 4

$y^2-y-\sin x$

$y=\frac{1+\sqrt{1+4\sin x}}{2}$ y cannot be -ve

$2\frac{dy}{dx}=\frac{1}{2\sqrt{1+4\sin x}}\cdot 4\cos x$

$\frac{dy}{dx}=\frac{\cos x}{\sqrt{1+4\sin x}}$ option 2

$y(y-1)=\sin x$

$(y-1)=\frac{\sin x}{y}$

$\frac{dy}{dx}=\frac{y\cos x-\sin x\frac{dy}{dx}}{y^2}\Rightarrow\frac{dy}{dx}=\frac{y\cos x}{y^2}-\frac{\sin x\frac{dy}{dx}}{y^2}$

$\frac{dy}{dx}+\frac{\sin x\frac{dy}{dx}}{y^2}=\frac{y\cos x}{y^2}$

$\frac{dy}{dx}\left(1+\frac{\sin x}{y^2}\right)=\frac{\cos x}{y}\Rightarrow\frac{\cos x}{y}\times\frac{1}{1+\frac{\sin x}{y^2}}$

$=\frac{\cos x}{y}\times\frac{y^2}{y^2+\sin x}=\frac{y\cos x}{y^2+\sin x}$

TAH 3

Neha (Gorakhpur)

Question

#Q. Let $f : (0, \infty) \to (-\infty, \infty)$ be defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$, then

A $g''(e) = \frac{1-e}{(1+e)^3}$

B $g''(e) = \frac{e-1}{(1+e)^3}$

C $g'(e) = e + 1$

D $g'(e) = \frac{1}{e+1}$

TAH 4

Sr No. _____ Date _____

TAH-4

Que Let $f:(0,\infty) \to (-\infty,\infty)$ be defined as $f(x) = e^x + \ln x$ and $g = f^{-1}$, then

Solution: $f(x) = e^x + \ln x \quad \therefore g = f^{-1}$

at $x=1$ $f(1) = e$

$\therefore f'(x) = e^x + \frac{1}{x} \rightsquigarrow f'(1) = e+1$

& $f''(x) = e^x - \frac{1}{x^2} \rightsquigarrow f''(1) = e-1$ ...

$\therefore g'(x) = \frac{1}{f'(x)} \Rightarrow g'(x) = \frac{1}{e^x + 1/x}$

$g'(x) = \frac{x}{xe^x+1}\Big|_{x=1}$

$g'(e) = \frac{1}{e+1}$ Now $g''(x) = -\frac{f''(x)}{(f(x))^3}\Big|_{x=1}$

$g''(e) = -\frac{(e-1)}{(1+e)^3}$

Prabhu Tiwari
Bareilly (UP)

$g''(e) = \frac{1-e}{(1+e)^3}$ ⇝ Agra

Ans...

$g'(e) = \frac{1}{e+1}$ ⇝ IIT Delhi

Ans...

Question

#Q. If $f(x) = \begin{cases} e^x + b; & x \leq 2 \\ kx^3; & x > 2 \end{cases}$ be a twice differentiable function with inverse function g, find the value of $\left|36\ k\ b\ g''\left(\frac{2e^2}{3}\right)\right|$.

TAH 5

Sr No.

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TAH-5

Que If $f(x) = \begin{cases} e^x + b & x \le 2 \\ kx^3 & x > 2 \end{cases}$ be a twice derivable fn.
with inverse fn g. find the value of $|36kbg''(\frac{2e^2}{3})|$

Solution: $\because f(x)$ is twice derivable $\Rightarrow f'(x)$ is derivable & continuous
$\hookrightarrow f(x)$ is also derivable & continuous.

Now
at $x=2$ LHL: $\lim_{x\to 2^-} e^x + b = e^2 + b$
$e^2 + b = 8k$ ... ①

RHL: $\lim_{x\to 2^+} kx^3 = 8k$

$\because f'(x) = \begin{cases} e^2, & x<2 \\ 3kx^2, & x>2 \end{cases}$ $\Rightarrow$ L.H.D : e^2
R.H.D: $12k$
$12k = e^2$
$\Rightarrow k = \frac{e^2}{12}$ ... ②

from ① & ②
$e^2 + b = \frac{e^2}{12} \times 8$
$\Rightarrow b = -\frac{e^2}{3}$

Now we know $g''(\beta) = \frac{-f''(\alpha)}{(f'(\alpha))^3}$
here $\beta = \frac{2e^2}{3} \Rightarrow \alpha = 2$

$\Rightarrow g''(\frac{2e^2}{3}) = \frac{-f''(2)}{(f'(2))^3}$

Now $f'(x) = \begin{cases} e^x & x<2 \\ \frac{e^4}{4}x^2, & x>2 \end{cases}$

$f''(x) = \begin{cases} e^x & x<2 \\ \frac{e^4}{2}x, & x>2 \end{cases}$

$\Rightarrow g''(\frac{2e^2}{3}) = \frac{-e^2}{(e^2)^3} \Rightarrow e^{-4}$

So if we have, $k = \frac{e^2}{12}$, $b = -\frac{e^2}{3}$, $g''(\frac{2e^2}{3}) = e^{-4}$

$\therefore |36kbg''(\frac{2e^2}{3})| = |36 \times \frac{e^2}{12} \times \frac{-e^2}{3} \times e^{-4}|$

$|36kbg''(\frac{2e^2}{3})| = 1$

Ans ...

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TAH 5
Remain



THANK YOU